

Hip and Countersteered Motorcycle Turn

Parameters definitions

λ center of mass lean angle

σ steering angle

ϕ is the hip angle

θ is the bike lean angle

I_0 Moment of wheel around axis I_0

I_s Moment of wheel around steering axis

$I_{\lambda w}$ Moment of wheel around point of contact with ground (lean direction)

I_y Moment of inertia of center of mass (not including wheel)

$M.T$ is the center of mass

$M.1$ is the bike+ lower body mass

$M.2$ is the upper body mass

H height off ground of the hip pivot

$l.1$ is the height off the ground of $M.1$

$l.2$ is the distance between the pivot point and $M.2$

ω wheel rotational velocity

v bike velocity

N_s steering torque

g gravitational acceleration

b distance from the rear wheel of the center of mass

Δ trail

L wheel base

β damping from σ_t

$g := 9.8$

Motorcycle parameters

Geometrical

$$L := 1.54 \quad b := \frac{L}{2}$$

$$\Delta := .117$$

$$r := .29 \quad I_0 := \left(\frac{20}{2.2}\right) \cdot r^2 \quad I_s := I_0 \cdot .75 \quad I_{\lambda w} := 4$$

$$M_1 := 240 \quad M_2 := 60 \quad H := .73 \quad l_1 := .5 \quad l_2 := .25$$

$$\Gamma := 4$$

Turn

$$v := 20 \quad \frac{v}{.447} = 44.743 \quad V := v \cdot \frac{m}{s} \quad V = 20 \frac{m}{s}$$

$$\omega := \frac{v}{r} \quad V = 72 \text{ kph}$$

$$V = 44.739 \text{ mph}$$

Turning Radius $\rho := 200$

Required steer angle $\Sigma_c := -\frac{L}{\rho} \quad \Sigma_c \cdot \frac{180}{\pi} = -0.441$

Required tilt angle

Simple $\Lambda_c := \frac{v^2}{9.8 \cdot \rho} \quad \Lambda_c \cdot \frac{180}{\pi} = 11.693 \quad \frac{v^2}{\rho} = 2$

Taking into account wheel angular momentum

$$\Lambda_c := \frac{v^2}{9.8 \cdot \rho} \cdot \left[1 + \frac{2 \cdot \omega \cdot I_0}{\dots} \right] \quad \Lambda_c \cdot \frac{180}{\pi} = 12.038$$

Hip bending parameters

$$A_1 := 30 \cdot \frac{\pi}{180} \quad A_2 := 4 \cdot \frac{\pi}{180} \quad A_3 := -24 \cdot \frac{\pi}{180}$$

$$\Delta T := 0.5 \quad T_1 := 0.62 \quad T_2 := 6$$

☞ Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeHipBendSubroutines.mcd(R)

Steering Torque parameters

$$N_1 := 12.7 \quad N_2 := 0 \quad N_3 := -12.7$$

$$\Delta T_N := 0.5 \quad T_{N1} := 0.62 \quad T_{N2} := 6$$

☞ Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeTorqueProfileSubroutines.mcd(R)

☞ Reference:C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeDiffEQRoutines.mcd(R)

Initial Conditions

$$\theta := 0 \quad \theta_t := 0 \quad \sigma := 0 \quad \sigma_t := 0$$

$$t_f := 10 \quad \text{Points} := 1500$$

Find Trajectory

$$\text{Tr} := \text{Traj}\Gamma(0, t_f, \theta, \theta_t, \sigma, \sigma_t, \text{Points})$$

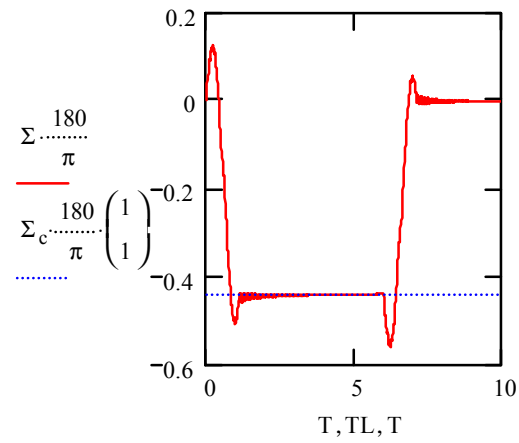
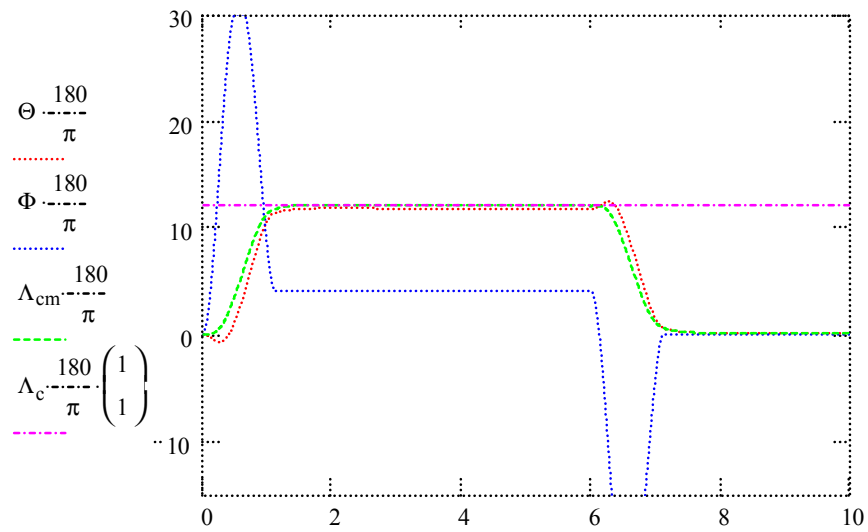
$$T := \text{Tr}^{\langle 0 \rangle} \quad \Theta := \text{Tr}^{\langle 1 \rangle} \quad \Theta_t := \text{Tr}^{\langle 2 \rangle} \quad \Sigma := \text{Tr}^{\langle 3 \rangle} \quad \Sigma_t := \text{Tr}^{\langle 4 \rangle}$$

