Countersteered Motorcycle Turn

Parameters definitions

λ center of mass lean angle
σ steering angle
ϕ is the hip angle
θ is the bike lean angle

I0 Moment of wheel around axis I0
Is Moment of wheel around stearing axis
Iλw Moment of wheel around point of contact with ground (lean direction)
Iy Moment of inertia of center of mass (not including wheel)
M.T is the center of mass
M.1 is the bike+ lower body mass
M.2 is the upper body mass
H height off ground of the hip pivot
l.1 is the height off the ground of M.1
l.2 is the distance between the pivot point and M.2
ω wheel rotational velocity
v bike velocity
Ns steering torque
g gravitational acceleration
b distance from the rear wheel of the center of mass
Δ trail
L wheel base
β damping from σ_t

$g := 9.8$

Motorcycle parameters

Geometrical

$L := 1.54$ $\quad b := \frac{L}{2}$

$\Delta := .117$

$r := .29$ $\quad I_0 := \left(\frac{20}{2.2}\right) \cdot r^2$ $\quad I_s := I_0 \cdot .75$ $\quad I_{\lambda w} := 4$

$M_1 := 240$ $\quad M_2 := 60$ $\quad H := .73$ $\quad l_1 := .5$ $\quad l_2 := .25$

$\Gamma := 3$

$I_\lambda := M_1 \cdot l_1^2 + M_2 \cdot (H + l_2)^2$

$I_\lambda = 117.624$

Turn

$v := 20$ $\quad \frac{v}{.447} = 44.743$ $\quad V := v \cdot \frac{m}{s}$ $\quad V = 20 \frac{m}{s}$

$\omega := \frac{v}{r}$

$V = 72\ \text{kph}$

$V = 44.739\ \text{mph}$

Turning Radius $\quad \rho := 200$

Required steer angle $\quad \Sigma_c := -\frac{L}{\rho}$ $\quad \Sigma_c \cdot \frac{180}{\pi} = -0.441$

Required tilt angle

$\frac{v^2}{\rho} = 2$

Simple $\quad \Lambda_c := \frac{v^2}{9.8 \cdot \rho}$ $\quad \Lambda_c \cdot \frac{180}{\pi} = 11.693$

Taking into account wheel angular momentum

$\sqrt{\;\ldots\; (M_1 + M_2)^2}$

$\Lambda_c := \frac{v^2}{\ldots} \cdot \left[1 + \frac{2 \cdot \omega \cdot I_0}{\ldots} \right]$ $\quad \Lambda_c \cdot \frac{180}{\pi} = 12.038$

Hip bending parameters

$A_1 := 0 \cdot \frac{\pi}{180}$ $\quad A_2 := 0 \cdot \frac{\pi}{180}$ $\quad A_3 := 0 \cdot \frac{\pi}{180}$

$\Delta T := 0.75$ $\quad T_1 := 2$ $\quad T_2 := 6$

Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeHipBendSubroutines.mcd(R)

Steering Torque parameters

$N_1 := 12.7$ $\quad N_2 := 1.03$ $\quad N_3 := -11.7$

$\Delta T_N := 0.5$ $\quad T_{N1} := 1$ $\quad T_{N2} := 6$

Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeTorqueProfileSubroutines.mcd(R)

Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeDiffEQRoutines.mcd(R)

