Penning-Malmberg and Minimum-B Trap Compatibility: the Advantages of Higher-Order Multipole Traps

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Abstract. The ATHENA and ATRAP collaborations have recently created large numbers of untrapped anti-hydrogen atoms. The most commonly suggested scheme for trapping the anti-hydrogen is to use a Minimum-B trap. Unfortunately, the Minimum-B fields are very likely to destroy the confinement of the anti-hydrogen constituents; the positrons and anti-protons, which are themselves held in double-well Penning-Malmberg traps. The reasons for the loss of confinement, and modifications to the Minimum-B trap that may alleviate this problem, are discussed in this paper.

INTRODUCTION

Both ATHENA and ATRAP have had remarkable success generating anti-hydrogen ($\overline{\mathrm{H}}$) [1, 2, 3]. The two experiments differ in details, but both experiments employ double-well, Penning-Malmberg traps to hold and cool the anti-hydrogen constituents: positrons (e^+), and anti-protons ($\overline{p}$). In the experiments considered here, temperature effects or "sloshing" are used to make the anti-hydrogen constituents overlap, and, occasionally, form anti-hydrogen through radiative or three-body recombination. Being neutral, the anti-hydrogen is not confined in the Penning-Malmberg trap. Consequently, the anti-hydrogen lasts only until it is carried into the trap wall by its initial momentum.

Because the anti-hydrogen is generated inside the Penning-Malmberg trap, the Minimum-B fields must be superimposed on the Penning-Malmberg fields. Unfortunately, simple Minimum-B fields (see Fig. 1) are likely to destroy the confinement of the anti-hydrogen constituents, e^+ and $\overline{p}$, in the Penning-Malmberg traps; anti-hydrogen may not have enough time to be formed before the positrons and anti-protons are lost. However, using high-order multipole fields may minimize the deleterious effects of the Minimum-B fields on the constituent confinement.

ATHENA AND ATRAP PARAMETERS

For both ATHENA and ATRAP, the positron Debye length is much less than the positron column radius or length; thus the positrons are in the collective, or plasma regime. The anti-proton Debye lengths are only somewhat smaller (factors of two to seven) than the anti-proton column lengths and radii. Thus, the anti-protons are also in the plasma regime, but less robustly than the positrons. Note that both the positrons and the anti-

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protons are far from the Brillouin limit, and diamagnetic effects are unimportant.

EFFECT OF THE MIRROR FIELD

Axial $\bar{H}$ confinement in Minimum-B traps is provided by a "mirror" field that peaks the field at the trap ends. Such fields can be generated by two end coils, as shown in Fig. 1. Both the e^+ and $\bar{p}$ equilibria and transport will be affected by the mirror fields. Equilibria in mirror fields have been studied experimentally [4] and theoretically [5]. The equilibria are substantially more complicated than those in a simple solenoidal field. They may not be as advantageous. For example, the constituent densities will be highest at the ends of the trap, underneath the mirror coils, not in the center, where the $\bar{H}$ generation takes place.

A more serious consequence of the mirror field is increased transport. Kabantsev and Driscoll [6] have shown that transport increases dramatically with axial magnetic field variations. The transport is thought to be due to "Trapped Particle Scattering:" [7] scattering across trapped/untrapped particle separatrices. Kabantsev and Driscoll measured a five-fold increase in the transport for a 0.1% field variation. Given that a satisfactory Minimum-B trap must have at least a 100% variation in the field, this increase could be devastating; a linear extrapolation predicts that the transport would increase by a factor of 5000. It is unlikely that such high transport could be controlled by the rotating wall technique [8]. However, a 100% field variation is beyond the linear regime, and the nonlinear transport scaling is currently unknown.

Moving the mirror field coils out beyond the extent of the Penning-Malmberg trap would prevent this transport. This solution is not ideal as the volume into which the $\bar{H}$ is trapped would increase.

