Countersteered turn on a bicycle

Parameters definitions

λ center of mass lean angle
σ steering angle
ϕ is the hip angle
θ is the bike lean angle

I0 Moment of wheel around axis I0
Is Moment of wheel around stearing axis
Iλw Moment of wheel around point of contact with ground (lean direction)
Iy Moment of inertia of center of mass (not including wheel)
M.T is the center of mass
M.1 is the bike+ lower body mass
M.2 is the upper body mass
H height off ground of the hip pivot
l.1 is the height off the ground of M.1
l.2 is the distance between the pivot point and M.2
ω wheel rotational velocity
v bike velocity
Ns steering torque
g gravitational acceleration
b distance from the rear wheel of the center of mass
Δ trail
L wheel base
β damping from σ_t

$$g := 9.8$$

Bicycle parameters

Geometrical

$L := 1.$ $\quad b := 0.33$

$\Delta := 0.02$

$r := .33 \quad I_0 := 0.095 \quad I_s := 0.079 \quad I_{\lambda w} := 0.844$

$M_1 := 50 \quad M_2 := 50 \quad H := 1.25 \quad l_1 := 1 \quad l_2 := .25$

$\Gamma := 0.65$

Turn

$v := 7 \qquad \frac{v}{.447} = 15.66 \qquad V := v\cdot\frac{m}{s} \qquad V = 7\frac{m}{s}$

$\omega := \frac{v}{r}$

$V = 25.2\,\text{kph}$

$V = 15.659\,\text{mph}$

$I_\lambda := M_1\cdot l_1^{\,2} + M_2\cdot(H + l_2)^2$

$I_\lambda = 162.5$

Turning Radius $\quad \rho := 25$

Required steer angle $\quad \Sigma_c := -\frac{L}{\rho} \qquad \Sigma_c\cdot\frac{180}{\pi} = -2.292$

Required tilt angle

$\frac{v^2}{\rho} = 1.96$

Simple $\quad \Lambda_c := \frac{v^2}{9.8\cdot\rho} \qquad \Lambda_c\cdot\frac{180}{\pi} = 11.459$

Taking into account wheel angular momentum

$$\sqrt{\left[\ldots\left(\ldots\right)\right]^2} \qquad (M_1 + M_2)^2$$

$$\Lambda_c := \frac{v^2}{\ldots}\cdot\left[1 + \frac{2\cdot\omega\cdot I_0}{\ldots(M_1 + M_2)}\right] \qquad \Lambda_c\cdot\frac{180}{\pi} = 11.512$$

Hip bending parameters

$A_1 := 0 \cdot \frac{\pi}{180}$ $A_2 := 0 \cdot \frac{\pi}{180}$ $A_3 := 0 \cdot \frac{\pi}{180}$

$\Delta T := 0.75$ $T_1 := 2$ $T_2 := 6$

➜ Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeHipBendSubroutines.mcd(R)

Steering Torque parameters

$N_1 := 0.44$ $N_2 := .0065$ $N_3 := -0.436$

$\Delta T_N := 0.5$ $T_{N1} := 1$ $T_{N2} := 6$

➜ Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeTorqueProfileSubroutines.mcd(R)
➜ Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeDiffEQRoutines.mcd(R)