Initial Conditions

$\theta := 0 \quad \theta_t := 0 \quad \sigma := 0 \quad \sigma_t := 0$

$t_f := 10 \quad \text{Points} := 500$

Find Trajectory

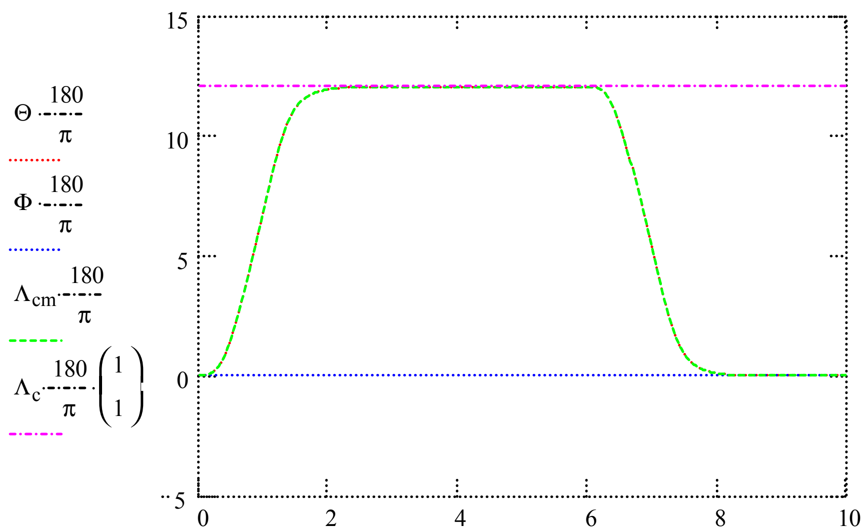
$\text{Tr} := \text{Traj}\Gamma\left(0, t_f, \theta, \theta_t, \sigma, \sigma_t, \text{Points}\right)$

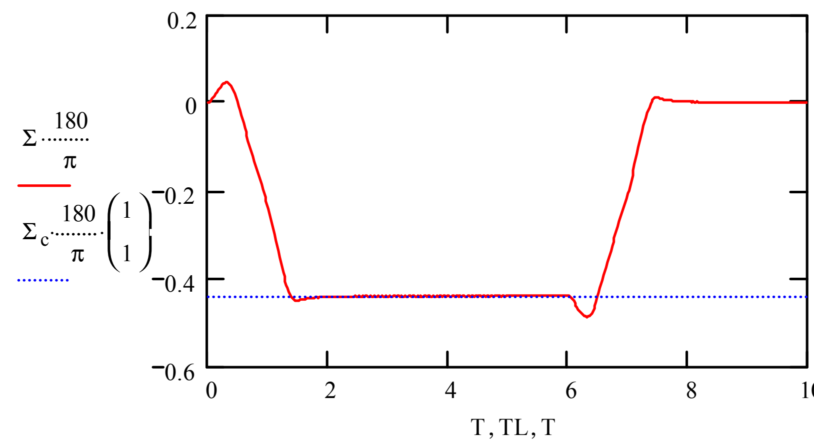
$T := \text{Tr}^{\langle 0\rangle} \quad \Theta := \overrightarrow{\text{Tr}^{\langle 1\rangle}} \quad \Theta_t := \text{Tr}^{\langle 2\rangle} \quad \Sigma := \text{Tr}^{\langle 3\rangle} \quad \Sigma_t := \text{Tr}^{\langle 4\rangle} \qquad TL := \begin{pmatrix} 0 \\ t_f \end{pmatrix}$

$\Phi := \phi_f(T) \quad q := 0..\,\text{last}(T)$

$\text{Data} := \overrightarrow{\text{Tilt}\left(T, \Theta, \Theta_t\right)} \quad h_q := \left(\text{Data}_q\right)_0 \quad \Lambda_{cm_q} := \left(\text{Data}_q\right)_1 \quad N_{\phi_q} := \left(\text{Data}_q\right)_3$

$\Lambda_{cm_q} := \text{if}\left(\Lambda_{cm_q} > \pi, \Lambda_{cm_q} - 2\cdot\pi, \Lambda_{cm_q}\right)$





T , T , T , TL

Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeAnalyzeTorques.mcd(R)

