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Three-dimensional non-neutral plasma shapes

A. J. Peurrung and J. Fajans
Department of Physics, University of California at Berkeley, Berkeley, California 94720

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The three-dimensional shapes of azimuthally asymmetric non-neutral plasmas are investigated numerically, and a simple theory is developed that predicts these shapes in many circumstances.

Knowledge of the full, three-dimensional shape of non-neutral plasmas subject to asymmetries is necessary to understand certain results in non-neutral plasma physics. For example, asymmetric confinement potentials are known to cause radial transport and plasma loss.[1–8] A "viscous" mechanism damps the diocotron motion of an off-center plasma.[9] Misalignment of the trap magnetic field affects radial transport and helps to form a highly reproducible plasma.[10] Finally, the alignment procedure devised by Hart and Spencer depends on the shape change of a misaligned and off-center plasma as it rotates around the trap axis.[11,12]

Non-neutral plasmas are generally confined in a trap similar to the one shown in Fig. 1. A strong magnetic field provides radial confinement and a negative voltage V_g applied to the conducting cylinder at the plasma's end provides axial confinement. When the plasma temperature is near zero, the high mobility of the electrons combined with Boltzmann's relation requires that the interior plasma density be constant along a magnetic field line. Thus, since the trap geometry is known, the complete three-dimensional plasma shape can be uniquely reconstructed from the axial integrated, two-dimensional plasma profile. This profile is commonly obtained by measuring the charge profile as the plasma is allowed to flow out of the trap along the magnetic field lines and into an imaging device.[13,14] The shapes and edge profiles of azimuthally symmetric, centered plasmas have been studied by Prasad and O'Neil.[15] When the plasma radius r_p is much less than the wall radius r_w, a simple and accurate method of calculating the shapes has been obtained.[16] Spencer and Hart presented numerical and theoretical studies of the 3-D shape of the Brigham Young University plasma.[11,12] However, the diameter of this plasma is only a few Debye lengths (λ_D), so the plasma edge thickness is of the order of the plasma radius. Consequently it is difficult to generalize their results. The goal of this present work is the reconstruction of the 3-D shapes of low-temperature plasmas ($r_p \gg \lambda_D$) when the plasmas are not axially centered.

While we here study only plasmas that are displaced from the trap axis, these plasmas arise in a wide variety of experimental situations. Plasmas may be off-centered as a result of growth of the diocotron mode,[10] growth of the ion resonance instability,[17] or the application of electric field asymmetries.[18] In each of these cases, the plasma's end shape is precisely that of the displaced plasmas studied here. In addition, magnetic asymmetries such as "tilt" cause the bulk of the plasma to be misaligned with the trap axis and may cause the plasma's ends to be displaced.

We assume that the plasma is semi-infinite in axial extent and that the equilibrium 2-D profile is circular. Other transverse cross sections are permissible; indeed, the equilibria-along-field-lines constraint discussed in this paper has no bearing on the cross section. Fine[19] has shown that the plasma tends to an ellipse as the plasma is displaced outwards. Fortunately, however, numerical studies of noncircular transverse profiles show that the detailed 2-D profile does not greatly influence the end shape.

Since $r_p/\lambda_D \gg 1$ the plasma has such a thin edge region[15] that the plasma shape can be calculated simply by finding the unique axial plasma boundary for which the electrostatic potential is constant along magnetic field lines within the plasma. (Otherwise the plasma's charge would quickly redistribute along field lines.) Rows (a) and (b) of Fig. 2 show the end shapes of centered and displaced plasmas with various radii. Row (c) shows the change between the shapes of the plasmas in Figs. 2(a) and 2(b). Although the end shape of a centered plasma depends dramatically on the radius, the shape change resulting from displacement is similar in each case. As the plasma moves off-center, the plasma shape becomes generally "slanted." Analysis of these shapes indicates that, to a good approx-

