1. Calculate the rate of spontaneous emission for hydrogen 2p to 1s transitions. (Does this rate depend on the initial value of the $m$ quantum number?) You can check that you’re on the right path by looking at the same calculation done in Bransden, p. 530-531.

For $N$ atoms initially in the 2p state, what is the initial radiated power?

2. Suppose now that a hydrogen atom is initially in the 2p state with quantum number $m = 0$, i.e., the eigenstate of $L_z$ is 0. Along what direction(s) should incident radiation be polarized to generate transitions to the 1s state? Why? For this polarization, what intensity per unit frequency interval $I(\omega_{ba})$ will generate one stimulated emission transition from 2p to 1s per nanosecond?

3. Suppose that instead of moving in the Coulomb field of a nucleus, an electron is confined in a 3D harmonic oscillator potential:

$$H = \frac{p^2}{2m} + \frac{k|\mathbf{r}|^2}{2}. \quad (1)$$

Now assume that an electromagnetic wave with polarization along $\hat{x}$ is incident on the atom. If the system is initially in a state with harmonic oscillator quantum numbers $n_x, n_y, n_z$, and energy

$$E = \hbar \omega (3/2 + n_x + n_y + n_z) \quad (2)$$

where $\omega$ is the classical oscillation frequency $\sqrt{k/m}$, what transitions are allowed within the dipole approximation? Hint: your goal is to find which states $n'_x, n'_y, n'_z$ have $|D_{ba}|^2 \neq 0$. It may help to think about how $x$ is expressed in terms of the raising and lowering operators for the harmonic oscillator.

4. Bransden 11.8. (Hint: Proving this “Thomas-Reiche-Kuhn sum rule” is a little tricky. It may help you to think about how the action of the expression $\sum_k |\psi_k\rangle E_k \langle \psi_k|$ on a state is exactly the same as that of an operator you know well, and to remember that $\sum_k |\psi_k\rangle \langle \psi_k|$ is the identity operator.)

5. Bransden 13.2. The “laboratory coordinate system” means the one in which B particles are initially stationary.