

## **Workshop on Non-neutral Plasmas\***

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### **Abstract**

A summary of the Workshop on Non-neutral Plasmas, held in Boulder, Colorado, on July 29 through August 1, 1997, is given. This workshop provided an opportunity for the exchange of results and scientific information among the diverse areas of plasma physics, atomic physics, and chemistry, which use non-neutral plasmas.

### **Key Words**

non-neutral plasmas, anti-matter plasmas; dusty plasmas; vortex dynamics, plasma thermal equilibrium; strongly coupled plasmas

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A Workshop on Non-neutral Plasmas took place on the Boulder campus of the University of Colorado from July 29 to August 1, 1997. The workshop was sponsored by the US Office of Naval Research, the National Science Foundation, the Center for Integrated Plasma Studies of the University of Colorado, and the National Institute of Standards and Technology (NIST). This workshop was the 4th in a series of workshops on this subject.<sup>1,2,3</sup> Like its predecessors, it focused on a broad range of topics on non-neutral plasmas in the diverse areas of plasma physics, atomic physics, and chemistry. Subjects which were discussed included transport, 2 dimensional (2D) fluid studies, thermodynamics of non-neutral plasmas, anti-matter plasmas, strongly coupled plasmas including laser-cooled ion plasmas and dusty plasmas, ion cyclotron resonance (ICR) mass spectroscopy, and non-neutral beams. Significant progress since the last workshop was noted in the above areas. We summarize some of this progress below. (The complete workshop program can be obtained from the conference abstract booklet.<sup>4</sup>)

**Transport.** With a beautiful, intertwined series of experiments and analytic results, Francois Anderegg and Dan Dubin [with others at the University of California at San Diego (UCSD)] presented a broad understanding of collisional transport in non-neutral, magnetized plasmas. Here the focus is on the transport of heat and particles across the magnetic fields, due to binary “collisions” between particles in a quiescent plasma. Experiments have directly measured the diffusion of “tagged” particles, the bulk transport of particles due to viscous forces, and the relaxation of thermal gradients. Comparison with theory demonstrates the importance of novel long-range collisions in these processes.

Non-neutral plasmas are typically in the guiding-center regime ( $r_c \ll \lambda_D$ , where  $r_c$  is the radius of the cyclotron orbit and  $\lambda_D$  is the Debye length). In this regime the classical collisional transport coefficients, derived decades ago for neutral plasmas, are irrelevant because classical transport theory implicitly assumes the opposite ordering ( $r_c \gg \lambda_D$ ). The classical theory envisions a transport process wherein a guiding center steps by a cyclotron radius following a close collision that scatters the velocity vector. Such collisions occur only with impact parameter  $\rho < r_c$ . In the guiding-center regime, however, there are many more collisions with impact parameters in the range  $r_c \ll \rho \leq \lambda_D$  than collisions with impact parameters in the classical range  $\rho < r_c$ . These guiding-center interactions produce a larger (sometimes much larger) contribution to the transport than do the close collisions treated by the classical theory. In the guiding-center regime, the plasma particles make a series of uncorrelated steps across the magnetic field. The steps are caused by the  $\mathbf{E} \times \mathbf{B}$  drifts arising from Coulomb interactions between guiding centers.

The diffusion, thermal conductivity, and viscosity coefficients scale differently with density  $n$ , magnetic field  $B$ , and temperature  $T$ . Relative to the classical transport coefficients, the UCSD group observed a factor-of-10 enhancement in test particle diffusion,<sup>5</sup> an enhancement of up to  $10^4$  in bulk viscous transport,<sup>6</sup> and an enhancement of up to 200 in heat transport.<sup>7</sup>

Because all of these transport processes occur in cylindrically symmetric systems, they all conserve angular momentum, and cannot produce any global (expansive) transport. Although experimentalists have learned to control global transport by applying rotating asymmetric potentials, the mechanisms underlying global transport remain mysterious. Recent single particle measurements<sup>8</sup> by Dennis Eggleston (Occidental College) suggest that the transport may be due to resonant particles, but the question remains wide open, both experimentally and theoretically.

**2D fluid studies.** Fred Driscoll (UCSD) gave an overview of 2D fluid experiments<sup>9,10</sup> undertaken

with pure electron plasmas. Experiments on diocotron waves, the Kelvin-Helmholtz instability, and vortex merger have established that, at least under some conditions, the behavior of pure electron plasmas is similar to that of inviscid 2D fluids. Experiments with pure electron plasmas have several advantages over experiments with real fluids like water: the effective viscosity of the plasmas is much lower than that of most real fluids, complex initial conditions are easier to set up, and plasma “vorticity” is directly measurable. However, the behavior of plasma systems is not absolutely identical to that of inviscid 2D fluids,<sup>11</sup> and the important differences were summarized.