$$\Phi := \phi_f(T) \quad q := 0.. \text{last}(T)$$

$$TL := \begin{pmatrix} 0 \\ t_f \end{pmatrix}$$

$$\text{Data} := \text{Tilt}(T, \Theta, \Theta_t) \quad h_q := (\text{Data}_q)_0 \quad \Lambda_{cm_q} := (\text{Data}_q)_1 \quad N_{\phi_q} := (\text{Data}_q)_4$$

$$\Lambda_{cm_q} := \text{if}(\Lambda_{cm_q} > \pi, \Lambda_{cm_q} - 2 \cdot \pi, \Lambda_{cm_q})$$



T, T, T, TL

☞ Reference: C:\Documents and Settings\Joel\My Documents\Paper Archive\WebPage\Teaching\MoreBikeFiles\Temp\BikeAnalyzeTorques.mcd(R)

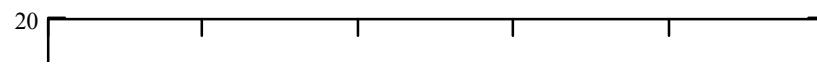
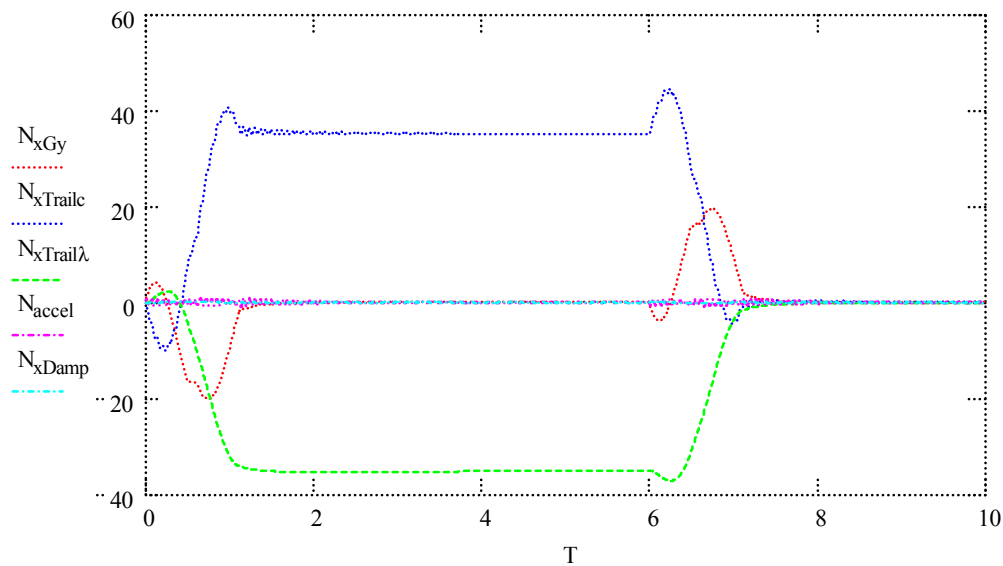
$$N_{xGy} := \overrightarrow{\text{Gyroscopic}N_x(\Theta_t, \omega, I_0)} \quad N_{\text{accel}} := \text{Acceleration}N_x(T, \Sigma_t)$$

$$N_{x\text{Trailc}} := \overrightarrow{\text{Trail}N_{xc}(\Sigma, b, L, M_T, v, \Delta)} \quad N_{x\text{Damp}} := \text{Damping}N_x(\Sigma_t)$$