Initial Conditions

$\theta := 0 \quad \theta_t := 0 \quad \sigma := 0 \quad \sigma_t := 0$

$t_f := 10 \qquad \text{Points} := 500$

Find Trajectory

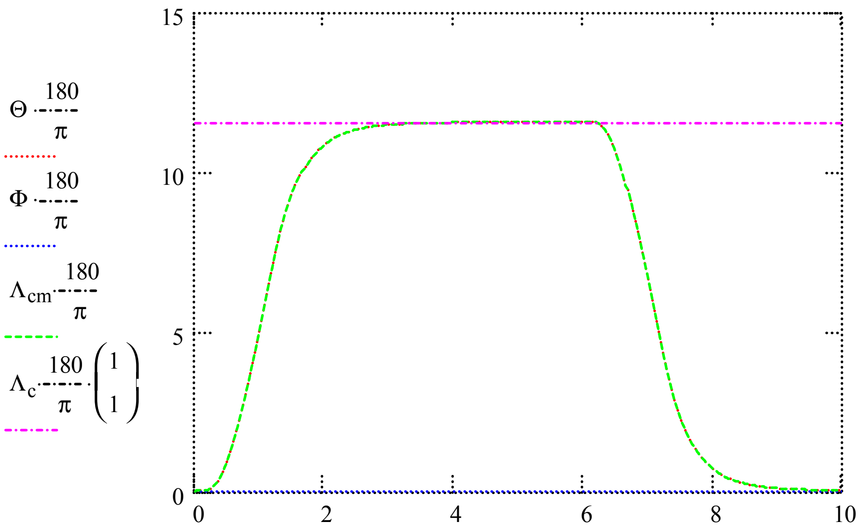
$$Tr := Traj\Gamma\left(0, t_f, \theta, \theta_t, \sigma, \sigma_t, Points\right)$$

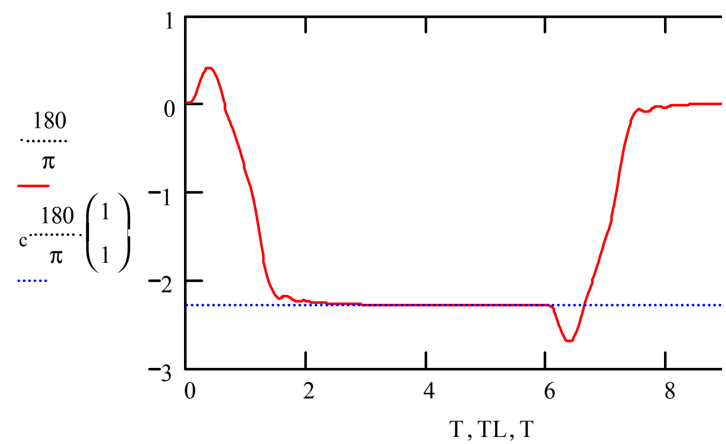
$$T := Tr^{\langle 0 \rangle} \quad \Theta := \overrightarrow{Tr^{\langle 1 \rangle}} \quad \Theta_t := Tr^{\langle 2 \rangle} \quad \Sigma := Tr^{\langle 3 \rangle} \quad \Sigma_t := Tr^{\langle 4 \rangle} \qquad TL := \begin{pmatrix} 0 \\ t_f \end{pmatrix}$$

$$\Phi := \phi_f(T) \quad q := 0..\,last(T)$$

$$Data := \overrightarrow{Tilt\left(T, \Theta, \Theta_t\right)} \quad h_q := \left(Data_q\right)_0 \quad \Lambda_{cm_q} := \left(Data_q\right)_1 \qquad N_{\phi_q} := \left(Data_q\right)_3$$

$$\Lambda_{cm_q} := if\left(\Lambda_{cm_q} > \pi, \Lambda_{cm_q} - 2\cdot\pi, \Lambda_{cm_q}\right)$$





T, T, T, TL

Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeAnalyzeTorques.mcd(R)