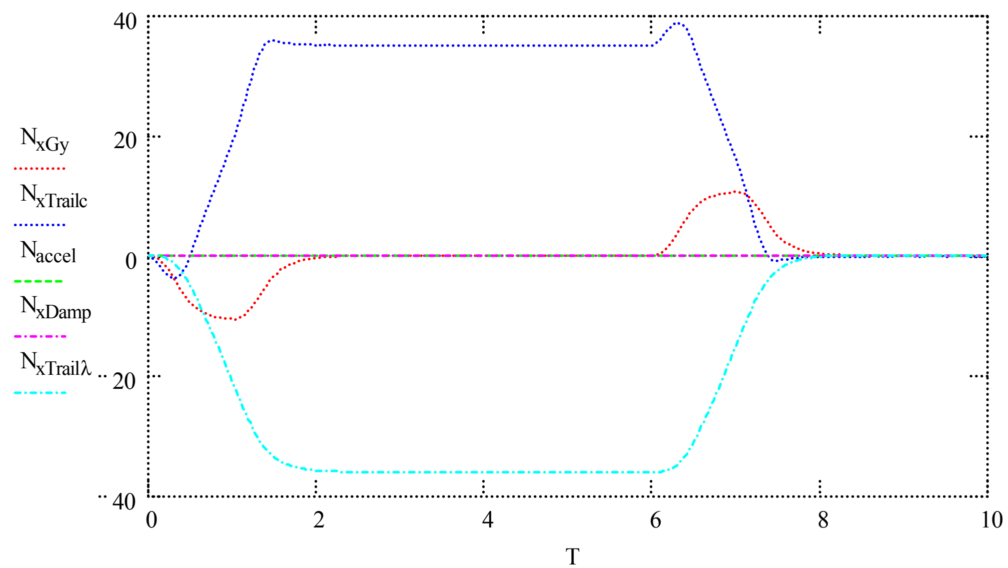
$N_{xGy} := \overrightarrow{\text{GyroscopicN}_x(\Theta_t, \omega, I_0)}$ $\quad N_{accel} := \text{AccelerationN}_x(T, \Sigma_t)$

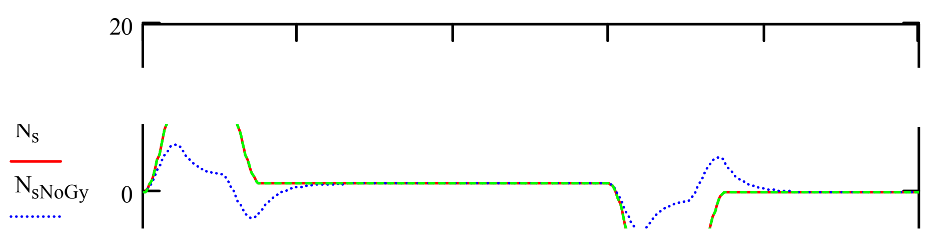
$N_{xTrailc} := \overrightarrow{\text{TrailN}_{xc}(\Sigma, b, L, M_T, v, \Delta)}$ $\quad N_{xDamp} := \text{DampingN}_x(\Sigma_t)$

$N_{xTrail\lambda} := \overrightarrow{\text{TrailN}_{x\lambda}(\Theta, b, L, M_T, v, \Delta)}$

Total Steering Torque $\quad N_s := \overrightarrow{N_{sf}(T)}$

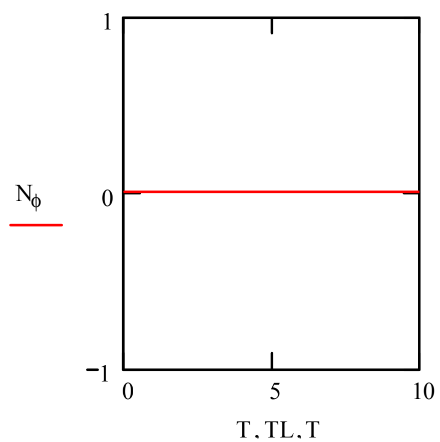
$N_{sNum} := N_{accel} - N_{xTrailc} - N_{xGy} - N_{xTrail\lambda} - N_{xDamp}$ $\quad N_{sNoGy} := N_{accel} - N_{xTrailc} - N_{xTrail\lambda} - N_{xDamp}$