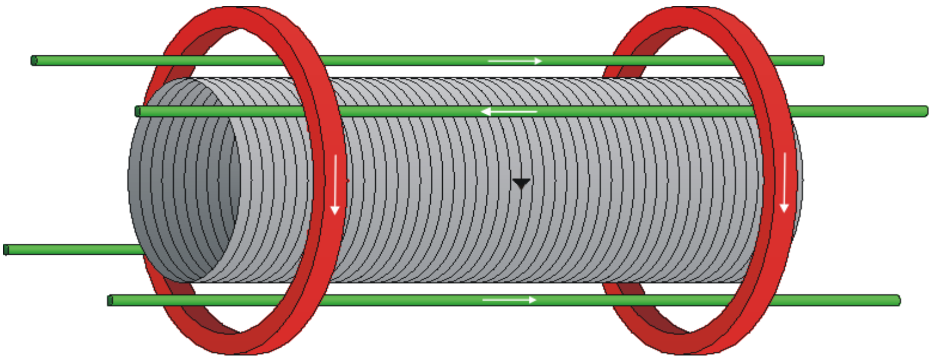

FIGURE 1. A quadrupole Minimum-B/Malmberg-Penning trap. The solenoid produces the axial field for the Penning-Malmberg trap, the two end coils generate the mirror field that confines $\bar{H}$ axially, and the four quadrupole wires generate the field that confines $\bar{H}$ radially. This configuration is similar to a Joffe configuration. To trap $\bar{H}$, the mirror coils must be positioned so that the minimum in the field extends over the overlap region.

EFFECT OF THE MULTIPOLE FIELD

Radial neutral-particle confinement requires a magnetic field that increases with radius. Such fields can be generated by quadrupole or higher-order magnetic multipoles. The equilibria and transport in multipole fields are quite complicated. The magnetic field lines associated with the quadrupole superimposed on the Penning-Malmberg trap are shown in Fig. 2. The shape of the magnetic field will create elliptically shaped plasma, which may be unstable.

As with the mirror field, the most significant effect of the multipole field is likely to be increased transport. Experiments have demonstrated sharply increased transport with very small quadrupole fields; quadrupolar fields at the plasma edge 4000 times lower than solenoidal field have been shown to double the outward diffusion [9]. Preliminary data indicates that the diffusion increases linearly with quadrupole field strength when the quadrupole is strong [10]. To form a significant Minimum-B well, the quadrupole field would have to be comparable to the solenoidal field. Thus, the quadrupole might enhance diffusion by a factor of 4000.

The quadrupolar induced transport is strongest in the neighborhood of a resonance. Experimental data [9, 11] suggests that the resonance occurs when the ratio nL/Bv_{T} remains constant, where n is the plasma density, L is the plasma length, B is the solenoidal field strength, and $v_{\mathrm{T}} = \sqrt{kT/m}$ is the thermal velocity. This scaling is consistent with an orbital resonance; if a particle rotates by 90 degrees during the time it takes to make one trap transit, then it can be on a trajectory that goes ever outwards or inwards [9]. While such resonant particles would be lost, there are too few precisely-resonant particles to induce significant transport. However, there is a broad class of particles that are near resonance, which undergo large radial excursions. If these particles collide, their excursions will cause them to diffuse. The detailed theory of this Resonant Particle Transport is difficult to construct, nonetheless, a back-of-the-envelope theory

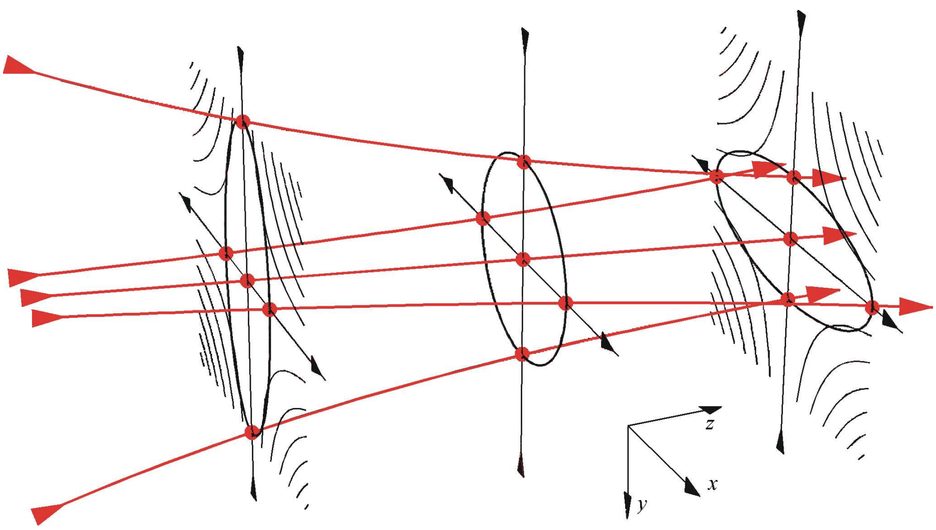

FIGURE 2. Net field lines in the presence of a quadrupole and a solenoid.

predicts transport of the right order of magnitude [11]. However, the observed position of the resonance is off by a factor of two from the expected position. This discrepancy has lead Kabantsev and Driscoll to suggest [6] that the transport is more properly described by their Trapped Particle Scattering mechanism.

