Direct Excitation of High-Amplitude Chirped Bucket-BGK Modes

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Abstract. For the first time, high amplitude ($\Delta n/n \approx 40\%$), high Q (up to 100,000) BGK modes have been controllably excited in a plasma. The modes are created by sweeping an excitation voltage downwards in frequency, thereby dragging a phase space “bucket” of low density into the bulk of the plasma velocity distribution. The modes have no linear limit, and differ markedly from plasma waves and Trivelpiece-Gould modes.

Plasma waves [Trivelpiece-Gould (TG) waves in finite geometry [1]] are easy to generate and ubiquitous in nature. However, plasma waves are Landau damped, often quickly. At first glance this damping seems unavoidable, so it was quite surprising when Bernstein, Greene and Kruskal (BGK) predicted that there exists a broad class of waves that do not damp. These BGK modes [2] are undamped because the distribution of the particles in the wave is already in the Landau relaxed form. In principle, BGK modes can have very large amplitude.

BGK modes underpin much of kinetic wave theory, but experimental verification of the existence of undamped BGK modes has proved difficult. It is easy to create transient large-amplitude waves or structures, but the waves are typically short-lived or unstable. For instance, waves created by Wharton, Malmberg and O’Neil [3] were unstable due to a sideband instability. More recent work has not been much more successful [4, 5]. Long-lasting structures can be created by continuous drives; driven double layers, which are closely related to BGK modes, have been observed in the earth’s auroral zone [6]. Danielson [7] recently reported that plasma waves eventually decay into low amplitude, but long-lasting, BGK modes.

Here we report that we can excite very high amplitude BGK modes that differ markedly from the more common TG modes. The modes are excited by an oscillating voltage applied to one end of a pure-electron plasma column confined in a standard Penning-Malmberg trap [8] [see Fig. 1(a).] The resulting density fluctuations are detected by monitoring the image charge on another cylinder, typically at the opposite end of the trap. When the plasma is cold, TG waves are observed as expected. A typical spectrum is shown in Fig. 2. But as the plasma temperature $T$ is increased, the TG waves become so heavily damped that they essentially disappear. Nonetheless, undamped waves can be excited in hot plasmas by low amplitude drives that sweep downward in frequency, from some frequency $f_s$ to a lower frequency $f_e$. Typical response curves are shown in Fig. 3. Large amplitude waves are excited for a very broad range of $f_s$ and $f_e$.

We believe that these waves are BGK modes. Table 1 highlights the differences
FIGURE 1. (a) Penning-Malmberg Trap geometry. The pure-electron plasma column is confined axially by the large negative end potentials, and radially by a 1500G magnetic field. Typical plasma densities are $\sim 10^7$ cm$^{-3}$, lengths $\sim 27$ cm, and radii $\sim 1$ cm, and the plasma is confined in cylinders with radius 1.905 cm. (b) Density fluctuations due to the BGK mode at the column end, and (c) near the column center. The positive fluctuation corresponds to an electron hole. Because the BGK mode must pass through the center twice (approaching and leaving) each time the pulse reaches the end, there are twice as many pulses in (c) as in (b). As expected for an open-ended reflection, the mode is twice as large at the end (b) as in the center (a).

FIGURE 2. Observed response from a pure 0.05V drive, for three different plasma temperatures. Also shown is the background noise from external sources. As predicted by the TG dispersion relation, $\omega^2 = a^2 k^2 \omega_p^2 (1 + a^2 k^2) + 3 v_{th}^2 k^2$, (where $\omega$ is the wave frequency, $a$ is the radial geometric constant, $k = m \pi / L$ is the wavenumber, $m$ is a positive integer, $L$ is the plasma length, $\omega_p$ is the plasma frequency and $v_{th}$ is the electron thermal velocity,) the wave dispersion relation is slightly concave down, and the frequencies increase with plasma temperature.
between these modes and TG modes in our experiments. First, TG modes only exist when the plasmas are sufficiently cold, while our BGK modes exist only when the plasmas are sufficiently hot; there is only a small overlap region. The TG modes occur at distinct frequencies; even when thermally broadened, the TG modes possess a well-defined linear limit. In contrast, the BGK modes have no linear limit. As can be seen in Fig. 3, they can be excited over a broad range of frequencies. The TG modes are typically excited by single frequency drive. The BGK modes can only be excited by a swept frequency drive. Using a swept frequency drive, Yamazawa and Michishita [9] determined that TG modes possess a hard nonlinearity: as their amplitude increases, their frequency increases slightly (< 5%). BGK modes have a soft nonlinearity: as their amplitude increases, their frequency decreases by nearly 50%. Even for cold plasmas, the Q’s (ω/γ, where γ is the damping rate) for undriven, low amplitude TG modes are never much higher than 600, and the Q’s get very low for hotter plasmas. Once excited by a swept frequency drive, the Q’s for the undriven BGK modes are remarkably high: typically around 6000, but occasionally as high as 100,000. Indeed, Q is often a poor measure of the lifetime of these modes as their amplitude does not decay exponentially. For example, the very-high-Q mode amplitudes are often nearly flat, except for small fluctuations, followed by a sudden collapse. High amplitude TG modes are unstable; Hart and Peterson [5], and Yamazawa and Michishita [9] demonstrated that they mode convert quickly. As one might expect from their high Q’s, the BGK modes are very stable even when their density fluctuations exceed 40%. Finally, low amplitude TG modes have little harmonic content, but the harmonic content of the BGK modes is rich even at low amplitude.

