

Breaking the Azimuthal Symmetry- Jumping off-axis or Staying Away from the Axis?

Eli Sarid,¹ Catalin Teodorescu², Phil Marcus³ and Joel Fajans

Physics Department, University of California, Berkeley, Berkeley, CA 94720-7300

¹*Permanent address: NRCN, P.O. 9001, Beer Sheva 84190, Israel*

²*West Virginia University, P.O. 6315, Morgantown, WV 26506-6315*

³*Mechanical Engineering, University of California, Berkeley, CA 94720*

Abstract. We performed experiments with electrons in a Malmberg-Penning trap, as well as 2D fluid simulations, and found conditions that lead to off-axis equilibrium states. We found that unstable initial distributions whose average distance from the axis is larger than about half the wall radius end up in off-axis states. These states consist of a small strong vortex and a large diffuse background, each having about half of the initial charge. We present a simple model showing that for small initial distributions, on-axis solutions maximize the background radius and thus the entropy. For extended initial distributions, no on-axis solutions are available and staying off-axis is necessary for the conservation of angular momentum.

GOING OFF-AXIS

Several theoretical studies discussed the possibility of off-axis equilibrium states of a cylindrically bounded guiding-center plasma [1-3]. Those solutions were obtained in the frame of statistical theories, based on the assumption of ergodicity, constrained by conservation laws. Those studies were not followed by experimental verification. We demonstrate that such off-axis solution can indeed be obtained, but under different conditions than those implied by previous studies. We also give a simple model, examining a limited phase space, that explains the behavior we observed, and emphasize the importance of the conservation of angular momentum as the reason for “going off-axis”.

Here we show that sufficiently extended, unstable initial distributions of electrons in a Malmberg-Penning trap can end up in off-axis states. We corroborate our experiments with 2D fluid simulations showing the same behavior. The transition between initial distributions that lead to on-axis and off-axis solutions is quite sharp: the average distance of the electrons from the trap axis needs to be about half the wall radius, r_w , for off-axis solutions to be obtained. The exact geometry of the initial distribution is not crucial: we obtained the same behavior whether we started with rings, bars (elongated rectangles) or tri-bars (three rectangles merging in the center of the trap). All the initial distributions had on-axis center-of-mass, and led to off-axis solutions when they were sufficiently extended.

The electron experiments were performed with the photo-cathode machine [4]. With this machine, various initial distributions of electrons could be easily obtained. The time evolution is studied with repeated experiments, damping the electrons on a phosphor screen after varying hold times.

Figure 1 shows an example for a typical time evolution, starting with a large ring of electrons. The ring breaks due to diocotron instability into discrete “vortices”. After a short period of mixing and merging these vortices end up in a single strong vortex, and an extended “background”. In the case of Fig. 1, this background reached the trap wall and has also a “hole” in it. The smaller coherent vortex, with density comparable to the initial one, is shifted off-axis. So is, although to a lesser degree, the center of mass of the total distribution of the electrons. All this happens on a short time scale (30 ms) over which it can be assumed that 2D fluid-like physics is sufficient to understand the phenomena. Over a much longer time scale (hundreds of ms) the center of mass slowly approaches the axis of the trap.

Figure 2 shows an example for a time evolution, with a smaller ring of electrons as the initial distribution. Qualitatively, the evolution is similar to that of the larger distribution: the ring breaks into discrete “vortices”, the vortices merge and eventually end up in a single strong vortex with an extended “background”. Here, however, the smaller coherent vortex is on-axis. The background is more uniform than in Fig. 1 and does not reach the wall (for even smaller rings, we find backgrounds with smaller radius in the final state).