One of the most unexpected results in recent years was the observation of the decay of initially turbulent vortex states into vortex crystals.<sup>10</sup> Here, the expected relaxation of 2D turbulence is “arrested” by the formation of an ordered lattice of vortices. This phenomenon is now understood to be an effect which should occur in all low-viscosity, incompressible 2D fluids. David Schecter (UCSD) reported on computer simulations that exhibit vortex crystal formation and indicate that the phenomenon is caused by vortex cooling on the low background electron density (vorticity).

**Shielding.** Shielding in collisionless, highly magnetized plasmas is paradoxical. Since electrons speed up in the vicinity of a positive test charge, the local electron density near the test charge would appear to *decrease*. However shielding requires that the electron density *increase*. Thus the test particle would seem to be anti-shielded rather than shielded. This paradox is resolved by including the effects of electrons trapped near the test particle. The mechanism for trapping particles in collisional plasmas is immediately apparent, but the trapping mechanism in collisionless particles is less apparent. Joel Fajans and colleagues (University of California at Berkeley) have investigated<sup>12</sup> these effects in pure electron plasma experiments (their results apply to neutral plasmas as well) and have found that the response to a positive test particle can be divided into four different regimes: Anti-shielding—Under proper circumstances, the plasma will indeed antishield a positive test particle. The plasma response enhances the field of the test particle. Instantaneous Shielding—When the test particle is introduced quickly compared to the plasma electron transit time, the test particle will be shielded, but this shielding can be weak. Adiabatic Shielding—When the test particle is introduced slowly compared to the electron transit time but fast compared to the electron collision time, electrons will be trapped near the test charge adiabatically. The test charge will be shielded. Debye Shielding—The standard collisional regime which is well understood.

**Thermal equilibrium.** An important difference between magnetically confined neutral plasmas and non-neutral plasmas is that non-neutral plasmas can relax to a confined global thermal equilibrium state. This results in long confinement times and the possibility of using thermodynamics in the study of non-neutral plasmas. Tom O’Neil reported on a study with Dan Dubin (UCSD) on the role thermodynamics can play in trapped non-neutral plasmas.

O’Neil emphasized that a thermodynamic approach can greatly simplify the description of slow plasma evolution due to such effects as collisions with neutrals, interaction with field errors, and interaction with laser cooling and torque beams. If the change in the plasma energy and angular momentum ( $E$  and  $L$ ) is slow compared to the time required for Coulomb collisions to bring the plasma to thermal equilibrium, then the plasma evolves through a sequence of thermal equilibrium states, and the slow change in  $E$  and  $L$  translates to a slow change in temperature and rotation frequency ( $T$  and  $\omega$ ). The thermodynamic approach describes the evolution using two

first-order ordinary differential equations for the time evolution of  $T$  and  $\omega$ , which is a major simplification from the partial differential equations typically required to describe transport.

Ross Spencer (Brigham Young University) discussed theoretical work he and his colleagues have done modeling the equilibrium and modes of flat, pancake-shaped plasmas in Penning traps. This work is potentially important for atomic physics experiments using Penning traps because, over long periods of time, trapped, charged particles evolve into pancake-shaped plasmas.

**Anti-matter plasmas.** Non-neutral plasma techniques have been used to trap anti-particles that form anti-matter plasmas. Rod Greaves (UCSD) reported on work done in collaboration with Cliff Surko and their coworkers (UCSD) on positron plasmas.<sup>13</sup> They are able to accumulate up to  $10^8$  positrons in a Penning-Malmberg trap and store them for several hours. Of interest to plasma physics is the study of positron/electron plasmas. Greaves described the first laboratory electron-positron plasma experiment by transmitting an electron beam through a stored positron plasma.<sup>14</sup> Notable observations included a two-stream instability leading to rapid positron heating and a transit time instability.

