C-2. LR Circuits

Teaching Notes

THE MAIN POINT

1. Analyze LR circuits using the loop rule, focusing on the $t=0^+$ and $t=\infty$ regimes.

2. Analyze LR circuits using the loop rule and the differential equation.

3. Analyze the energetics of LR circuits.

SAMPLE LESSON PLAN

Discussion Questions 1,2 ... 10 minutes in groups, 10 minutes GSI-led discussion & boardwork.
Problem 1 .... 25 minutes in groups, 15 minutes GSI-led discussion & boardwork.
Problem 2 .... 20 minutes in groups, 10 minutes GSI-led discussion & boardwork.
Problem 3 .... 15 minutes in groups, 15 minutes GSI-led discussion & boardwork.

GENERAL TEACHING SUGGESTIONS

"Immediately afterwards" and "A long time later." As with RC circuits, there are basically two approaches to an LR circuit. The first is to write down the loop rules and junction rules, and then solve them as differential equations. The second is to be content with understanding the behavior of the circuit "immediately after" some switch is closed and "a long time later." Most of the problems on this worksheet were written from the point of view that there is easily enough physics in the latter approach.

On the other hand, we want to give our students a good feel for the *dynamics* of LR circuits, and it seems clear that if we focus entirely on "two-time" calculations, then not everyone will gain this kind of understanding. For this reason, the worksheet problems seldom treat $i(0^+)$ and $i(\infty)$ as ends in themselves; these quantities are usually put to use in drawing an accurate sketch of the *full behavior* of $i(t)$. Likewise, when you present problems at the board, treat these *labeled graphs* as the "final answers," not the two-time values themselves.

It *is* instructive to be able to get an actual solution to LR Circuits, but we don't want to re-teach ways of solving differential equations. There is a suppliment included with the worksheets that gives all of the relevant information. Namely, it is sufficient for a student to be able to use the loop rule to *get* a differential equation and manipulate it into one of the forms listed, and then to plug in the 'general solution' and initial conditions. This way, we can fully see what is *going on*in an LR circuit without bothering to go into the details of *solving* the ODEs.

Sample Mini-Lecture and Board Summary

You should not need to lecture more than a few minutes at the start of section. The only thing your students really need to know to get started is how to deal with inductors in the loop rule. Probably the most that will go wrong here is a sign error.

Remarks on the Discussion Questions and Problems

Discussion Question 1

The goal here is for your students to be able to tell a *story* about what happens to an LR circuit over time. (The initial rise in current causes a change in the magnetic flux through the inductor, which creates a back-EMF as per Faraday's Law, which prevents the current from reaching its final value instantaneously.) The fewer formulas they invoke, the better.

Discussion Question 2

All the students need to understand is that after a time τ, the current and other quantities will have gone roughly two thirds of the way towards their asymptotic values. After another time τ, these quantities will have gone roughly another two thirds towards their final values. And so on.

An example will help them to visualize this. Let τ be one millisecond. Let the initial value of the current be 0 A, and suppose that the final value of the current is 1 A. Then after one millisecond, the current will be around 2/3 A. After another millisecond, the current will be around 8/9 A. After another millisecond, the current will be around 26/27 A. As you can see, after one second, the current will be indistinguishable from its final value of 1 A. And this is all because one second is "many times longer than" τ.

Discussion Question 3

The goal here is to get students thinking about the short- and long-term behaviors of capacitors in circuits. Understanding these behaviors, an LR circuit at $t=0$ or $t=\infty$ can be reduced to a much simpler DC circuit. Knowing the behavior at the beginning and end of the LR circuit is usually sufficient to extrapolate all of the other important information!

Problem 1

This problem serves as an introduction to LR circuit analysis (see the Sample Lesson Plan above). Explanations are very important here, so grill people on the "Why" parts.

Problem 2

Part (d) is important, since it encourages the students to visualize the circuit dynamics as an energy flow from the inductor to the environment through the resistor. This is a wonderful physical picture.

This kind of problem also helps to make "real" the notion that magnetic fields contain energy in each cubic centimeter of space. After all, in an LR circuit you can feel this energy leaking out through the resistor as heat. (With your own hands yet!)

As you discuss the energetics of this problem at the board, draw a picture of the initial situation, with a strong magnetic field shown. Point to the magnetic field and comment on all of the energy stored in the coil. Then draw an "intermediate time" picture, with a weaker magnetic field shown. Include some arrows emanating from the resistor to represent the heat dissipation. Finally, draw a "final" picture, with no field and no current: all of the energy is now gone.

Problem 3

Problem 3 asks the students to write the loop rule for a simple LR circuit, valid at all times, expressing their result as a differential equation for the current through the inductor. Then they are given the solution and asked to verify that it satisfies the differential equation.

The most important parts of this problem are (g) and (h). Here the students verify that the total energy dissipated as heat through the resistor is equal to the energy initially stored in the inductor. This is a very important idea, and we want the students to be able to analyze the energetics of circuits. (This will be especially true of LRC circuits.) If your students do not solve Problem 3, make sure that they can convincingly answer parts (f) - (h) of Problem 2.

Problem 4

Problem 4 asks the students to write the loop rule for another simple LR circuit, valid at all times, expressing their result as a differential equation for the current through the inductor. Then they are given an ansatz and asked to solve the differential equation using the loop rule and the behavior of the circuit just after the switch is closed.

Problem 5

Multi-loop LR circuits are a favorite on exams. For a reasonably involved problem like this one, I recommend that the students begin by writing out loop and junction rules that are valid at all times. Then they can specialize them to the various time regimes.

As a matter of board technique (see diagram below), when you discuss this problem you might want to start by writing your loop and junction rules on the center board. (These are the versions that are valid for all times. You can write a title at the top: "Any time".)

Then, when you want to specialize the rules to $t=0^+$, do so on the left-hand board. (You can write a title at the top of the left-hand board: "Immediately after".) Be sure to re-write *all* of the formulas from the middle board, even you know ahead of time that they don't read any differently at $t=0^+$. This is so that you can solve your system of equations in a self-contained way while remaining at the left-hand board. (This is also a good organizational technique for your students to use on exams.)

Finally, when you want to specialize to $t=\infty$, do so on the right-hand board. (You can write a title at the top of the right-hand board too: "A long time later".) Again, re-write all of the formulas from the middle board, even if you know ahead of time that they don't read any differently at $t=\infty$.

(The way I've been speaking, it sounds as if *you're* doing all the work here. But of course it goes without saying that you should make the students do most of the work. For example, don't really write a loop rule all by yourself; instead, tell the class what loop you have in mind, and then make someone tell *you* what to write.)

At the very end of your discussion of this problem, it would be a good idea to summarize the behavior of the circuit with three graphs: $i_1(t)$, $i_2(t)$, and $i_3(t)$. Be sure to label the axes with given information from the problem. Draw these graphs one above the other, so you can point out how $i_2(t)$ and $i_3(t)$ add up to $i_1(t)$ at all times.