M-6. Inductance

Teaching Notes

THE MAIN POINT

1. Understand that inductance is determined by the geometry of the conductors.
2. Calculate inductance for simple geometries.
3. Introduce magnetostatic energy density and energy storage in inductors.

SAMPLE LESSON PLAN

Discussion Question 1 .... 10 minutes in groups, 5 minutes GSI-led discussion & boardwork.
Mini-Lecture I .... 5 minutes. (Introduce inductance if necessary.)
Discussion Question 2 .... 10 minutes in groups, 5 minutes GSI-led discussion & boardwork.
Problem 1, parts (a)-(d) ...... 15 minutes in groups, 10 minutes GSI-led discussion & boardwork.
Mini-Lecture II.................... 5 minutes. (Introduce magnetostatic energy density if necessary.)
Problem 1, parts (e), (f) ...... 10 minutes in groups, 10 minutes GSI-led discussion and boardwork.
Problem 2 or 3 .... 20 minutes in groups, 10 minutes GSI-led discussion & boardwork.

GENERAL TEACHING SUGGESTIONS

Geometric nature of inductance. Discussion Question 2 emphasizes the geometric nature of inductance, in much the same way that Discussion Question 1 of Worksheet E-6 emphasized the geometric nature of capacitance.

Strategy for calculating inductance. As with capacitance problems, emphasize a standard strategy for finding the inductance of an arrangement of conductors (such as a solenoid, a coaxial cable, or what have you). Again, Elby's book *The portable TA*, Volume 2, does a good job with this. (See Chapter 43.) Here's one version of the strategy:

Imagine that the conductors carry currents ±i.
Find the resulting magnetic field.
Find the flux of this magnetic field through the arrangement.
Find the induced EMF by differentiating the flux with respect to time.
It will always turn out that the induced EMF is proportional to di/dt. The proportionality constant $EMF_{induced}$ / (di/dt) is called the *inductance*, and depends only on geometric factors.

Problems 1 and 2 are laid out according to this strategy---but do not assume that your students will notice this on their own. Summarize the steps on the board.

Energy in magnetic fields. Do not neglect the problems on this worksheet dealing with energy storage in inductors. (See the Sample Lesson Plan above.) As to why inductors store energy in the first place, emphasize two points of view:

Work done to get the current going. Because of back-EMFs, you will have to do some work to get a current flowing through an inductor. When you do 75 J worth of work on the inductor, you can think of those 75 J as being stored in the inductor.

Energy stored in the magnetic field of the inductor. When you set up a current in an inductor, you create a magnetic field in the inductor. Since magnetic fields store energy in each cubic centimeter of space, the region in which the inductor's field is non-zero yields a certain store of energy.

Correspondences. There are many correspondences between electrostatic / capacitance ideas and magnetostatic / inductance ideas. Some students find that these analogies help them to keep the numerous formulas organized in their minds. I sometimes write a table on the board, showing the analogous formulas side by side.

Mutual inductance vs. self-inductance. In most of the problems on Worksheet M-5 *Faraday's Law*, we concentrated on the flux of one object's magnetic field through *another* object. Now, however, we are mainly concentrating on *isolated* objects whose magnetic fields pierce *themselves*. To put it another way, in 7B we almost always calculate *self*-inductances rather than *mutual* inductances.

SAMPLE MINI-LECTURE AND BOARD SUMMARY

It should not be necessary for you to lecture much at the start of section, especially if you are beginning with Discussion Question 1, which is a bit of Faraday's Law review. However, in between Discussion Questions 1 and 2, you will probably want to introduce the notion of inductance. (See the Sample Lesson Plan above.)

Mini-Lecture I. To introduce inductance, I like to observe that **the "back-EMF" in Discussion Question 1 only arises when we try to *change* the current in the loop**. Make sure everyone understands this.