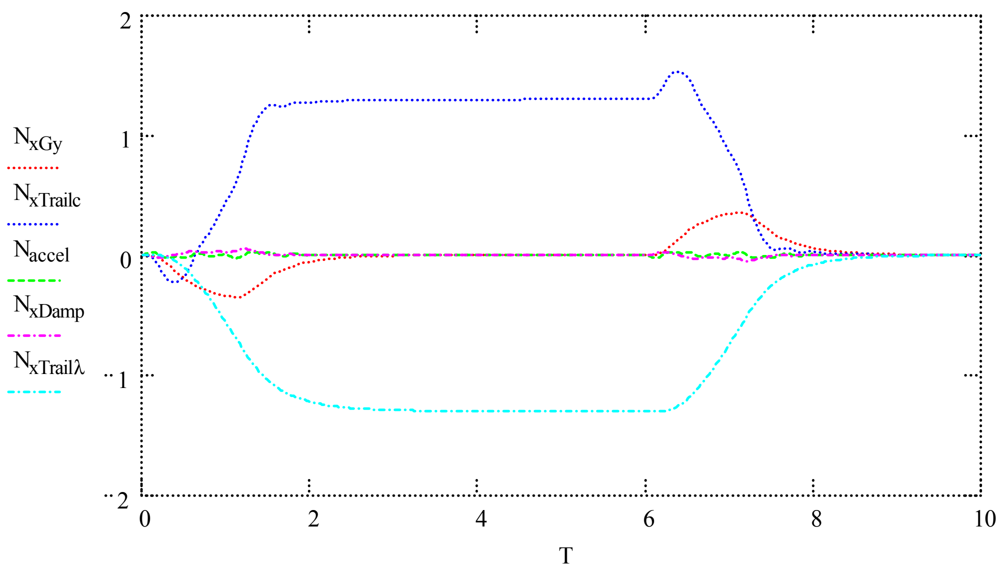
$N_{xGy} := \overrightarrow{GyroscopicN_x(\Theta_t, \omega, I_0)}$ $N_{accel} := AccelerationN_x(T, \Sigma_t)$

$N_{xTrailc} := \overrightarrow{TrailN_{xc}(\Sigma, b, L, M_T, v, \Delta)}$ $N_{xDamp} := DampingN_x(\Sigma_t)$

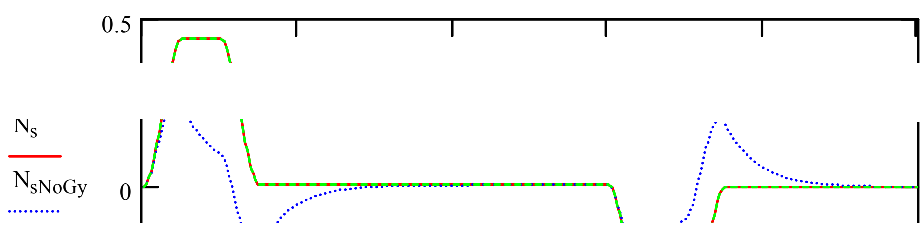
$N_{xTrail\lambda} := \overrightarrow{TrailN_{x\lambda}(\Theta, b, L, M_T, v, \Delta)}$

Total Steering Torque $N_s := \overrightarrow{N_{sf}(T)}$

$N_{sNum} := N_{accel} - N_{xTrailc} - N_{xGy} - N_{xTrail\lambda} - N_{xDamp}$ $N_{sNoGy} := N_{accel} - N_{xTrailc} - N_{xTrail\lambda} - N_{xDamp}$



