Econ 100A: Intermediate Microeconomic Analysis Lecture 11

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Plan of Today's Lecture

- About the Midterm Exam
- Next, we will define:
  - isoquants
  - isocosts
- We will cover Appendix to Ch. 7:
  - Duality of
  - 1. Cost minimization
  - 2. Production maximization
  - We will use the Lagrangian Method to demonstrate the equivalence of 1 & 2
  - Example: Cobb-Douglas Production Function

Choosing Inputs

- Let us choose the output we wish to produce and then determine how to do that at minimum cost
  - **Isoquant** is the quantity we wish to produce
  - **Isocost** is the combination of K and L that results in a given cost
- One can address how to minimize cost for a given level of output by combining isocosts with isoquants

Duality of Production and Cost Theory

- 1. Cost Minimization
  - To choose the lowest isocost tangent to a given isoquant (Q_0):
- Min $C = \omega L + rK$, s.t. $F(K,L)=Q_0$

Compare with:

- 2. Maximal production
  - to choose the highest isoquant tangent to a given isocost (C_0):
- Max $F(K,L)$, s.t. $\omega L + rK = C_0$

Cost Min versus Production Max

- Use the Lagrangian method:
- Cost Min:
- $\Phi = \omega L + rK - \lambda\,[F(K,L) - Q_0]$
- Production Max:
- $\Phi = F(K,L) - \mu[\omega L + rK - C_0]$
- To solve, we will differentiate with respect to K, L and the Lagrange multiplier (λ or μ)
- In each case, we will have three equations with three unknowns ➔ they can be solved to give us optimal K and L (and also (λ or μ))

Cost Minimization

- $\Phi = \omega L + rK - \lambda[F(K,L) - Q_0]$
- Differentiate wrt K, L and λ
- 1. $\partial\Phi/\partial K = r - \lambda MP_K = 0$
- 2. $\partial\Phi/\partial L = \omega - \lambda MP_L = 0$
- 3. $\partial\Phi/\partial\lambda = F(K,L) - Q_0 = 0$
- From 1 and 2 ➔

$$MP_K/r = MP_L/\omega$$

- Also $\lambda = r/MP_K = \omega/MP_L$

Production Maximization

- $\Phi = F(K,L) - \mu[\omega L + rK - C_0]$
- Differentiate wrt K, L and μ
- 1. $\partial\Phi/\partial K = MP_K - \mu r = 0$
- 2. $\partial\Phi/\partial L = MP_L - \mu\omega = 0$
- 3. $\partial\Phi/\partial \mu = \omega L + rK - C_0 = 0$
- From 1 and 2 ➔

$$MP_K / r = MP_L / \omega$$

This is almost identical to slide 6 (cost minimization)

- Also $1/\mu = r/MP_K = \omega / MP_L = \lambda$

MRTS along isoquant(s)

- Along the isoquant: dQ=0

$MP_K\, dK + MP_L\, dL = 0$ ⇐ ⇒

- $MP_L / MP_K = - dK/dL = MRTS_{LK}$
- $MRTS_{LK}$ Marginal rate of technical substitution between labor and capital (of labor for capital)
- And from slide 6 (or 7) at the optimal point $MP_L / MP_K = - dK/dL = \omega/r$
- OR $MP_L/\omega = MP_K / r$
  - marginal products adjusted by the unit cost of each input are equal

Cobb-Douglas Production Function

- Let $F(K,L) = AK^{\alpha}L^{\beta}$
- How much L and K should be used for optimal production of output Q_0?
- In optimum,
  - L = some function of Q_0
  - K = some other function of Q_0
- We will apply the Lagrangian Method:
- $\Phi = \omega L + rK - \lambda[AK^{\alpha}L^{\beta} - Q_0]$
- Differentiate wrt K, L and λ

Optimal L and K for Cobb-Douglas Production Function

- 1. $\partial\Phi/\partial K = r - \lambda\alpha A K^{\alpha-1} L^{\beta} = 0$
- 2. $\partial\Phi/\partial L = \omega - \lambda\beta A K^{\alpha} L^{\beta-1} = 0$
- 3. $\partial\Phi/\partial\lambda = A K^{\alpha} L^{\beta} - Q_0 = 0$
- From 1 & 2: $L/\beta r = K/\alpha\omega$ or $L = \beta r K/\alpha\omega$
- Using 3: $A K^{\alpha} (\beta r K/\alpha\omega)^{\beta} = Q_0$, and
  $K^{\alpha+\beta} = Q_0 (\beta r/\alpha\omega)^{-\beta}/A$
- ➔K, L are expressed as the functions of Q_0 (see A7.24 and 25, p. 259)

Returns to Scale: how to …

- If $F(\lambda K, \lambda L) > \lambda F(K, L)$ ➔ increasing returns to scale
- $F(\lambda K, \lambda L) = \lambda F(K, L)$ ➔ constant returns to scale
- $F(\lambda K, \lambda L) < \lambda F(K, L)$ ➔ decreasing returns to scale
- If doubling is easier for you, just double

Examples:

- Determining the returns to scale:
- 1. $F(K,L)=K^2L$
- ➔ $F(\lambda K, \lambda L) = (\lambda K)^2 (\lambda L) = \lambda^3 K^2 L = \lambda^3 F(K, L) > \lambda F(K, L)$, i.e., increasing returns to scale
- 2. $F(K,L)=10K+5L$
- ➔ $F(\lambda K, \lambda L) = 10\lambda K + 5\lambda L = \lambda F(K, L)$, i.e., constant returns to scale
- 3. $F(K,L)=(KL)^{0.5}$
- ➔ $F(\lambda K, \lambda L) = (\lambda K \lambda L)^{0.5} = (\lambda^2)^{0.5} (KL)^{0.5} = \lambda (KL)^{0.5} = \lambda F(K, L)$, i.e., constant returns to scale
- For Cobb-Douglas production function:
- If $\alpha+\beta = 1$ ➔ constant returns
- If $\alpha+\beta > 1$ ➔ increasing returns
- If $\alpha+\beta < 1$ ➔ decreasing returns

Summary of Today & Plan of Next Lecture

- Concepts:
  - isoquants (same quantity)
  - isocosts (same cost)
- Appendix to Ch. 7
- Duality of Production and Cost Theory
- Cobb-Douglas production function & its returns to scale
- Next Lecture
- We will be covering Ch. 7