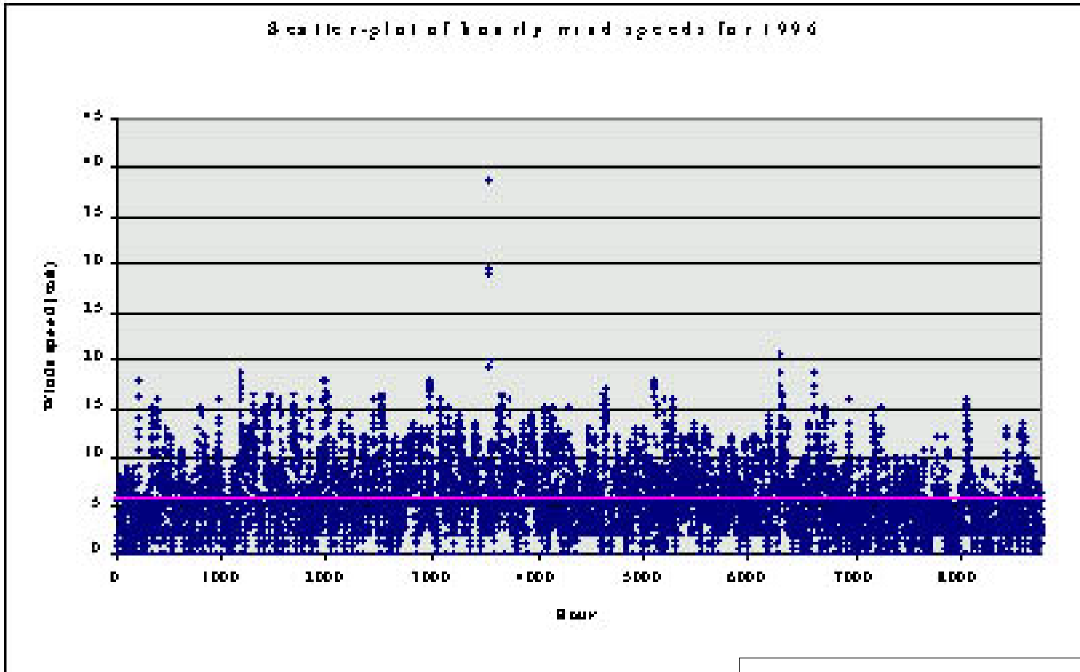
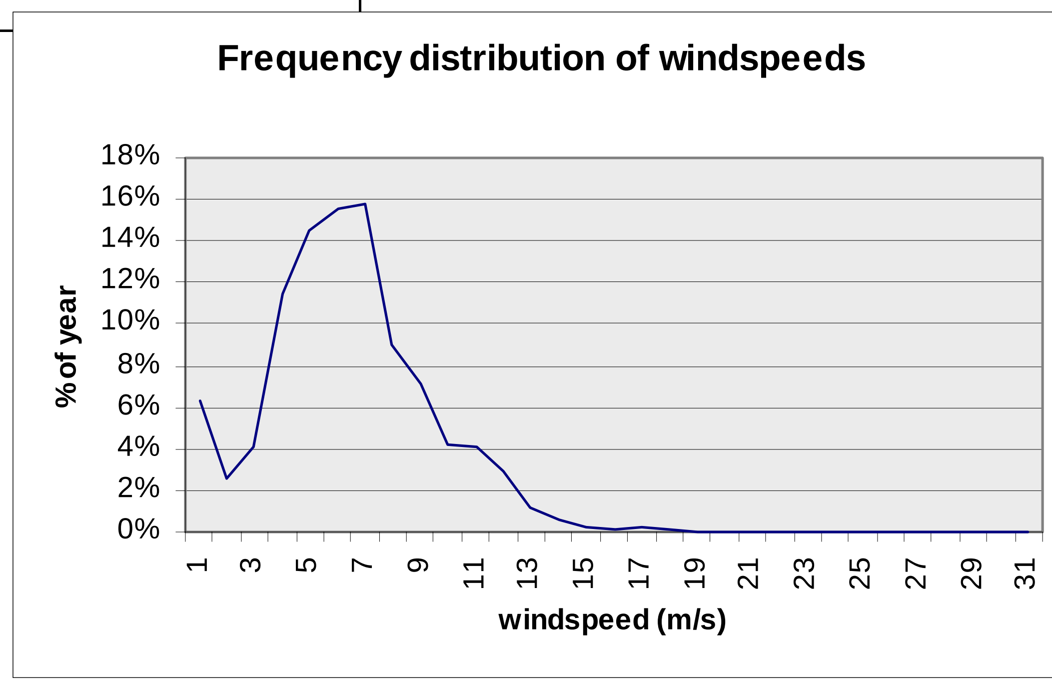


←Relatively meaningless plot of hourly data… average windspeed is given by the pink line.