Gerald Gabrielse (Harvard University and the ATRAP collaboration) and Michael Holzschneider (Los Alamos National Laboratory and the ATHENA collaboration) reported on their plans for producing low-energy anti-hydrogen. Non-neutral plasma physics will play an important role in making anti-hydrogen. This is because significant positron and anti-proton plasmas must be formed and stored before they are brought together for recombination. Much progress on creating and cooling separate positron and anti-proton plasmas has taken place during the past decade.<sup>15,16</sup> Recently (December 96, before the Low Energy Anti-proton Ring at CERN closed) the ATRAP collaboration made a quick try of bringing low energy positrons and anti-protons together in the laboratory for the first time. Although this attempt to make low energy anti-hydrogen was unsuccessful, both collaborations have experiments planned for when the CERN Anti-proton Decelerator becomes operational in a few years

**Laser-cooled ion plasmas.** With laser cooling, the temperature of a non-neutral ion plasma can be reduced to a few mK while the Coulomb potential energy per particle can be several orders of magnitude higher. These ultra-cold plasmas are called strongly coupled and exhibit liquid-like or solid-like behavior. Pei Huang (NIST) reported on a study of the ion correlations in a strongly coupled  $\text{Be}^+$  plasma in a Penning trap by analyzing the cooling laser light Bragg scattered by the ions. Crystal-like patterns were observed,<sup>17</sup> and with approximately spherical plasmas with greater than 200 000 ions, the observed patterns were always body-centered-cubic (bcc). This result is significant because bcc is the predicted minimum energy state of the bulk strongly coupled one-component plasma.

Huang also reported on the use of a “rotating wall” technique to control and even phase lock the rotation of the laser-cooled  $\text{Be}^+$  plasmas. This technique was first successfully used to stabilize the rotation frequency in  $\text{Mg}^+$  ion plasmas in work done at UCSD.<sup>18</sup> The idea of this technique is to apply a rotating electrostatic perturbation. This perturbation produces a distortion of the plasma boundary which rotates at the applied rotating wall frequency. Particles near the plasma surface experience a torque due to the rotating boundary. Through viscous effects, angular momentum can be transmitted to the plasma interior and the plasma will tend to rotate at the applied rotating wall frequency. Huang was able to use the Bragg scattered laser light to

determine that the orientation of the plasma crystals was phase locked to the rotating perturbation. This precise control of the plasma rotation frequency is important for Penning trap atomic clocks. This is because the time-dilation shift due to the plasma rotation is one of the largest systematic shifts in such a standard.<sup>19</sup>

Penning traps have been used to laser cool dimensionally larger plasmas than in rf (Paul) traps because of the problem of rf heating due to the time dependent fields. However, Michael Drewsen (University of Aarhus) discussed a significant increase in the size of a  $Mg^+$  ion plasma which was laser cooled to low temperature in a linear rf trap. He reported on laser cooling structures consisting of as many as 11 concentric shells. Based on the Penning trap results, this may be within a factor of 2 of the dimension where bcc crystals will start to form.

Rudi Grimm (Heidelberg Test Storage Ring) and Jeffrey Hangst (ASTRID storage ring in Aarhus) described the progress in the laser cooling of  $Be^+$ ,  $Li^+$ , and  $Mg^+$  ions in these rings, where the ion velocities are on the order of a few percent of the speed of light.<sup>20,21</sup> In a frame of reference moving with the beam, temperatures parallel to the beam direction of a few mK have been measured. However the coupling is limited by the perpendicular energy spread, which is a few orders of magnitude higher. Nevertheless, space charge effects are very important in these beams and the possibility of making a crystallized string of ions remains open.

**Dusty plasmas.** The formation of ordered structures of micrometer sized particles in plasmas ("plasma crystals") were first observed and reported on in 1994.<sup>22,23</sup> Alexander Piel (University of Kiel) reported on the impressive understanding of these crystals that has been obtained over the past three years. Experiments commonly use ~10 micron spheres levitated in a thin horizontal layer by an rf electric sheath. This system has been used as a test ground for studying the solid-liquid phase transitions in a nearly 2D system. The plasma crystal undergoes a solid-liquid phase transition when the density of the ambient gas is reduced, or when the applied power of the rf discharge is increased.

Observations have been compared to two melting models. The KTHNY theory for 2D melting predicts the formation of free disclination defects as a solid melts, but that has never been observed in the experiments. KTHNY also predicts a gradual and homogeneous phase transition, while the experiments reveal a transition that occurs locally, with the formation of disordered defect clusters. Phase diagrams of the first-order melting of 3D Yukawa systems, predicted by molecular dynamics simulations,<sup>24</sup> show better agreement with experiment. This comparison requires measurements of particle charge and shielding length, which have been reported using wave propagation and modulation of the vertical confining potential in the trap.<sup>25,26,27</sup>

Strongly coupled behavior depends closely on the nature of the inter-particle Coulomb interaction. The repulsion between the particles is shielded by the ambient ions. It has been realized however that a model based on shielding by thermal ions is not appropriate.<sup>28</sup> In the space charge sheath between the plasma and the electrode, where the plasma crystal is trapped, ions stream with supersonic velocity through the plasma crystal. Deflection of ions by Coulomb collisions leads to the accumulation of positive space charge in the wake of the obstacle<sup>29,30</sup> and results in attractive forces in the vertical direction that favor the vertically aligned structures observed in experiments.

