1. **Planetary atmospheres** (from Sethna): Treat diatomic oxygen for now as a monatomic ideal gas with mass $5.3 \times 10^{-23}$ g, which is twice that of a single oxygen atom (ignore all excitations, which is OK here because they do not modify the escape probability). Assume that the probability distribution for the $z$ component of momentum is as expected for an ideal gas,

$$
\rho(p_z) = \frac{1}{\sqrt{2mk_BT}} e^{-p_z^2/(2mk_BT)}. \quad (1)
$$

Assume a uniform temperature of 300 K. The Earth’s escape velocity at sea level is about 11 km/s and gravitational constant at sea level is 9.8 m/s$^2$.

(a) What is the RMS $z$-directed velocity $\sqrt{\langle v_z^2 \rangle}$? If a collisionless molecule started at the Earth’s surface with this RMS vertical velocity, how high would it go, and how much time would pass before it hit the ground again?

(b) Use integration by parts to show that

$$
\int_{x}^{\infty} e^{-t^2} dt \approx e^{-t^2/(2t)}, \quad \text{for } t \gg 1. \quad (2)
$$

Apply this result to compute the probability that an O$_2$ molecule has sufficient upward velocity to escape the Earth if there are no collisions.

(c) Assuming that the particles thermalize each time they collide with the ground, and that the time between collisions is the time calculated in (a), what fraction of the oxygen molecules will be left each year?

(d) Repeat the calculation to find the leakage rate for hydrogen. You should find that most of the hydrogen will have escaped the Earth’s atmosphere during the Earth’s lifetime; Jupiter retains its mostly hydrogen atmosphere because its gravity is higher.

2. **Entropic spring**: Consider a polymer with one end fixed, modeled as a random walk of $N$ steps on a square lattice of side $a$. Suppose that the polymer is at temperature $T$. Ignoring any energy of the polymer, i.e., considering only entropic forces, what is the force on the other end of the polymer when it is at a distance $L$ from the origin, and how is it directed? You will want to take a continuum limit of small $a$ in order to define a continuous force, and you can assume that $L$ is of order the typical displacement $\sqrt{Na}$, so that $Na \gg L \gg a$, and develop a Gaussian approximation.

3. **Pressure**: Derive the ideal gas law $p = nk_BT$ microscopically by considering the Maxwellian distribution $\propto e^{-\beta p^2/2m}$ of momentum in an ideal gas and computing the momentum per time imparted to a surface by elastic collisions between a particle and the surface. Be careful to include all factors that affect how often particles collide with the surface.

4. Reif 1.12 (For this, it will be useful to refer back to Reif 1.9 on the Poisson distribution)

5. Reif 1.16 (“Consider a gas...”)

6. Reif 6.5
7. Use the partition function of the ideal gas to verify that the Maxwell relation (Reif equation 5.5.20) is satisfied by taking appropriate derivatives:

\[
\left. \frac{\partial T}{\partial V} \right|_{S,N} = - \left. \frac{\partial p}{\partial S} \right|_{V,N}.
\] (3)

This is a little tricky because we do not normally think of \((S, V, N)\) as the set of independent variables.