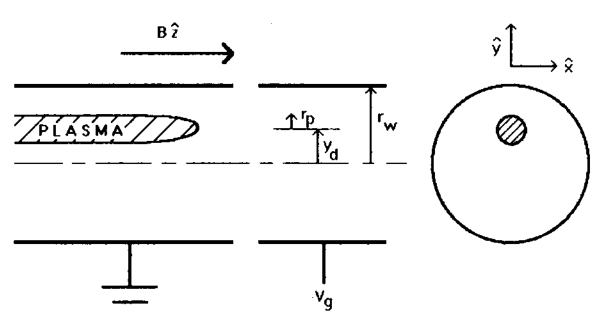

FIG. 1. Trap geometry.

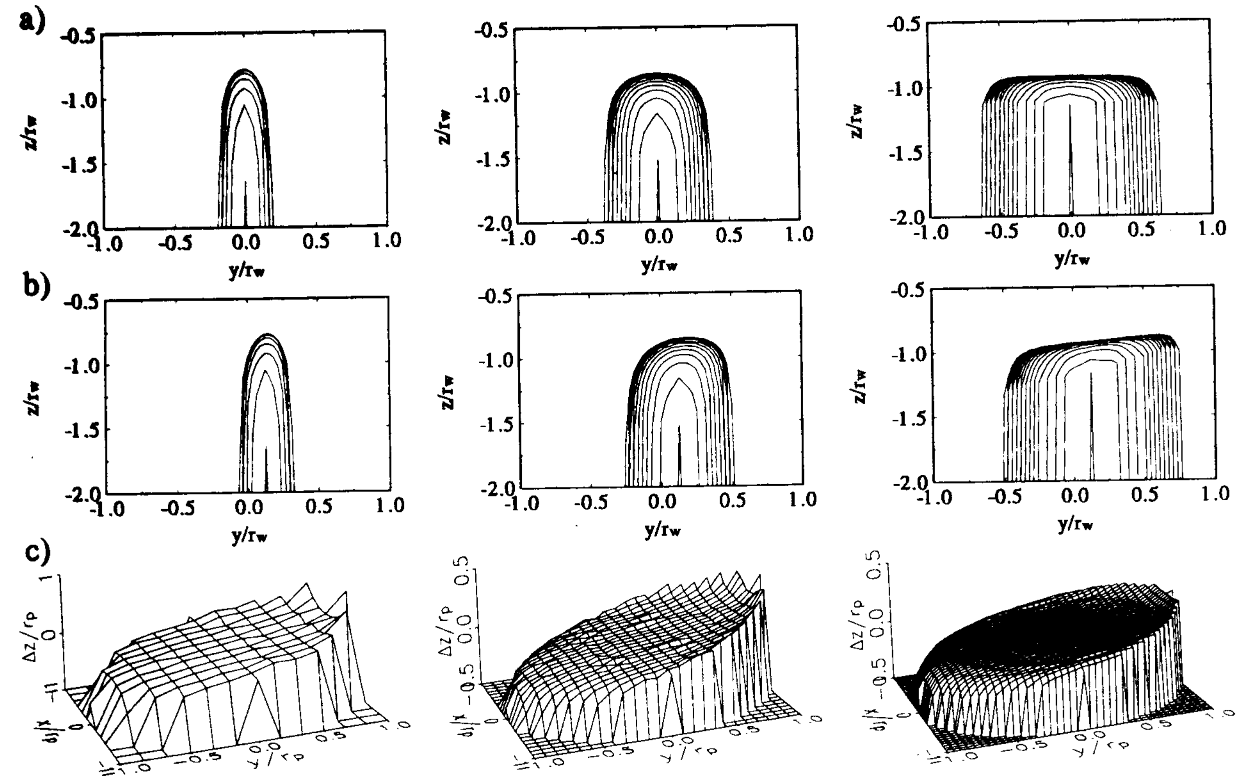

FIG. 2. End shapes of plasmas with $r_p/r_w = 0.187$, 0.375, and 0.625. The shapes for $y_d/r_w=0$ (a), $y_d/r_w=0.125$ (b), and the corresponding change in the axial boundary (c). The contour lines in (a) and (b) are boundary lines of constant x, where y measures distance in the displacement direction and x measures distance in the perpendicular direction.

