Problem set 8: gauge invariance; radiative transitions
Assigned Saturday, 27 October. Due in box Friday, 2 November.

1. First, do Bransden 11.1. The energy of a single photon of frequency $\omega$ is $\hbar \omega$, and one watt is one joule per second. Then read the definition of the absorption cross section (Bransden 11.58) and compute its value in square angstroms for $1s\rightarrow 2p$ transitions of a hydrogen atom. You may use the value of the dipole matrix element in this case, Bransden 11.86.


3. An electron in a hydrogen atom is initially in a 5d state. Ignore spin-orbit coupling, list all the states to which this electron is allowed to decay by spontaneous emission, using the electric dipole selection rules. You do not need to compute the rate or the photon energy. Assume that the emitted photon can have any polarization.

4. The adiabatic approximation predicts that for nondegenerate eigenstates, a very slow change in the Hamiltonian should not induce transitions: a system that begins in the ground state will remain in the ground state. Show that this is consistent with the transition rate (Bransden 9.61) for a two-level system with harmonic perturbation. (Previously we focused on this equation when we were near the resonance of one of the two terms. Now you have to show that as $\omega \to 0$, $P_{ab}^{(1)} \to 0$, using the relationship between $A$ and $A^\dagger$.)

5. (a) Find a solution that is stationary on average for motion of a nonrelativistic classical particle of charge $q$ and mass $m$, under the motion of a periodic electrical field $E_0 \hat{y} \sin(\omega t)$. You should find simple harmonic motion of frequency $\omega$ in the $y$ direction. (b) Now calculate the total energy radiated per period of this oscillation, using the Larmor formula for instant radiated power of a classical charged particle of acceleration $a$: in SI units,

$$P = \frac{e^2 |a|^2}{6\pi\epsilon_0 c^3}.$$  \hspace{1cm} (1)