Asymmetric Stable Equilibria of Non-Neutral Plasmas

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A pure electron plasma, confined within an azimuthally symmetric boundary by a coaxial magnetic field, has an equilibrium shape which is cylindrical. We apply perturbations which break the azimuthal symmetry and deform the plasma into a noncircular shape that is stationary in the laboratory frame. These asymmetric equilibria form a broad new class of stable equilibria. A theoretical model correctly predicts the plasma shapes and explains their stability.

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Pure electron plasmas exhibit many unusual confinement properties, and have been extensively studied [1]. The typical plasma confinement geometry is azimuthally symmetric, and, consequently, the plasmas assume an azimuthally symmetric equilibrium shape. We find that when the wall boundary conditions are asymmetric, that is, when the boundary is not an equipotential surface, the plasma equilibrium shapes are themselves asymmetric. These new equilibria exhibit several remarkable features, and require a new confinement paradigm to explain their long confinement times.

Our plasma trap (Fig. 1) consists of a set of three conducting cylinders (radii of 1.9 cm). The plasma is confined within the center cylinder. The electrons are radially confined by a strong (1 kG) coaxial magnetic field, and axially confined in the potential well formed by negatively biasing the two end cylinders. The grounded center cylinder is equipped with three electrically isolated angular patches. Biasing these patches produces the asymmetric wall boundary conditions which result in the asymmetric equilibria [2]. The trap described above is similar to other pure electron plasma traps [3].

The plasma electron dynamics is characterized by several time scales. The cyclotron orbit time scale ($\tau \sim 10^{-10}$ s) is so fast that we can use the guiding center approximation. The axial bounce motion along the magnetic field is also fast ($\tau \sim 10^{-7}$ s); consequently, the electric field experienced by the particle can be approximated by the bounce-averaged electric field. Thus, the dynamics is primarily the two-dimensional, bounce-averaged $\mathbf{E}\times\mathbf{B}$ drifts. For example, when the angular patches are grounded, the plasma's self-electric-field is entirely radial, so that the plasma rotates due to the azimuthal $\mathbf{E}\times\mathbf{B}$ drifts.

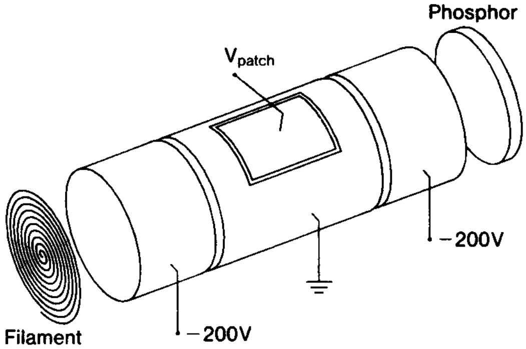

FIG. 1. Diagram of the confining cylinders showing one of the angular patches. The two end cylinders are negatively biased so as to confine the plasma within the center cylinder.

The experimental procedure for obtaining noncircular equilibria is as follows. First, one of the negatively biased end cylinders is momentarily grounded, allowing a cylindrical plasma to flow from a thermionically emitting tungsten filament into the central confining cylinder. Next, the plasma is trapped by restoring the grounded end cylinder to its usual negative bias. The boundary is then perturbed by slowly biasing the desired patches. (Here "slow" means that the biases change by a small fraction during the plasma $\mathbf{E}\times\mathbf{B}$ drift period.) The plasma responds to these biases by assuming an asymmetric shape. Finally, this shape is imaged by briefly grounding one of the negatively biased end cylinders. The unconfined electrons then rapidly spill out along the magnetic field lines, and are accelerated onto a phosphor screen. The resulting image is recorded with a charge-coupled-device camera. Some typical images are shown in Fig. 2. In these experiments the electron density is $n \approx 3\times10^{7}$ cm^{-3}, the temperature is $T \approx 3$ eV, the plasma length is $l \approx 3$ cm, the initial plasma radius is $r_0 = 1.2$ cm, and a typical patch bias is 20 V.

The asymmetric plasma equilibria have many interesting properties. For example, although the plasma itself is stationary in the laboratory frame, individual electrons follow long and convoluted paths. Since these paths are determined by the $\mathbf{E}\times\mathbf{B}$ drifts, the electrons move perpendicular to the electric field, i.e., along the equipotential contours. These contours result from the sum of the self-consistent plasma potentials and the externally applied boundary wall potentials, and may be very irregu-