We identify these modes as BGK modes because of the method by which they are excited. When the drive is first applied, electrons whose end-to-end bounce frequencies are close to the drive frequency will be captured into a trapping bucket if they have the right phase relative to the drive; approximately half the resonant electrons are captured
TABLE 1.

<table>
<thead>
<tr>
<th></th>
<th>TG Modes</th>
<th>BGK Modes</th>
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<tbody>
<tr>
<td>Mode Frequencies:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hot</td>
<td>Distinct</td>
<td>Nonexistent</td>
</tr>
<tr>
<td>Hot</td>
<td>Nonexistent</td>
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</tr>
<tr>
<td>Excitation</td>
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<tr>
<td>Pure or Swept</td>
<td>Swept Only</td>
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</tr>
<tr>
<td>-50%</td>
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<tr>
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</tr>
<tr>
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<td>3</td>
<td></td>
</tr>
<tr>
<td>High Amplitude Stability</td>
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<td></td>
</tr>
<tr>
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<td>Excellent</td>
<td></td>
</tr>
<tr>
<td>Harmonic Content</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single Harmonic (low amp.)</td>
<td></td>
<td>Many Harmonics (all amp.)</td>
</tr>
</tbody>
</table>

(see Fig. 4). The untrapped electrons are perturbed, and may form a TG mode, but this TG mode damps out quickly because of the high plasma temperature.

As the drive frequency is swept downward to frequency $f_r$, the resonant electron velocity $\bar{v} = 2Lf$ decreases. To the first approximation, few electrons can cross the separatrix between trapped and untrapped electrons if the sweep is slow; few new electrons enter the trapping bucket, and few initially-trapped electrons leave it. Thus, the trapped electrons are dragged to lower velocities, and the density of electrons in the bucket remains fixed. But for typical distribution functions, the initial density of electrons is lower at velocity $\bar{v}_r$ than at $\bar{v}_e$, and dragging the bucket downward creates a hole in phase space. Since the spatially-localized bucket bounces from end to end in phase with the drive, the phase space hole oscillates as well. The hole creates an electrostatic perturbation, which constitutes the postulated BGK modes. Because the excitation process involves dragging a bucket through velocity space, we call the generated waves “Chirped Bucket-BGK” modes.

Strong evidence for this model comes from experiments which perturb the distribution function. For example, a strong, fixed frequency drive at $f_f$ will phase-mix the velocities in the region centered around the velocity $\bar{v}_f$ resonant with the drive, effectively

![FIGURE 4. Spatially-averaged distribution function $f(v)$ showing the evolution of the trapping bucket as the frequency is swept from $f_i$ to $f_r$. The dark and light regions indicate the trapped and untrapped electrons, respectively.](image-url)
FIGURE 5. Response from a 1 GHz/s, 100 mV sweep from $f_s = 3$ MHz to $f_s = 2.2$ MHz that is preceded by a 100 μs long, 500 mV, fixed frequency $f_f$ drive. The fixed frequency drive flattens the distribution function around the resonant velocity linked to its frequency. Near (a), the fixed frequency drive is below 3 MHz, near (b) it is above 3 MHz, and the dotted line (c) is the response without the fixed frequency drive.

flattening the distribution function there. The flat in the distribution function increases the number of particles with velocity slightly greater than $\bar{v}_f$, and decreases the number of particles with velocity slightly less than $\bar{v}_f$. If a sweeping drive is then initiated, the difference between the flattening frequency $f_f$ and the swept drive start frequency $f_s$ will affect the final amplitude of the BGK mode (see Fig. 5.) If $f_s < f_f$, the density will be diminished at $\bar{v}_s$, so fewer particles will be trapped into the sweeping bucket, the phase space hole will be proportionally larger, and the final amplitude will increase. In contrast, if $f_s > f_f$, the density of particles near $\bar{v}_s$ will be increased, and the final amplitude will decrease.