$\max(N_s) = 12.7$

$\frac{\max(N_s)}{\ldots} = 2.592$



$$N_{yCent} := \overrightarrow{\mathrm{CentrifugalN_y}(\Sigma, h, L, M_T, v)}$$

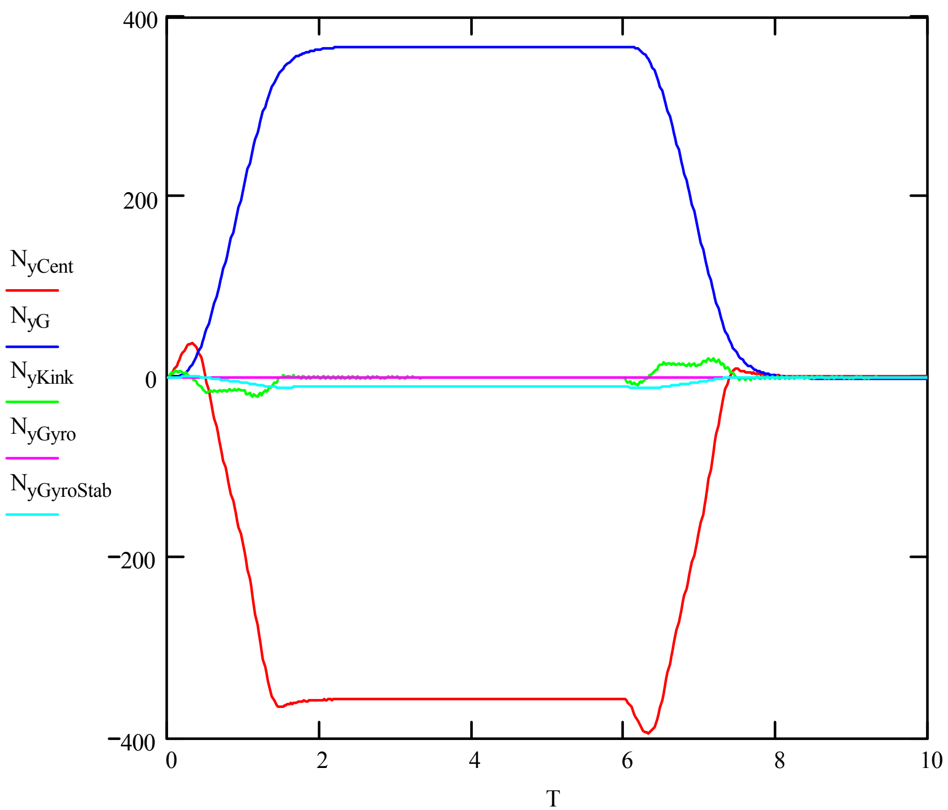
$$N_{yG} := \overrightarrow{\mathrm{GravitationalN_y}(\Lambda_{cm}, h, M_T, v)}$$

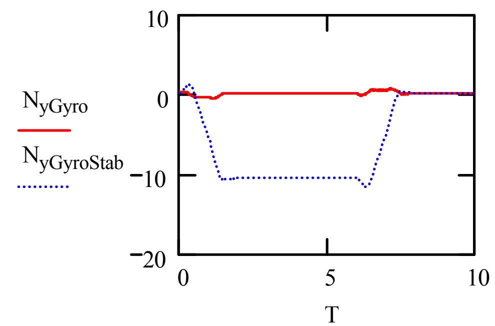
$$N_{yKink} := \overrightarrow{\mathrm{KinkN_y}(\Sigma_t, h, b, L, M_T, v)}$$

$$N_{yGyro} := \overrightarrow{\mathrm{GyroN_y}(\Sigma_t, \omega, I_0)}$$

$$N_{yGyroStab} := \overrightarrow{\mathrm{GyroStabN_y}(\Sigma, r, L, \omega, I_0)}$$

$$N_{yTotal} := N_{yCent} + N_{yG} + N_{yKink} + N_{yGyro} + N_{yGyroStab}$$


400
200
0
−200
−400
0
2
4
6
8
10
T
N_{yCent}
N_{yG}
N_{yKink}
N_{yGyro}
$N_{yGyroStab}$


10
0
−10
−20
0
5
10
T
N_{yGyro}
$N_{yGyroStab}$

Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikePathSubroutines.mcd(R)

$P := \mathrm{Path}(L, t_f, \mathrm{Points})$

$Xr := P^{\langle 1 \rangle}$ $Yr := P^{\langle 2 \rangle}$ $Xf := P^{\langle 3 \rangle}$ $Yf := P^{\langle 4 \rangle}$ $T := P^{\langle 0 \rangle}$

$\max(Yr) = 0.023$