imation, the slant angle θ_s depends linearly on the displacement y_d of the plasma's end. Notice that the point of the plasma which penetrates the farthest outward into the confining cylinders does not remain at the plasma center as the plasma is displaced. This point moves relative to the plasma's center in the direction *away* from the trap center by an amount that is proportional to the end displacement.

An approximate description of the plasma end shape can be obtained by carefully considering the potentials at the plasma's end. Inside the plasma, the total electrostatic potential is the sum of the confinement potential and the plasma potential. The confinement potential, $\phi_c(r,z)$ is found analytically by solving $\nabla^2\phi_c=0$ subject to the appropriate boundary conditions. The plasma potential can be found numerically, but if the plasma is assumed to have a flat, unslanted end shape, symmetry requires that the potential at the plasma's end be half of the potential of the corresponding infinite length plasma. Furthermore, since the total potential inside the plasma must be constant along a field line, at the end of the plasma the confinement potential must equal the plasma potential.

With these assumptions, the plasma potential at the plasma's end is

$$\phi_p(\mathbf{r}) = V_p/2+(\pi ne/2)\,|\mathbf{r}-y_d\hat{\mathbf{y}}|^2-\pi ne r_p^2 \ln(1-yy_d/r_w^2), \tag{1}$$

where $\mathbf{r}$ and y are measured from the trap center, n is the plasma density, V_p is the voltage at the device center when the plasma is centered, and the third term represents the potential arising from the plasma's image charge when $y_d \ll r_w$. Equation (1) can be simplified to yield

$$\phi_p(\mathbf{r}) = \frac{V_p}{2}\left(1-\frac{|\mathbf{r}-y_d\hat{\mathbf{y}}|^2}{r_p^2[1+2\ln(r_w/r_p)]}-\frac{2yy_d}{r_w^2[1+2\ln(r_w/r_p)]}\right). \tag{2}$$

The potential from the confining cylinder is

$$\phi_c(\mathbf{r}) = V_g e^{\chi_1 z/r_w} J_0(\chi_1 r/r_w), \tag{3}$$

where J_0 is a Bessel function of the first kind, χ_1 is the first root of the equation $J_0(x)=0$, and we assume that the plasma's end is sufficiently far from the confining cylinder

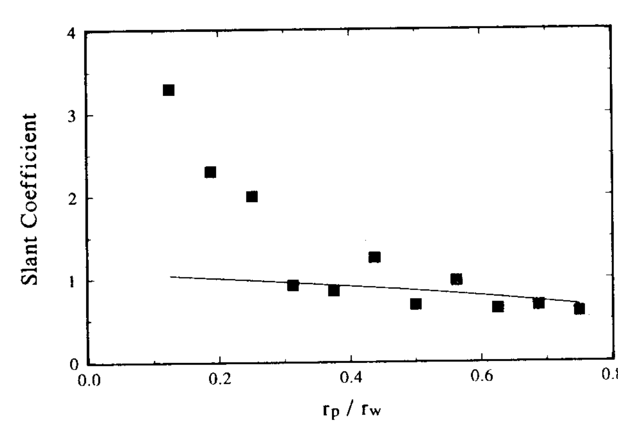

FIG. 3. Theoretically predicted and numerically calculated slant coefficient versus plasma radius.