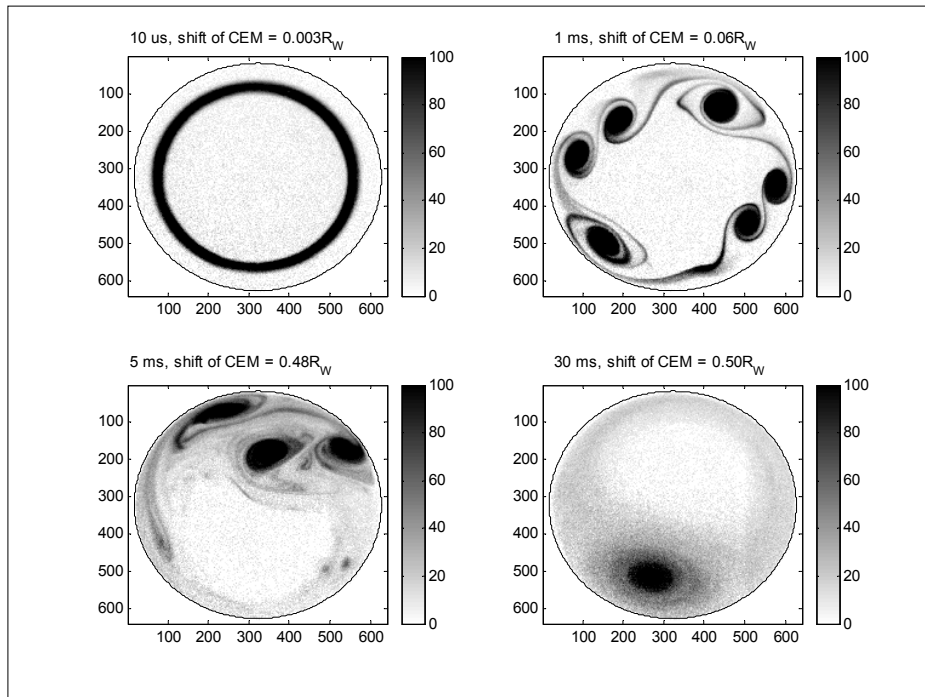


FIGURE 1. Going off-axis: with a large ring of electrons as the initial condition.

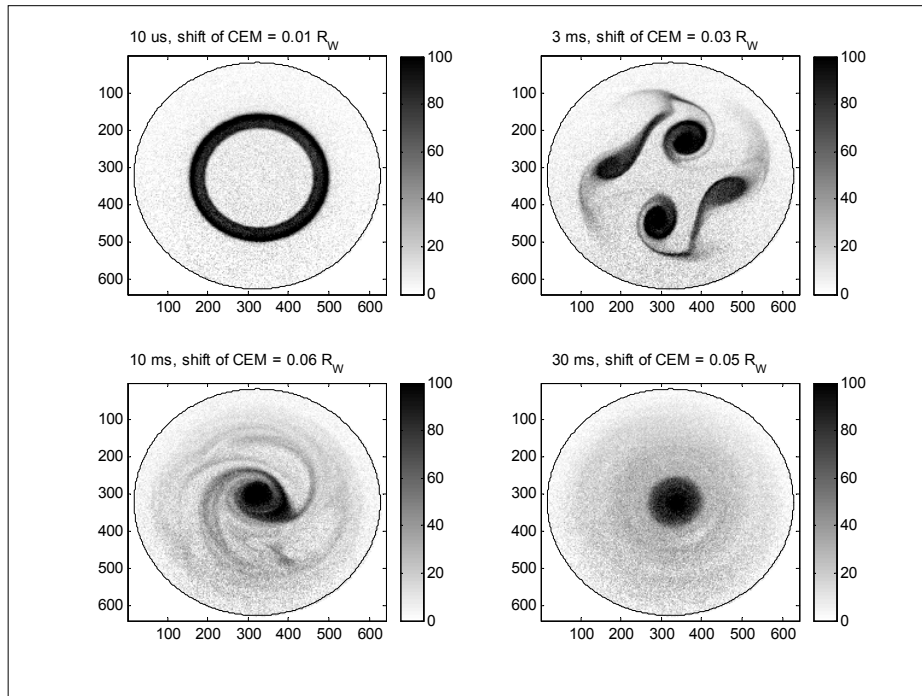


FIGURE 2. Staying on-axis: with a small ring of electrons as the initial condition.

There is a relatively sharp transition between the initial conditions that lead to on-axis solutions and those that lead to off-axis ones. Figure 3 shows the shift of the strong coherent vortex as function of the average initial radius of the ring. It can be seen that for rings whose average initial radius was smaller than about half the wall radius, on-axis vortices result. For larger rings, off-axis states appear, with a steep increase of the shift as function of the initial radius.

To verify our assumption that the observed phenomenon can be understood within the frame of 2D fluid dynamics, we ran a simulation solving the Euler equation using a spectral code. Indeed, the same transition from on-axis to off-axis states was observed in the simulation. The simulation was useful also in verifying that going off-axis happens with strict conservation of charge and angular momentum.