More meaningful plot of frequency distribution →



Weibull distribution:

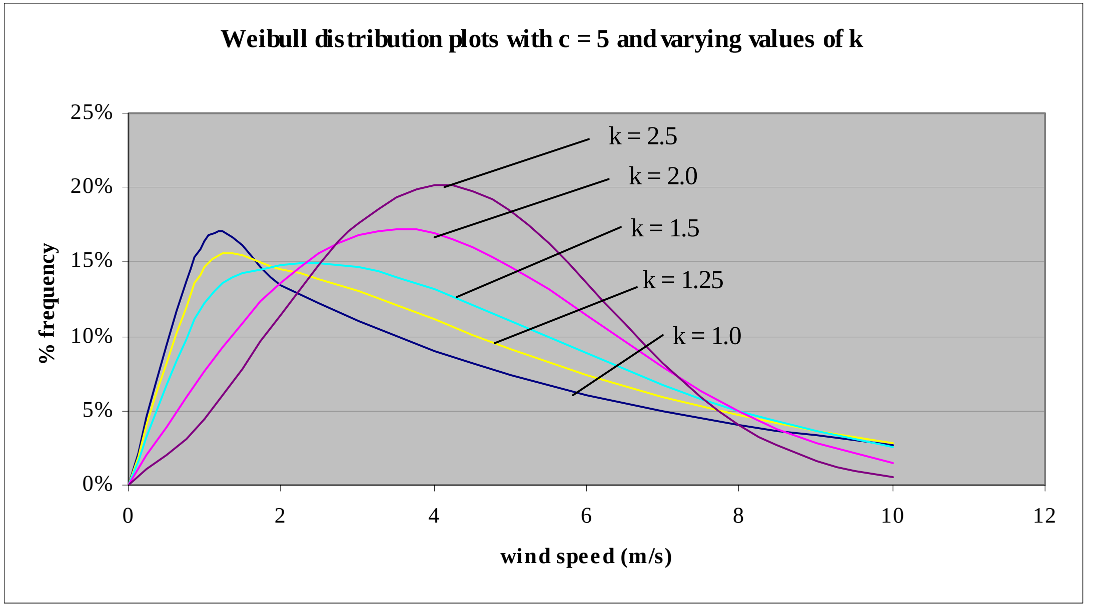
$$f(u) = \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} \exp\left[-\left(\frac{u}{c}\right)^{k}\right]$$

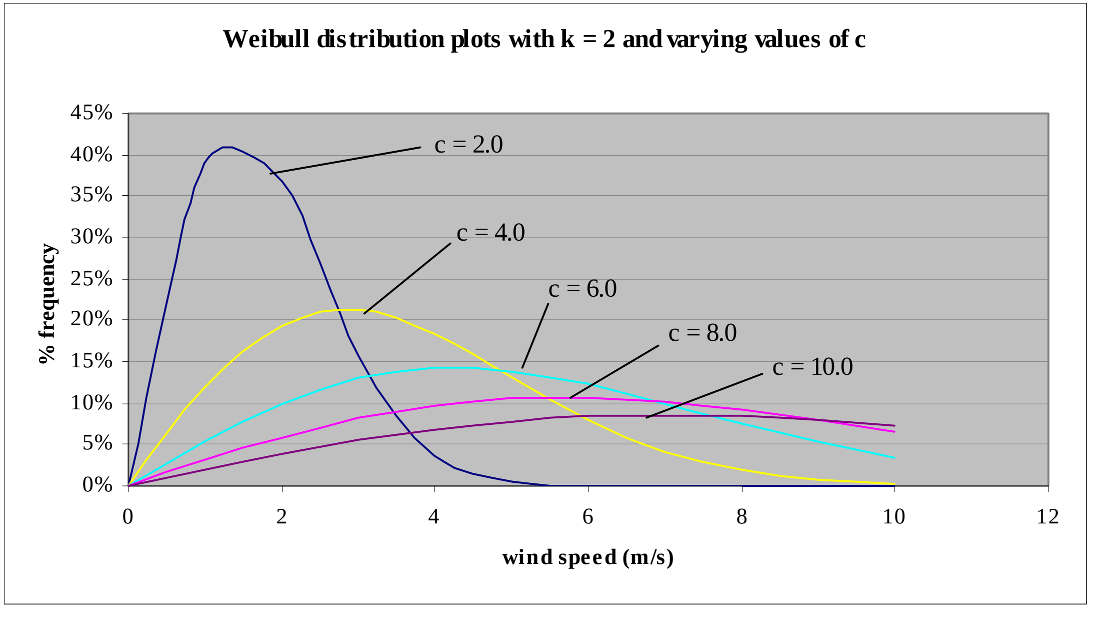
Most windspeed patterns fit the Weibull distribution reasonably well over long time periods.