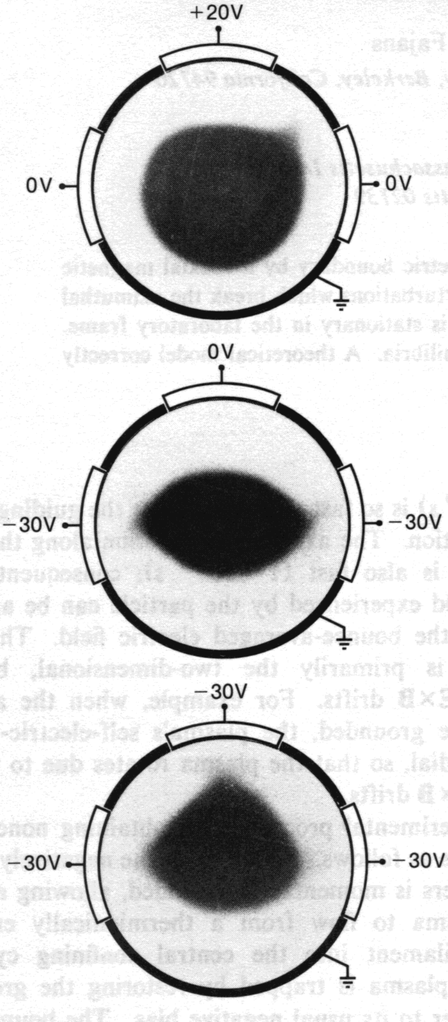

FIG. 2. Three experimentally observed noncircular plasma equilibrium shapes.

lar. In the experiments reported here, the electron density is constant along individual potential contours so that the equipotential contours and the bulk plasma shape remain stationary.

Figure 2 illustrates the surprising result that negative voltages on the angular patches attract the electron plasma, while positive voltages repel it. This phenomenon is explained by considering the behavior of the distorted equipotential contours. When a positive voltage is applied to an angular patch, there is a larger potential gradient between the plasma center (at a negative potential) and the angular patch than between the plasma center and the grounded outer wall. This increased gradient necessarily deforms the equipotential contours by pushing them away from the patch; hence the plasma edge is repelled from a positive angular patch. Similarly, a negatively biased patch diminishes the local potential gradient, decreasing the number of equipotential lines, and thereby attracting the plasma.

The unperturbed plasma equipotentials are closed; however, when biases are applied to the patches, the contours become unclosed. As shown in Fig. 4, a separatrix divides the closed and unclosed equipotentials. Electrons which cross this separatrix are immediately swept to the wall and lost. This effect is seen in long time confinement studies of these asymmetric equilibria. For several seconds the plasma expands slowly (perhaps by scattering off of background gas molecules) without loss of total charge. After the plasma has expanded to the separatrix, further expansion leads to immediate charge loss. Alternatively, if a sufficiently large perturbing bias is slowly applied to an angular patch, the separatrix constricts to enclose the same area as the initial plasma. Any larger bias causes the separatrix to cross into the plasma, resulting in the rapid loss of those electrons outside of the separatrix.

When a bias is applied to the angular patch, sufficient charge density is necessary for plasma confinement. In the limit of infinitesimal plasma density, a small patch potential makes all equipotential contours open. To obtain closed equipotentials, the plasma must have sufficient charge density to produce a local minimum in the combined plasma and boundary potential. Note that the minimum charge density is a complicated function of both the perturbing voltage and the initial plasma radius.

Asymmetric plasma equilibria can be used to fix the initial conditions for observing the subsequent evolution of an initially noncircular plasma. When the perturbing voltages on the angular patches are instantly reduced to zero, the plasma shape is no longer stationary; it proceeds to evolve within the symmetric confining voltages. Since the noncircular shapes can be decomposed into $l \geq 1$ diocotron [4,5] mode amplitudes, this technique can be used to observe the evolution of a diocotron mode of prescribed initial amplitude and phase.

The long confinement times of these asymmetric plasmas (typically 10 s, compared with 200 s for the unperturbed case) are not predicted by the usual non-neutral plasma confinement paradigm. Traditionally, the confinement of non-neutral plasmas is attributed to the azimuthal symmetry of the confinement geometry [6]. Such symmetry assures that the canonical angular momentum P_θ is conserved. In the limit of a strong magnetic field, $P_\theta = (-eB/2c)\sum_i r_i^2$, where the sum is over all plasma electrons. Thus, the constancy of angular momentum limits the radial expansion of the plasma. In the experiments described here, there is no such symmetry, and consequently angular momentum conservation cannot be used to explain the long confinement times. Future experiments will investigate the effect of asymmetries on the plasma confinement time.

The stability of these energy-conserving non-neutral plasmas has been investigated by O'Neil and Smith [7] using variational techniques. The plasma is locally stable to $\mathbf{E}\times\mathbf{B}$ dynamics if its energy is extremal when compared to the energy of all the nearby accessible states. Below, we derive an expression for the distortion of the unperturbed equilibrium as a function of the boundary perturbation [Eq. (4)]. We have also shown [8] that, in the limit of small perturbations, the distortions predicted by Eq. (4) yield energy maxima. That is, the plasma deforms to the shape that maximizes the energy consistent with keeping its area invariant. Consequently, **E**×**B** mechanisms are not responsible for the observed slow loss of plasma. This maximal energy principle also explains why the negatively charged plasma is attracted to negative imposed potentials and repelled from positive imposed potentials; such movements clearly maximize its energy.