The experiments using bars and the tri-bars as the initial distributions yielded similarly sharp transitions between the conditions that lead to on-axis solutions and those that lead to off-axis states. In the case of the bars and the tri-bars, the initial distribution naturally broke to two or three vortices, respectively. After the merging of these vortices, an on-axis strong vortex resulted in the case of the smaller shapes, an off-axis one in the case of the more extended initial distributions. . In terms of the average distance of the electrons from the axis, the transition occurred in a somewhat smaller average distance ($\geq 0.4 r_w$) than in the rings experiments.

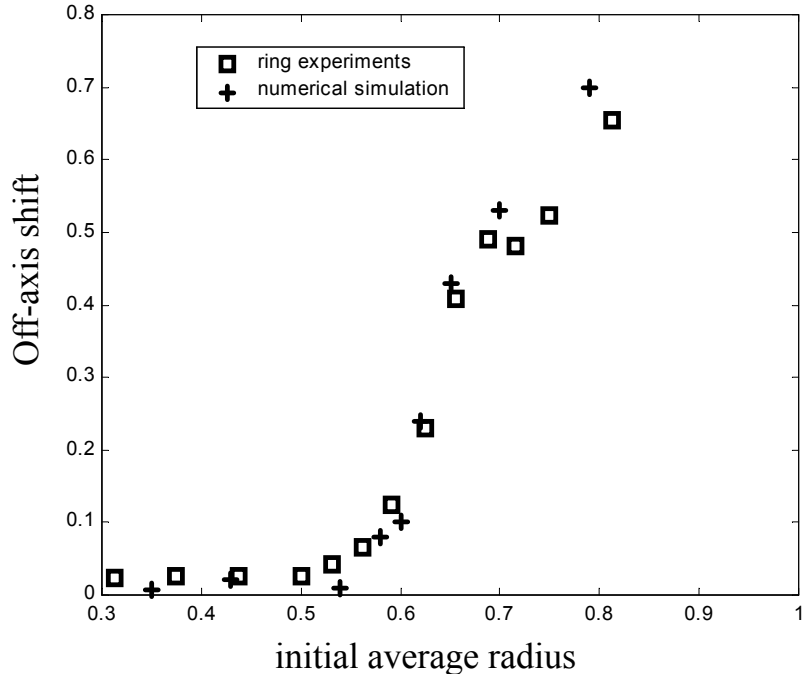


FIGURE 3. Off-axis shift of the coherent “disk” in the final state, as function of the initial average radius of the rings. Both the initial radius and the off-axis shift are normalized for $r_w=1$. There is a very good agreement between the results of the electron experiments and the 2D fluid simulation code.

A SIMPLE MODEL

We found that useful insights can be gained by looking at a very simple model, characterizing the final state with a minimum set of parameters, while restricting these states to those that roughly correspond to those observed in both experiments and simulations. A restricted phase space available to the final states is obtained under the assumption of conservation of total charge, energy and angular momentum.

We shall limit the discussion that follows to the case of a uniform density ring as the initial distribution of charge. The final states that we shall check consist of a high-density small disk and a large diffuse background. Both disk and background are assumed to be circular and of uniform density. The background is centered on the trap axis, while the small disk has a shift D from the axis (D might be zero). The background has radius of r_b (which might be as large as the wall radius, but can also be smaller).

In what follows we shall take two more assumptions, which will limit the number of parameters needed to describe the background to just two: D and r_b . They are:

- 1) The dynamics of the instability, the merging and the mixing, lead to a sizable, approximately constant, fraction of the initial charge to be eventually found in the diffuse background.

- 2) The density of the small “coherent” disk is close to the (uniform) density of the initial ring.

To make these assumptions concrete, we shall assign specific values to the fraction of the initial charge that goes to the final state background, and to the density of the final coherent disk. We choose to look at final states in which half of the initial charge is in the coherent disk and half in the background, with the density of the small disk being 0.7 that of the initial density. The two specific values we choose are based on the results of both the experiments and the simulations, but they are not crucial to the emerging physical picture. With these assumptions, only D and r_b remain as free parameters that characterize the final states. While the actual final states are richer in content than this simple model (see for example the hole in the background in Fig. 1), we find that this model is sufficient to illuminate the physical mechanism behind the going “off-axis”.

Figure 4 shows the families of solutions obtained for various values of r_b , from $r_b=0.6$ up to the cases where the background fills the trap up to the wall radius, $r_b=1$. For each value of the initial average radius of the ring, there is a continuum of possible solutions conserving charge, energy and angular momentum. These solutions have smaller shifts D of the small coherent objects for larger values of the background radius r_b . For average initial radius smaller than about half the wall radius, $D=0$ (on-axis) solutions are available with $r_b<1$. However, no such solutions exist for larger average initial radii. Such large rings have high angular momentum, and, in the frame of our model, conservation of angular momentum is not possible for on-axis states even if the background extends to the wall. The “missing” angular momentum results in the shift of the coherent disk from the axis of the trap.