**ICR mass spectroscopy.** Fourier transform ion cyclotron resonance (FT-ICR) mass spectrometry is becoming an increasingly important tool for analyzing heavy, biologically-

interesting molecular ions. However, it is also becoming clear that some ultimate limits to FT-ICR MS performance result from ion-ion interactions in the Penning trap, and that advanced plasma manipulation techniques can significantly improve the data. Alan Marshall (National High Magnetic Field Laboratory, Florida State University) reviewed some of these limits and new techniques.

Of particular interest and importance is the ICR peak coalescence due to the Coulomb coupling which can occur for ions of nearly the same charge-to-mass ratio,<sup>31</sup> as for the many isotopic constituents of a high-mass biological molecule. Electrospray-ionization (ESI) FT-ICR mass spectroscopy frequently starts with a heavy molecular ion that has acquired a substantial (positive or negative) charge. This molecule is broken up and the constituent parts analyzed. This process is then repeated  $n$  times (multi-stage  $MS^n$  analysis).  $MS$  and/or  $MS^n$  analysis can result in a series of heavy ions with closely spaced charge-to-mass ratios. The peak coalescence problem determines the resolution limit of the ICR technique for distinguishing between these heavy molecular ions.<sup>32</sup> A better understanding of the peak coalescence under realistic experimental conditions is an important plasma physics problem in ICR mass spectroscopy, especially if it could lead to a strategy which could improve this resolution limit.

**Non-neutral beams.** To an observer moving with the mean beam velocity, a bunch of charged particles in an accelerator or storage ring looks (to first order) like a nonneutral plasma confined in a trap. The workshop featured several talks on nonneutral beams (in addition to the laser-cooled storage ring talks discussed above). Martin Reiser (University of Maryland) and Ronald Davidson (Princeton University) each reported on their experimental and theoretical work on intense nonneutral beams. In addition, Linda Spentzouris (Fermilab) reported on recent progress in using plasma echoes to determine the collisionality of high energy particle beams.

A plasma echo is the time-delayed growth (and subsequent decay) of coherent particle motion at the difference frequency of previous sequential excitations, and relies on the fact that the correlation between the energy and phase of the particles remains after the initial coherent oscillations have damped away. Frequency and phase selection rules determine which oscillation modes participate in this weakly nonlinear wave coupling process. Echoes were first studied in neutral plasmas by Gould, O'Neil, and Malmberg in 1967,<sup>33</sup> and Su and Oberman,<sup>34</sup> and O'Neil<sup>35</sup> pointed out in 1968 that echoes could be used to determine collisional damping rates. Roy Gould discussed the possibility of observing echoes in a non-neutral plasma at the workshop.

In a stored-particle beam, the particle phase correlations required for echoes are eventually destroyed by random processes such as small angle Coulomb scattering. The echo amplitude decreases as the phase correlation breaks down. Since the time delay to the echo is dependent on the kick parameters of the initial excitations (which are controlled), it is possible to measure the diffusion rate in a beam.<sup>34,35</sup> Longitudinal echoes in an unbunched beam were first seen in the Fermilab Accumulator in 1994, by Linda Spentzouris and colleagues,<sup>36</sup> and Spentzouris reported on recent progress on these measurements.<sup>37</sup> This technique has also been successfully used on stored unbunched beams in the CERN SPS.<sup>38</sup> Longitudinal echoes in bunched beams have been studied and observed by Colestock and Assadi.<sup>39</sup>

**Dense non-neutral plasmas.** Dan Barnes (Los Alamos National Laboratory) described an experiment with the goal of creating a dense non-neutral plasma with potential fusion applications. The Penning Fusion eXperiment (PFX) has demonstrated spherical focusing of

electrons.<sup>40</sup> In this experiment electrons are injected at low energy along the axis of the trap. Only when the applied voltage is tuned to produce a spherical well, a nonthermal, spherically-focussed state is observed. Central densities up to 35 times the Brillouin density have been inferred from a combination of observations and bounce-averaged, Fokker-Planck calculations. The Los Alamos group is designing a new version of the experiment to test the applicability of non-neutral plasmas for producing fusion relevant ion densities and energies.

This summary gives only a brief discussion of some of the topics discussed at the Workshop. (The complete Workshop program can be obtained from the conference booklet of abstracts.<sup>4</sup>) However, we hope we have been able to convey the multi-disciplinary nature, the vitality of the current research, and some of the interesting applications of non-neutral plasma physics.

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