In the slowly rotating regime relevant here, the observed diffusion is inversely proportional to f_{R}^2. If the ATHENA and ATRAP plasmas were off resonance by a factor of more than 50, the diffusion would be down to tolerable levels. Unfortunately, they are much closer than this to resonance, and the diffusion is likely to be very large.

COLLISIONS

Both Trapped Particle Scattering Transport and Resonant Particle Transport occur only when the plasma particle orbits are disturbed by collisions. Unfortunately, the transport theories are not sufficiently well developed to predict the scaling with collision frequency. Moreover the collision frequencies are complicated to predict because of O'Neil's collisional adiabatic invariant [12, 13]. Nonetheless, the collision frequencies appear to be easily great enough to induce transport [14].

A recent paper [15] contends that, because particles in Minimum-B fields orbit on unique, stable trajectories, transport will be insignificant. But the collisions described here disturb the orbits; proof of the stability of individual particle orbits is largely irrelevant to the issues of transport and loss.

HIGH-ORDER MULTIPOLE ADVANTAGES.

As shown above, quadrupole fields pose profound problems for e^+ and $\bar{\mathrm{p}}$ confinement. These problems may be alleviated by using higher-order multipole fields. The field from an infinitely-long multipole of order s scales with radius r as

$$|B| = B_{\mathrm{max}} \left(\frac{r}{R_{\mathrm{w}}} \right)^{s-1}, \tag{1}$$

where B_{max} is the field at the wire radius R_{w}. For large order multipoles, the field is very small at the center. If a trap can be configured so that the e^+ and $\bar{\mathrm{p}}$ column radii are small compared to the wall radius, the constituents would be largely unaffected by the weak multipole field at the center. The resulting $\bar{\mathrm{H}}$ would still be trapped by the strong multipole field at the wall. For example, if the constituent radii were one third of the wall radius, the multipole field magnitude were equal to the solenoidal field magnitude at the wall, and the multipole order was ten, the maximum field experienced by the constituents would be twenty thousand times less than the solenoidal field. Such multipole fields would be on the same order as the fields used in the quadrupole experiments [9], a level that is probably tolerable. Finite length effects and the addition of the mirror field complicate this picture somewhat, and the fields are best calculated numerically. The fields from a typical trial configuration are shown in Fig. 3.

