
Econ 100A: Intermediate Microeconomic Analysis Lecture 11

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Plan of Today's Lecture

- About the Midterm Exam
- Next, we will define:
 - isoquants
 - isocosts
- We will cover Appendix to Ch. 7:
 - Duality of
 - 1. Cost minimization
 - 2. Production maximization
 - We will use the Lagrangian Method to demonstrate the equivalence of 1 & 2
 - Example: Cobb-Douglas Production Function

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Choosing Inputs

- Let us choose the output we wish to produce and then determine how to do that at minimum cost
 - **Isoquant** is the quantity we wish to produce
 - **Isocost** is the combination of K and L that results in a given cost
- One can address how to minimize cost for a given level of output by combining isocosts with isoquants

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Duality of Production and Cost Theory

- 1. Cost Minimization
 - To choose the lowest isocost tangent to a given isoquant (Q_0):
- $\text{Min } C = \omega L + rK, \text{ s.t. } F(K,L)=Q_0$
Compare with:
- 2. Maximal production
 - to choose the highest isoquant tangent to a given isocost (C_0):
- $\text{Max } F(K,L), \text{ s.t. } \omega L + rK = C_0$

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Cost Min versus Production Max

- Use the Lagrangian method:
- Cost Min:
 - $\Phi = \omega L + rK - \lambda [F(K,L) - Q_0]$
- Production Max:
 - $\Phi = F(K,L) - \mu [\omega L + rK - C_0]$
- To solve, we will differentiate with respect to K, L and the Lagrange multiplier (λ or μ)
- In each case, we will have three equations with three unknowns \rightarrow they can be solved to give us optimal K and L (and also (λ or μ))

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Cost Minimization

- $\Phi = \omega L + rK - \lambda [F(K,L) - Q_0]$
- Differentiate wrt K, L and λ
- 1. $\partial\Phi/\partial K = r - \lambda MP_K = 0$
- 2. $\partial\Phi/\partial L = \omega - \lambda MP_L = 0$
- 3. $\partial\Phi/\partial\lambda = F(K,L) - Q_0 = 0$
- From 1 and 2 \rightarrow
$$MP_K/r = MP_L/\omega$$
- Also $\lambda = r/MP_K = \omega/MP_L$

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Production Maximization

- $\Phi = F(K,L) - \mu[\omega L + rK - C_0]$
- Differentiate wrt K, L and μ
- 1. $\partial\Phi/\partial K = MP_K - \mu r = 0$
- 2. $\partial\Phi/\partial L = MP_L - \mu\omega = 0$
- 3. $\partial\Phi/\partial\mu = \omega L + rK - C_0 = 0$
- From 1 and 2 \rightarrow

$$MP_K/r = MP_L/\omega$$

This is almost identical to slide 6 (cost minimization)

- Also $1/\mu = r/MP_K = \omega/MP_L = \lambda$

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MRTS along isoquant(s)

- Along the isoquant: $dQ=0$
 $MP_K dK + MP_L dL = 0 \leftarrow \rightarrow$
- $MP_L/MP_K = -dK/dL = MRTS_{LK}$
- $MRTS_{LK}$ Marginal rate of technical substitution between labor and capital (of labor for capital)
- And from slide 6 (or 7) at the optimal point
 $MP_L/MP_K = -dK/dL = \omega/r$
- OR $MP_L/\omega = MP_K/r$
– marginal products adjusted by the unit cost of each input are equal

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Cobb-Douglas Production Function

- Let $F(K,L) = AK^\alpha L^\beta$
- How much L and K should be used for optimal production of output Q_0 ?
- In optimum,
 - L = some function of Q_0
 - K = some other function of Q_0
- We will apply the Lagrangian Method:
- $\Phi = \omega L + rK - \lambda[AK^\alpha L^\beta - Q_0]$
- Differentiate wrt K, L and λ

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Optimal L and K for Cobb-Douglas Production Function

- 1. $\partial\Phi/\partial K = r - \lambda\alpha AK^{\alpha-1}L^\beta = 0$
- 2. $\partial\Phi/\partial L = \omega - \lambda\beta AK^\alpha L^{\beta-1} = 0$
- 3. $\partial\Phi/\partial\lambda = AK^\alpha L^\beta - Q_0 = 0$
- From 1 & 2: $L/\beta r = K/\alpha\omega$ or $L = \beta r K/\alpha\omega$
- Using 3: $AK^\alpha (\beta r K/\alpha\omega)^\beta = Q_0$, and
 $K^{\alpha+\beta} = Q_0 (\beta r/\alpha\omega)^{-\beta}/A$
- \rightarrow K, L are expressed as the functions of Q_0 (see A7.24 and 25, p. 259)

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Returns to Scale: how to ...

- If $F(\lambda K, \lambda L) > \lambda F(K, L) \rightarrow$ increasing returns to scale
- $F(\lambda K, \lambda L) = \lambda F(K, L) \rightarrow$ constant returns to scale
- $F(\lambda K, \lambda L) < \lambda F(K, L) \rightarrow$ decreasing returns to scale
- If doubling is easier for you, just double

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Examples:

- Determining the returns to scale:
 - 1. $F(K,L)=K^2L$
 - $\rightarrow F(\lambda K, \lambda L) = (\lambda K)^2 (\lambda L) = \lambda^3 K^2 L = \lambda^3 F(K, L) > \lambda F(K, L)$, i.e., increasing returns to scale
 - 2. $F(K,L)=10K+5L$
 - $\rightarrow F(\lambda K, \lambda L) = 10\lambda K + 5\lambda L = \lambda F(K, L)$, i.e., constant returns to scale
 - 3. $F(K,L)=(KL)^{0.5}$
 - $\rightarrow F(\lambda K, \lambda L) = (\lambda K \lambda L)^{0.5} = (\lambda^2)^{0.5} (KL)^{0.5} = \lambda (KL)^{0.5} = \lambda F(K, L)$, i.e., constant returns to scale
- For Cobb-Douglas production function:
 - If $\alpha + \beta = 1 \rightarrow$ constant returns
 - If $\alpha + \beta > 1 \rightarrow$ increasing returns
 - If $\alpha + \beta < 1 \rightarrow$ decreasing returns

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Summary of Today & Plan of Next Lecture

- Concepts:
 - isoquants (same quantity)
 - isocosts (same cost)
- Appendix to Ch. 7
- Duality of Production and Cost Theory
- Cobb-Douglas production function & its returns to scale
- Next Lecture
- We will be covering Ch. 7

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