Given this fact, it is not too surprising that the magnitude of the back-EMF will depend on the *rate of change* of the current, di/dt:

$$\textbf{back-EMF} \propto \textbf{di/dt}.$$

The proportionality constant is called the *inductance*, L, and it is **completely determined by the shape of the conductor in question**.

$$\textbf{back-EMF = L di/dt}.$$

We will learn how to calculate inductances from scratch when we get to the problems. There will be a standard strategy for this, and it may remind you of how we used to calculate *capacitances* from scratch back in electrostatics.

Mini-Lecture II. After the students have completed parts (a) - (d) of Problem 1, you may need to say a few words about *magnetostatic energy density* before they can do part (e). Keep this brief. Remind them about how electric fields store energy in each cubic centimeter of space. Turns out, magnetic fields *also* store energy in each cubic centimeter of space. Even the formulas look similar:

$$u_E = (1/2)\, \varepsilon_0 E^2 \quad \text{for the electrostatic energy density}$$
$$u_M = (1/2)\, B^2 / \mu_0 \quad \text{for the magnetostatic energy density.}$$

Both of these energy densities have units of **Joules per cubic meter**.

REMARKS ON THE DISCUSSION QUESTIONS AND PROBLEMS

Discussion Question 1

This is really just a Faraday's Law question, a bit of "review" that has been inserted in an attempt to make sure that all of your students are up to speed before moving on.

You can also use this question to set up your remarks on inductance. (See the Sample Lesson Plan and Mini-Lecture I above.)

Discussion Question 2

This question isolates the geometric nature of inductance, in much the same way that Discussion Question 1 of Worksheet E-6 isolated the geometric nature of capacitance.

Problem 1

If you do only one problem on this worksheet, it should be Problem 1. (See the Sample Lesson Plan above.) One reason for this is the way the problem steps the students through a typical self-inductance calculation, outlining the basic strategy in the process. (After all is said and done, *summarize this strategy on the board*. See the General Teaching Suggestions above.)

Another reason you should do this problem is that it facilitates a discussion of energy storage in inductors. Note however that you may have to introduce magnetostatic energy density before the students can do part (e). See the Sample Lesson Plan, as well as Mini-Lecture II above.

After discussing parts (d) and (e) of this problem in front of the class, the board should contain somewhere the expressions

$$L = \mu_0 \pi R^2 n^2 \ell,$$

for the inductance of the solenoid, and

$$U = (1/2)\ \mu_0 \pi R^2 n^2 \ell\ \ i_0^2,$$

for the energy stored in the solenoid's B-field. Circle the junk in this middle of this last expression, so the students can see that it is just "L." This means that the energy stored in the inductor could be written as

$$\boxed{U_{\text{stored inductor}} = (1/2)\ LI^2}.$$

This relationship holds true for all inductors. It is the magnetic analogue of $U_{\text{stored in capacitor}} = (1/2)Q^2/C$.

Problem 2

Problem 2 is similar to Problem 1, the difference being that Problem 2 does not investigate the *magnetostatic energy* stored in the coaxial cable.

If you want to save time by doing Problem 2 *instead* of Problem 1 (rather than *in addition to* Problem 1, as in the Sample Lesson Plan above), then you might consider adding one more part:

f) When a current i_0 flows through the coaxial cable, how much energy is stored in the magnetic field of the cable?

You may have to say a few words about magnetostatic energy density before the students can answer this question. See Mini-Lecture II above.

Once you have the answer to part (f) in hand, point out that the energy stored in the inductor can be written as

$$\boxed{U_{\text{stored inductor}} = (1/2)\, LI^2}.$$

where L is the inductance and i is the current flowing through the inductor. This relationship holds true for all inductors. It is the magnetic analogue of $U_{\text{stored in capacitor}} = (1/2)Q^2/C$.

Problem 3

This problem is somewhat tangential, in that we find the inductance by methods *other* than the standard strategy. By all means, do this problem if you have time. But I would save it for last. (See the Sample Lesson Plan above.)