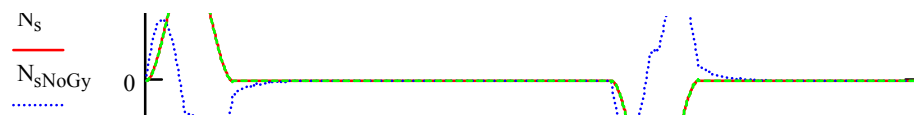
$$N_{x\text{Trail}\lambda} := \overrightarrow{\text{Trail}N_{x\lambda}(\Theta, b, L, M_T, v, \Delta)}$$

$$\text{Total Steering Torque} \quad N_s := N_{sf}(T)$$

$$N_{s\text{Num}} := N_{\text{accel}} - N_{x\text{Trailc}} - N_{xGy} - N_{x\text{Trail}\lambda} - N_{x\text{Damp}} \quad N_{s\text{NoGy}} := N_{\text{accel}} - N_{x\text{Trailc}} - N_{x\text{Trail}\lambda} - N_{x\text{Damp}}$$



$$\max(N_s) = 12.7$$



$$\frac{\max(N_s)}{4.9} = 2.592$$

$$N_{yCent} := \overrightarrow{\text{Centrifugal}N_y(\Sigma, h, L, M_T, v)}$$

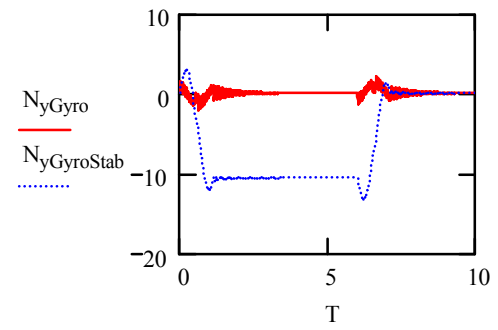
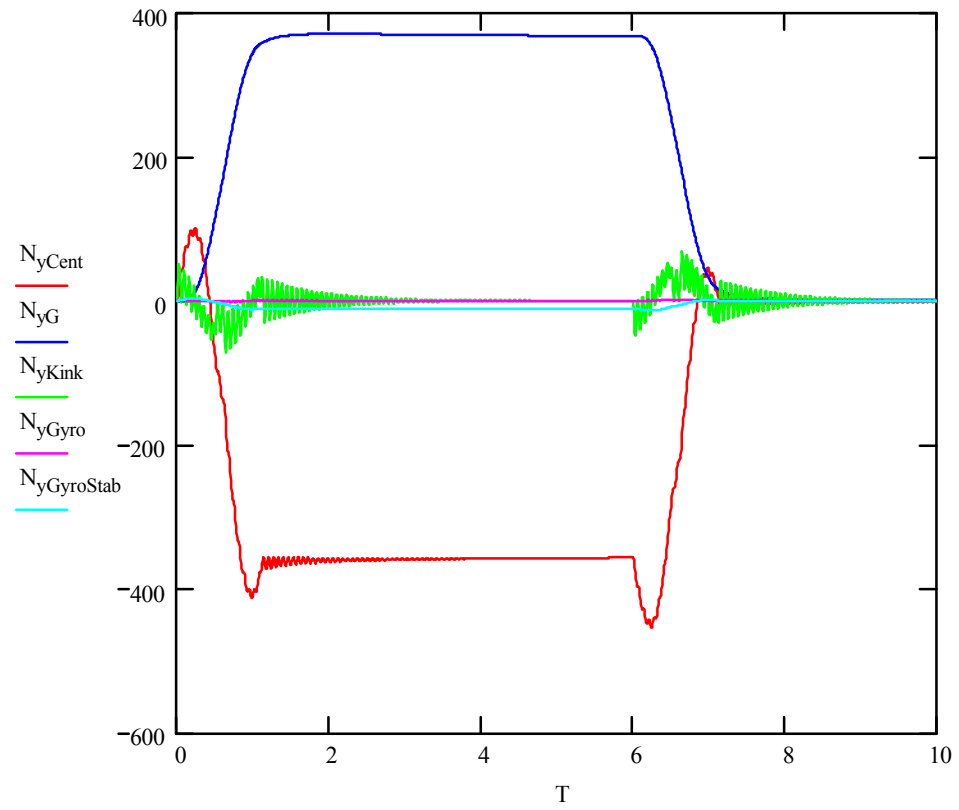
$$N_{yG} := \overrightarrow{\text{Gravitational}N_y(\Lambda_{cm}, h, M_T, v)}$$

$$N_{yKink} := \overrightarrow{\text{Kink}N_y(\Sigma_t, h, b, L, M_T, v)}$$

$$N_{yGyro} := \overrightarrow{\text{Gyro}N_y(\Sigma_t, \omega, I_0)}$$

$$N_{yGyroStab} := \overrightarrow{\text{GyroStab}N_y(\Sigma, r, L, \omega, I_0)}$$

$$N_{yTotal} := N_{yCent} + N_{yG} + N_{yKink} + N_{yGyro} + N_{yGyroStab}$$



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$P := \text{Path}(L, t_f, \text{Points})$

$X_r := P^{(1)}$ $Y_r := P^{(2)}$ $X_f := P^{(3)}$ $Y_f := P^{(4)}$ $T := P^{(0)}$ $\max(Y_r) = 0.056$