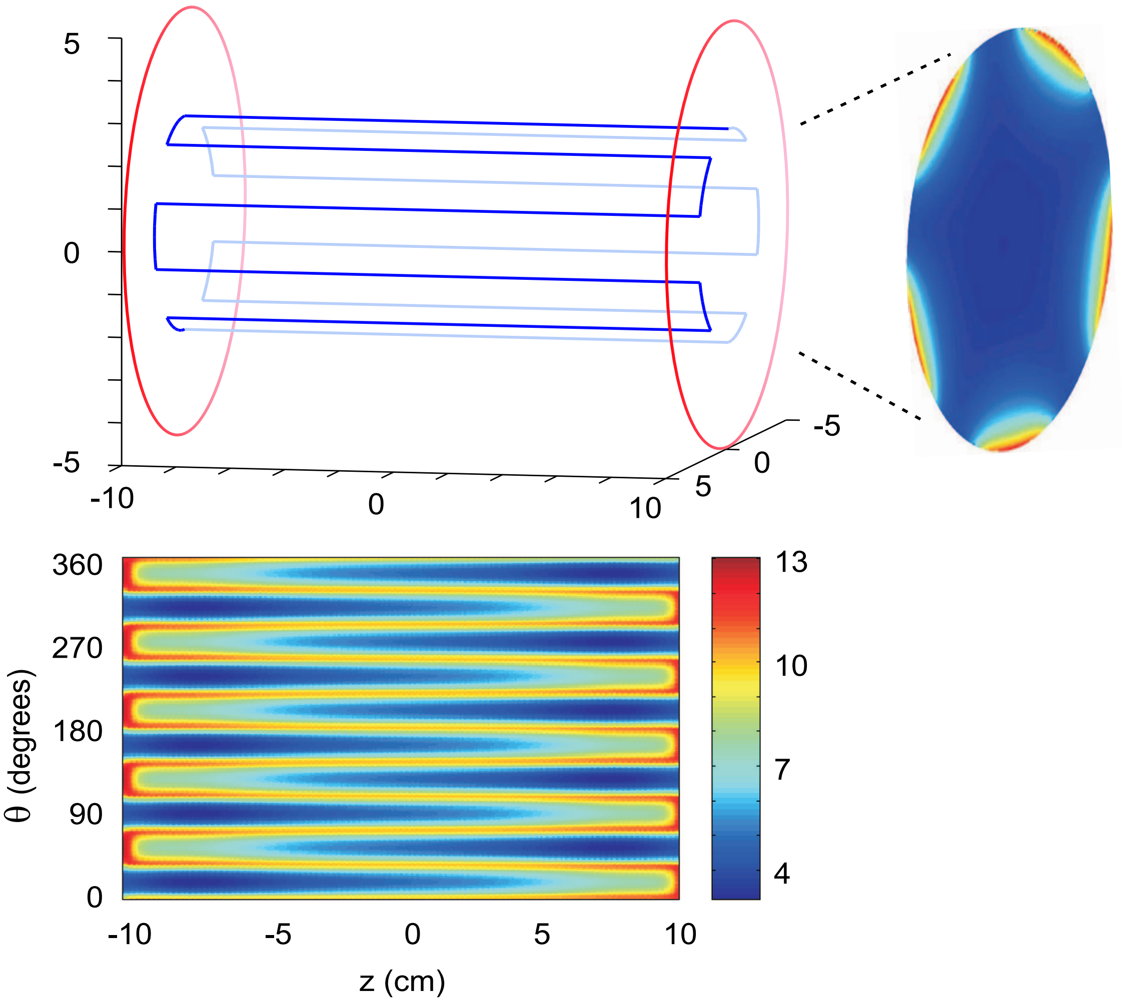

FIGURE 3. Boundary magnetic field values for the configuration shown at top left. All magnetic field values are normalized to the field at the center. The multipole is fifth order with wires at radius 2.5 cm. The mirror coils are of radius 5 cm. The ratio of current in the multipole to current in the mirror coils is 1.45. In addition, there is an infinite solenoid, oriented along the configuration axis, which contributes 80% of the field in the center of the trap. The plot at the left shows the field at the end, and the bottom plot shows the "unwrapped" field along on the sides at 90% of the radius of the multipole wires, i.e. a cylinder of radius 2.25 cm. The minimum field occurs about 14% of the way in from the end on the side, and is of magnitude 3.16.

One might worry that the greater degree of cancellation found in higher order multipoles would require larger currents for the same field increase. An elementary calculation shows that this is not true. Assuming a sinusoidal current distribution, the required current density J_λ, in (A/m), is

$$J_\lambda = \frac{2B_{\max}}{\mu_0} \sin s\theta, \tag{2}$$

where $B_{\max}$ is in Tesla. On gathering the current into $2s$ wires, the current in each wire drops proportional to $1/s$, a favorable outcome. As an added benefit, the magnetic forces on each wire also decrease proportionally. Nonetheless, the currents, and hence the forces, are quite large; a tenth order multipole that produces 1 T at wires arrayed at $R_w = 1$ cm requires currents on the order of 4000 A. It may be easier to create the required fields with permanent magnets; fortuitously, high-order multipole fields are easier to create with permanent magnets than are quadrupole fields.

Increasing the order of the multipole moves the multipole resonance to proportionally higher velocities. This can be favorable or unfavorable depending on the plasma parameters; it is likely to be favorable for ATHENA e^+, but unfavorable for ATRAP e^+.

A disadvantage of higher order multipoles is that the $\bar{H}$ trapping volume is relatively large; the multipole fields are not significant until the $\bar{H}$ move far off the axis. This problem could be solved by turning on an auxiliary quadrupole to more tightly confine the $\bar{H}$ once a sufficient number have been created and trapped in the high order multipole.

CAVEATS

The loss of confinement predicted here is based on significant extrapolations from the current experimental data. The transport could saturate. The experiments and theory to date have studied mirror and quadrupole configurations separately. It is conceivable, but unlikely, that each will ameliorate the effects of the other. The model for the multipole transport is based on orbital resonances. The effects of the magnetron fields are difficult to calculate. Further, particularly for the $\bar{p}$, the rotation frequency is difficult to determine in the $\bar{p}$-e^+overlap region. There may be very odd effects there. Finally, Kabantsev and Driscoll suggest that the proper mechanism for quadrupole induced transport is their Trapped Particle Scattering, not the Resonant Particle Diffusion mechanism presented here. However, the extrapolations are based on observational scalings, not theoretical predictions.

CONCLUSIONS

The tentative plans to trap $\bar{H}$ in ATHENA and ATRAP need to be refined. The proposed mirror and quadrupole field are very likely to destroy the confinement of the $\bar{H}$ constituents, e^+ and $\bar{p}$. Pushing the mirror field out beyond the constituent confinement region, and using a high order multipole, may alleviate the $\bar{H}$ constituent confinement problems. More exotic schemes are also available [16].

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