The model which predicts the shapes of these plasmas is based on the fact that the guiding centers drift along the equipotential contours. The stationary shape condition is satisfied if the plasma surface lies on an equipotential contour:

$$\phi(r_p(\theta),\theta)=\phi_p(r_p(\theta),\theta)+\phi_a(r_p(\theta),\theta)=\mathrm{const}\,, \qquad (1)$$

where $r_p(\theta)$ specifies the shape of the uniform density plasma, ϕ_p is the potential resulting from the charge of the plasma itself, and ϕ_a is the potential due to the applied patch perturbations. Although ϕ_p can be exactly expressed in integral form as a convolution of the density and a Green function, it is, for our parameters, well approximated by the potential of a circular plasma with the appropriate surface charge density, $\sigma(\theta)=n\delta r(\theta)$. Here $\delta r(\theta)$ is the deviation of the asymmetric plasma from a circular plasma of radius r_0, and may be Fourier decomposed as

$$r_p(\theta)=r_0+\delta r(\theta)=r_0+\mathrm{Re}\left[\sum_{l=1}^{\infty}\delta r_l e^{il\theta}\right]. \qquad (2)$$

This approximation is valid when $\delta r \ll r_0$. The potential field can then be easily found both inside and outside of the cylindrical column by employing Gauss's law to match the discontinuous electric field across the surface charge.

The applied electrostatic potential ϕ_a can also be expressed as a Fourier series,

$$\phi_a(r,\theta)=A_0+\mathrm{Re}\left[\sum_{l=1}^{\infty}A_l\left(\frac{r}{R}\right)^l e^{il\theta}\right], \qquad (3)$$

where the complex coefficients A_l are determined from the known patch biases. Thus the total field, $\phi=\phi_p+\phi_a$, can be expressed in terms of the applied perturbation, $\phi_a(R,\theta)$, and the shape of the plasma column, δr_l. By employing the equilibrium condition [Eq. (1)], and keeping only the lowest-order terms in δr, we find that the shape of the asymmetric plasma column in terms of the boundary perturbation is

$$\delta r_l=\frac{(r_0/R)^l}{2\pi enr_0\{1-[1-(r_0/R)^{2l}]/l\}}A_l\,. \qquad (4)$$

Figure 3 shows the plasma shape predicted by this approximate model. The model's success is illustrated by the close agreement of this shape with the corresponding shape of Fig. 2. The length of the angular patches is slightly less than the diameter of the confining cylinder; to compensate for this three-dimensional effect within the two-dimensional theory, the values of the patch potential used in the analytic calculations are reduced by a factor of 0.6 from the experimental values. This numerical factor was obtained by comparing the effect of an infinitely long sector to the bounce-averaged effect of a finite sector.

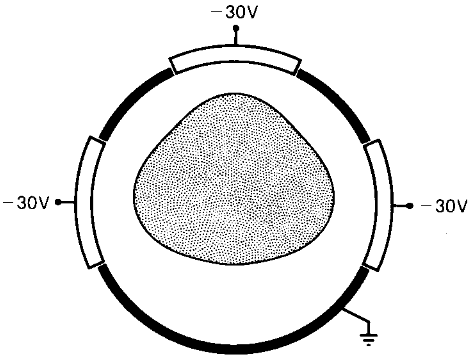

FIG. 3. Plasma edge as determined by the theoretical model.

In addition to the simple analytic theory presented here, a two-dimensional vortex-in-cell electrostatic code has been used to simulate the experiments. An assembly of 4096 line charges is used to model the plasma. The simulation is time advanced by first weighting the plasma to a uniform polar grid, then solving the Poisson equation, and finally, moving the line charges according to the **E**×**B** dynamics. The noncircular shape is obtained by slowly applying the external fields to the initially circular plasma. A comparison of Figs. 2 and 4 shows the good agreement between the experimental and simulated plasma shapes.

In conclusion, we find that stable asymmetric plasma equilibria are easily observed experimentally. The shapes agree with both theoretical models and numerical simulations. The long lifetimes of these asymmetric equilibria

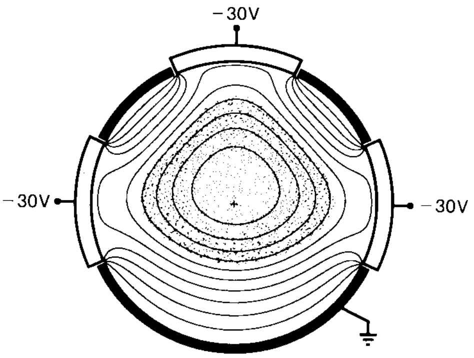

FIG. 4. Equipotential lines and particle positions obtained from the simulation. The separatrix lies between the closed and unclosed equipotential lines.

challenge the basic confinement paradigm of non-neutral plasmas, but may partially be explained by their stability within the $\mathbf{E}\times\mathbf{B}$ model. Since two-dimensional $\mathbf{E}\times\mathbf{B}$ plasma dynamics are identical to two-dimensional, inviscid, incompressible fluid dynamics, many of our results can be generalized to vortex dynamics. Our results may also explain the long confinement times observed in the square traps used in Fourier-transform mass spectrometers [9,10].

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