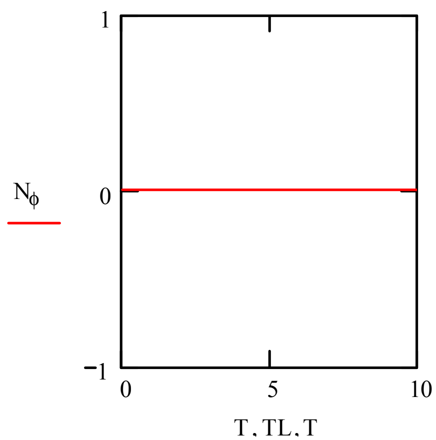
$\min(N_{xGy}) = -0.353$



$\max(N_s) = 0.44$

$\frac{\max(N_s)}{\text{[illegible]}} = 0.0898$

$$\frac{.44}{2 \cdot .25} = 0.88$$



$$N_{yCent} := \overrightarrow{\text{CentrifugalN}_y(\Sigma, h, L, M_T, v)}$$

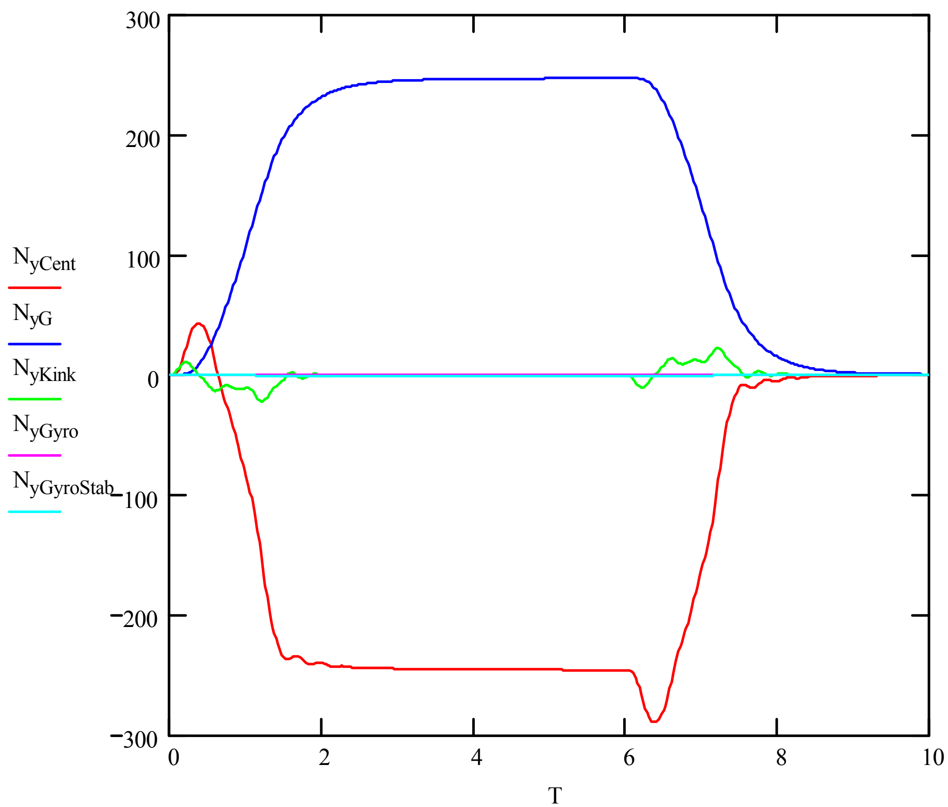
$$N_{yG} := \overrightarrow{\text{GravitationalN}_y(\Lambda_{cm}, h, M_T, v)}$$

$$N_{yKink} := \overrightarrow{\text{KinkN}_y(\Sigma_t, h, b, L, M_T, v)}$$

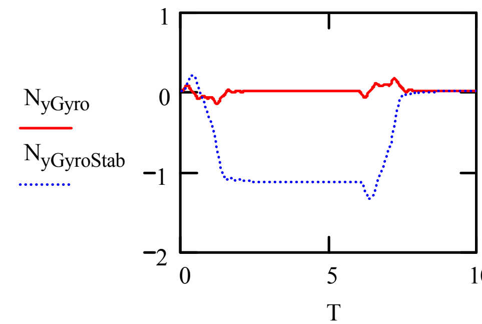
$$N_{yGyro} := \overrightarrow{\text{GyroN}_y(\Sigma_t, \omega, I_0)}$$

$$N_{yGyroStab} := \overrightarrow{\text{GyroStabN}_y(\Sigma, r, L, \omega, I_0)}$$

$$N_{yTotal} := N_{yCent} + N_{yG} + N_{yKink} + N_{yGyro} + N_{yGyroStab}$$



N_{yCent}
N_{yG}
N_{yKink}
N_{yGyro}
$N_{yGyroStab}$
300
200
100
0
−100
−200
−300
0
2
4
6
8
10
T



N_{yGyro}
$N_{yGyroStab}$
1
0
−1
−2
0
5
10
T

Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikePathSubroutines.mcd(R)

$P := \text{Path}(L, t_f, \text{Points})$

$Xr := P^{\langle 1 \rangle}$ $Yr := P^{\langle 2 \rangle}$ $Xf := P^{\langle 3 \rangle}$ $Yf := P^{\langle 4 \rangle}$ $T := P^{\langle 0 \rangle}$

$\max(Yr) = 0.057$