Among the available solutions for each initial average radius, the system chooses the one with maximum r_b . Maximizing the radius of the background is equivalent in the frame of our model to a requirement of maximum entropy or minimum enstrophy (in the frame of our model, we cannot not distinguish between these two requirements, as they yield the same solutions). With this principle, we can see how Figure 4 predicts the behavior observed in the experiments and in the simulations: up to about half the wall radius, maximizing the radius of the background leads to on-axis solutions. These solutions have larger background for larger initial average radius. The transition to off-axis shifts occurs when the background needs to extend to the wall to satisfy conservation of angular momentum. For larger rings, conservation of angular momentum is possible only for off-axis solutions. The observed shift D grows quickly, and the $r_b=1$ curve is very similar to the shifts observed in Figure 3 for rings whose average radius is larger than half the wall radius.

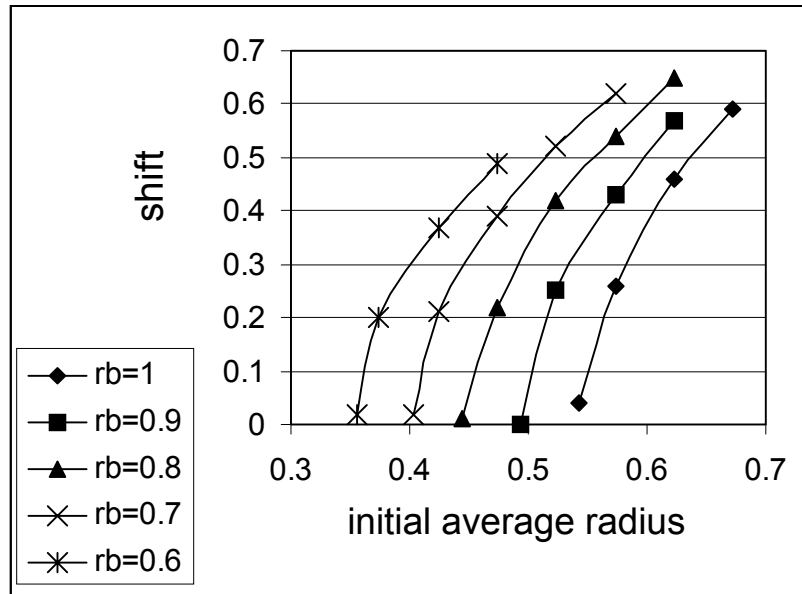


Figure 4. Solutions of the simple model for the final states, satisfying conservation of charge, energy and angular momentum, for various background radii.

DISCUSSION

In our model we discussed only the initial and final states, and we considered the states available under the constraints of conservation of charge, angular momentum and energy. In these we followed the attitude of previous studies [1-3]. Unlike those studies, however, we further restricted the final states considered to those that consist of both a strong coherent vortex and a diffuse background, each having half the charge. The exact fraction of the initial charge found in the background is not crucial. We verified that modified assumptions, putting a third or two thirds of the charge in the background, lead to the same transition between on-axis and off-axis solutions. Only the position of the transition point shifts somewhat as we vary our assumptions.

More can be said about the dynamics that leads to the production of the final states we considered. The evolution of the intermediate stages, after the initial unstable distribution breaks, has some resemblance to the behavior of a system of point vortices [5]. Studying the case of point vortices distributed on a ring, we found the even for Havelock unstable configurations, the center of mass stays on-axis if the vortices are distributed on a small enough ring. For vortices distributed over a circle with a radius larger than about half the wall radius, the center of mass goes off-axis. But such considerations are limited, because they do not consider the mixing and merging that lead to the extended background, carrying a sizable (about half) of the initial charge. Maximizing the extent of this background leads to on-axis states available only up to a certain initial radius.

In summary, we studied the production of off-axis equilibrium states, and for the first time presented both experiments and simulations in which they result. With a simple model, we showed that going off-axis could be understood as necessary to conserve angular momentum in a restricted family of solutions available to the system when the initial angular momentum is large. These solutions maximize the radius of the background, thereby maximizing entropy or minimizing enstrophy. Emphasizing the role of angular momentum threshold is in contrast to previous studies [1-3] that suggested that the transition occurs for distributions that have high energy for a given value of angular momentum.

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