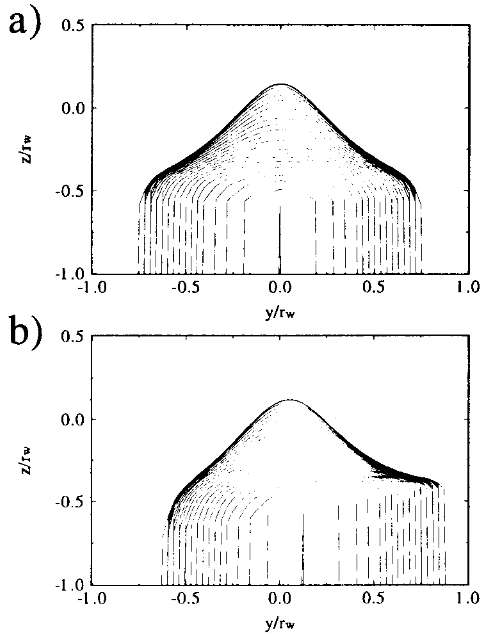

FIG. 4. Shapes of plasmas with $r_p/r_w=0.75$, $V_g=1.25V_p$, and $y_d/r_w=0.0$ (a) and $y_d/r_w=0.125$ (b).

that all but the first term in the Bessel series can be neglected. Further assuming that the Bessel function argument is small, we can write

$$\phi_c(\mathbf{r})\approx(V_p/2)(1-\chi_1^2r^2/4r_w^2). \tag{4}$$

Note that for a centered plasma ($y_d=0$), the end shape will be flat if the confinement and plasma potentials are equal for all $\mathbf{r}$ at a unique value of z. This condition is met when $\phi_p(\mathbf{r})-\phi_c(\mathbf{r})=0$, or $r_p\approx0.57r_w$, and is verified approximately by Fig. 2. The end shapes of plasmas with radii larger than this critical value are concave near the trap axis.[15]

The above formulas are exact only for flat, unslanted plasmas, but they are a reasonable approximation for plasmas with a variety of end shapes. The potential of a point fixed in the plasma changes as

$$\frac{\partial\phi_p}{\partial y_d}=\frac{-V_py}{r_w^2[1+2\ln(r_w/r_p)]}, \tag{5}$$

$$\frac{\partial\phi_c}{\partial y_d}=-\frac{V_p\chi_1^2y}{4r_w^2}, \tag{6}$$

when the plasma is displaced from the axis. To compensate, the plasma's end shifts by $\Delta z=\theta_sy$. The slant angle θ_s is found by requiring that the plasma and confinement potentials remain matched at the plasma's end: $\Delta y_d(\partial\phi_p/\partial y_d)=\Delta y_d(\partial\phi_c/\partial y_d)+\Delta z(\partial\phi_c/\partial z)$. Noting that $\partial\phi_c/\partial z=(\chi_1/r_w)\phi_c\approx(\chi_1V_p/2r_w)$, we find that

$$\theta_s=\frac{\Delta y_d}{r_w}\left(\frac{\chi_1}{2}-\frac{2}{\chi_1[1+2\ln(r_w/r_p)]}\right). \tag{7}$$

In essence, the first term on the right-hand side yields the local slope of the vacuum potential contours. The second term is the correction from the plasma potential, and has the same functional dependence on Δy_d because both the plasma confinement potential and the plasma potential are parabolic. From Eq. (7) we identify the slant coefficient as the factor multiplying $\Delta y_d/r_w$. Figure 3 shows the theoretically predicted and numerically calculated values of the slant coefficient as a function of the plasma radius. For wide plasmas the agreement is quite good given the many approximations used to obtain the theoretical result. The poor agreement found for narrow plasmas may occur because the end shapes of these plasmas are far from flat.

The above description of the plasma's end shape depends on the approximation used for the confinement potential in Eq. (3). If $V_g\geqslant3V_p$, this approximation is accurate and the plasma's end shape is independent of the plasma density and the confinement cylinder potential. Figure 4 shows the end shapes of a centered and displaced plasma for which $V_g=1.25V_p$. Such ill-confined plasmas never have a flat end; they always protrude into the confining cylinder near the trap axis.[16]

ACKNOWLEDGMENTS

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