Note: there are two parameters the define the characteristics of the distribution.

c **-** *the* ***scale*** *parameter (the Blue Book calls this v_c and also refers to it as the* ***characteristic velocity****).*

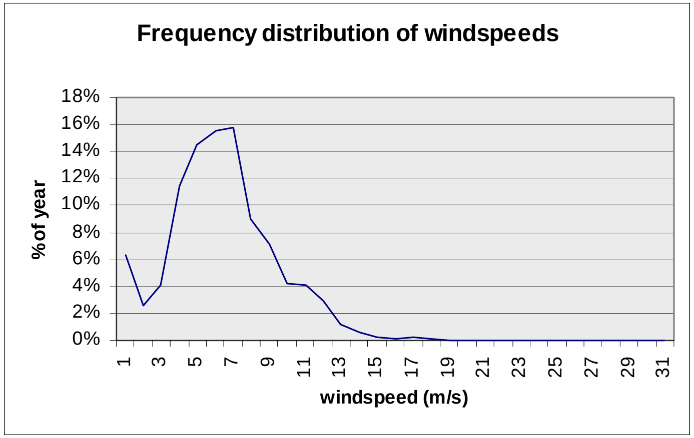
k - *the* ***shape*** *parameter (the Blue Book calls this b).*


Weibull distribution plots with c = 5 and varying values of k
25%
20%
15%
10%
5%
0%
% frequency
k = 2.5
k = 2.0
k = 1.5
k = 1.25
k = 1.0
0
2
4
6
8
10
12
wind speed (m/s)


Weibull distribution plots with k = 2 and varying values of c
45%
40%
35%
30%
25%
20%
15%
10%
5%
0%
% frequency
c = 2.0
c = 4.0
c = 6.0
c = 8.0
c = 10.0
0
2
4
6
8
10
12
wind speed (m/s)

How does this… $$f(u) = \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} \exp\left[-\left(\frac{u}{c}\right)^{k}\right]$$

Fit this?



And with what values of c and k?

First, relate the mean windspeed to these parameters…

for any variable x that occurs with a certain probability P(x), the mean of x, denoted by $\bar{x}$*, is given by the following integral:*

$$\bar{x} = \int_0^\infty x P(x) \mathrm{d}x$$

Similarly, for windspeed with a Weibull distribution, the average windspeed is given by...

$$\bar{u} = \int_0^\infty u \cdot \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} \exp\left[-\left(\frac{u}{c}\right)^k\right] du = \int_0^\infty k\left(\frac{u}{c}\right)^k \exp\left[-\left(\frac{u}{c}\right)^k\right] du$$

The solution of this integral is given by a set of functions called Gamma functions defined by the following relationship:

$$\Gamma(y) = \int_0^\infty x^{y-1} \exp[-x]\,dx$$

So, with some creative substitution, letting $x = (u/c)^k$ and $dx = (k/c)(u/c)^{k-1}du$ we get