These flattening experiments clearly demonstrate that the system response depends on the shape of the initial distribution function. Other experiments, not shown, show that we can transport particles in velocity space; for instance, an upwardly sweeping drive will carry particles to a higher velocity, leaving a hole behind in the distribution function at the drive’s initial resonant velocity $\bar{v}_s$. If a downward drive is then initiated starting from the same initial frequency, the resulting BGK mode will be larger. Repeating the upward-going drive several times before initiating the downward drive will dig an ever deeper hole, resulting in ever larger BGK modes. We can also observe trapping oscillations, proving that particles are trapped in the potential of the wave.

We have analyzed the excitation process using the multibeam model presented in Stix [10] and elsewhere, modified to include the swept excitation process. The details of the analysis will be presented in a later paper; the analysis predicts that the mode amplitude $a$ grows as

$$\sqrt{a} \propto \frac{\omega}{\omega - \omega_0} \Delta f(\omega/k),$$

(1)

where $\omega$ is the mode frequency in radians/sec, $\omega_0$ is the nearest lower linear TG mode frequency, and $\Delta f$ is the depth of the hole in the distribution function: $\Delta f(\bar{v}) \approx$
0.5 \left[ f(\tilde{v}) - f(\tilde{v}_s) \right]. This expression explains many aspects of the observed excitations: 
(1) The multiple peaks found in Fig. 3; as \omega is swept downwards towards any of the \omega_0, the mode amplitude increases. (2) The mode amplitude collapses near each \omega_0; as \omega sweeps past the relevant \omega_0, the RHS of Eq. 1 becomes negative. (3) The ability to excite the modes almost anywhere; while there are peaks near the \omega_0, the mode grows for almost any downward sweep. (4) The lack of modes below the last peak; for \omega less than the lowest TG mode frequency \omega_{00}, the RHS of Eq. 1 is necessarily negative. (5) The necessity to sweep downwards; sweeping downwards increases both terms on the RHS of Eq. 1, so the amplitude grows. Sweeping upwards makes both terms smaller, so the amplitude decreases.

Equation 1 also explains why Bucket-BGK modes and TG modes are generally not found simultaneously. For TG modes to exist, the plasma temperature has to be cool enough that the phase velocity \omega_{00}/k is much greater than the thermal velocity \nu_{th}. But Bucket-BGK modes exist only when \nu(\omega/k) is not trivially small, and when \omega is greater than \omega_{00}. Taken together, these rules requires that \omega_{00}/k \approx \nu_{th}. For our plasmas, the smallest k is about 0.1 cm⁻¹, and the smallest phase velocity is about 15 × 10⁷ cm/s. At 1 eV, \nu_{th} \approx 4 \times 10⁷ cm/s, and, as expected, only TG modes exist. At 6 eV, \nu_{th} \approx 10 \times 10⁷ cm/s and only Bucket-BGK modes exist. In a narrow range of intermediate temperatures, we do observe both TG and Bucket-BGK modes, but neither wave is well behaved in this regime.

A key assumption in the theory behind Eq. 1 is that particles do not escape the bucket as the driving frequency is swept. We find (Fig. 6) that this assumption is correct so long as the sweep rate is neither too slow nor too fast. If the drive is swept too slowly, particles escape out of the bucket because of collisions and/or loading in the external circuitry [7]. Preliminary evidence suggests that the time scales of these effects are related to the time scale for the decay of the undriven modes.

If the sweep rate is too high, the Bucket-BGK modes disappear. Using action-angle variables, we find that the buckets disappear entirely when the sweep rate is too high; the drive strength must be increased linearly with the sweep rate for the buckets to exists.
FIGURE 7. Maximum Sweep Rate that generates Bucket-BGK modes as a function of drive strength. The sweep rates extends from 3 → 2.2 GHz

This prediction is proved by experimental measurements (Fig. 7.) The analysis is similar to that for autoresonance for Diocotron modes [11, 12].

In conclusion, we have generated very high amplitude Bucket-BGK modes. The modes have no linear limit and can be excited over a very broad range of frequencies. They are very stable, can be produced controllably, and respond appropriately to changes in the distribution function. The longevity of the modes is remarkable. Not only do the mode Q’s reach 100,000, but the modes survive despite up to 60,000 reflections from the trap ends. We have developed a simple theory that explains many of the results. Our Chirped Bucket-BGK modes live much longer than the BGK modes found in earlier experiments. The reason may be related to the fact that most of the previous experiments used strong, or even impulsive, drives. Our gentle excitation process allows us to create seemingly perfect BGK modes.

This work was supported by the NSF. We thank B. Afeyan, T. O’Neil, and J. Wurtele for their comments.

REFERENCES