$$\bar{u} = c\int_0^\infty (x)^{1/k} \exp[-x^k]\,dx$$

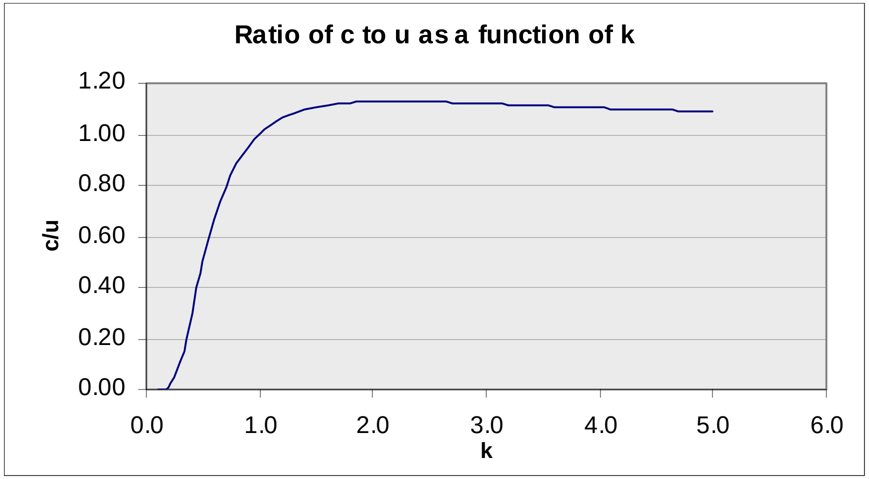
Which can also be written as a *Gamma function* like this...

$$\bar{u} = c\Gamma\left(1 + \frac{1}{k}\right)$$

But why is this useful????

- One option is to calculate $\bar{u}$ directly and use it to find the parameters c and k for example:

$$\frac{c}{\bar{u}} = \frac{1}{\Gamma\left(1+\frac{1}{k}\right)}$$



Note: the ratio of c to u is relatively constant (~1.12) over a wide range of shape parameter k so, to a good approximation, we can assume c ~ 1.12 $\bar{u}$

- Another option is to linearize the distribution function and do a regression to determine the parameters…

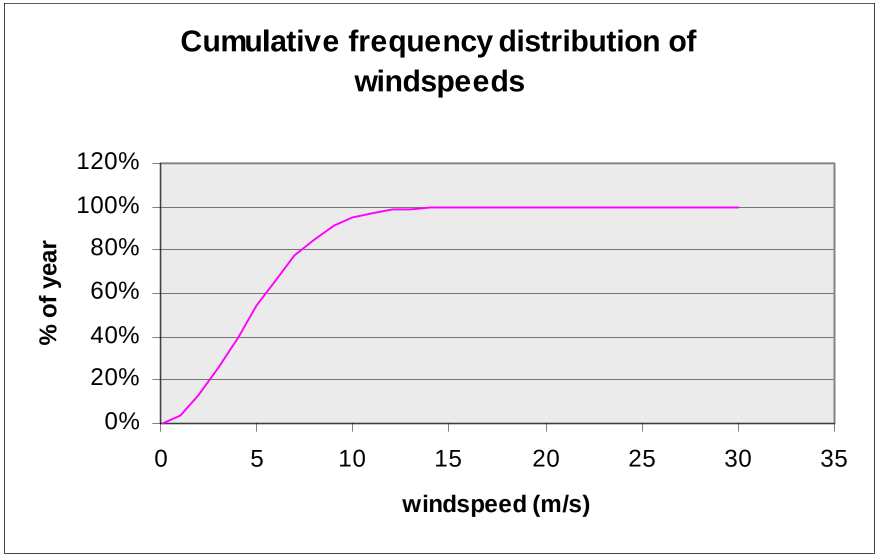
$$f(u) = \frac{k}{c}\left(\frac{u}{c}\right)^{k-1} \exp\left[-\left(\frac{u}{c}\right)^{k}\right]$$

Take the integral of f(u) - this gives you the *cumulative distribution function* F(u)...

$$f(u) = \frac{d}{du}F(u) \textit{ so that (without going through all of the details)}$$

$$F(u) = 1 - \exp\left[-\left(\frac{u}{c}\right)^{k}\right]$$

which looks more or less like this →



Now take the natural log two times (to linearize bothe the exponential and the power functions). With some rearrangement this gives us…

$$\ln[\ln(1-F(u))] = k\ln(u) - k\ln(c)$$

This can be plotted as a straight line: y = ax + b (*don't confuse the b in this equation with the b that the Blue Book uses to represent the* ***shape parameter****, which is k in this derivation*).

Here, the slope of the line 'a' will be equal to the **shape parameter** k and the intercept of the line, 'b' is equal to -k ln(c) so that $c = e^{-b/k}$.

This treatment is pretty simple to do in excel. Plot it, and do a linear curve fit...

…and the result of the regression looks something like this, though the fit parameters depend strongly on which data points you include or exclude. *That's for you to decide...*