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Advice:

Today: lec 2 Linear Circuits I – lab 1.2
lec 3 TH Jan 25
lec 4 TUE Jan 30
lec 5 TH Feb 1

Notes: Do pre-lab questions before labs.
Attempt to answer all of them.
Don't hand in incomplete labs.
–3 per day for late reports.

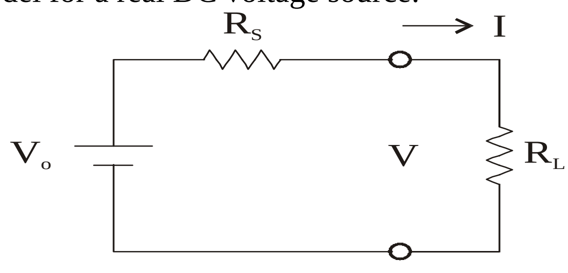
Office Hours: – by appointment – JLSiegrist@lbl.gov

Introduction & conclusion content guidelines outside TA office.
+2 points for experiment evaluation forms on time.
Get pre-lab questions signed off < Friday.

Circuit Analysis Example

Equivalent Circuits, Input & Output Impedance

Divide circuit into subunits – interested in terminal characteristics (I vs. V) of subunits
⇒ replace them with a simpler equivalent circuit.
In general, for a linear system, (original contains voltage & current sources, resistances) an equivalent circuit is one with the same I-V characteristic as the original, at all frequencies of interest.
Model for a real DC voltage source:



$R_S = 0$ for an ideal source (V independent of I)
The relation between V across R_L and the open-circuit voltage V_o is the voltage transfer

$$V = I(R_L) = \frac{V_o}{R_S + R_L}(R_L) \Rightarrow \frac{V}{V_o} = \frac{R_L}{R_S + R_L} \qquad \textit{voltage divider}$$

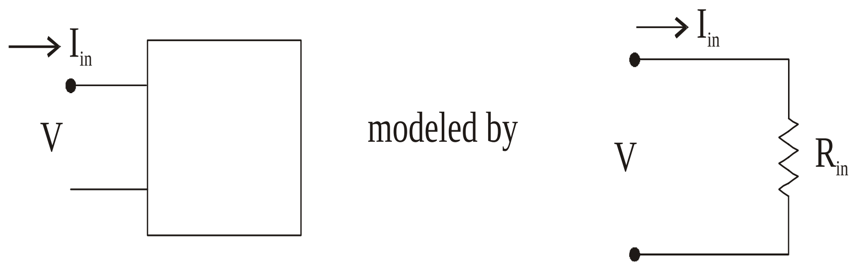
$R_L \gg R_S \Rightarrow V = V_o \quad \Rightarrow$ ~ ideal voltage source

$R_L \ll R_S \Rightarrow$ current through the load ~ V_S/R_S (short circuit current of the input network)

$\Rightarrow$ ~ current source

Find R_S by varying R_L to measure I-V characteristic – output impedance (resistance) of the source.

Likewise, measure i/v for R_L (or the 2-terminal linear device and find is input impedance [resistance, for now])

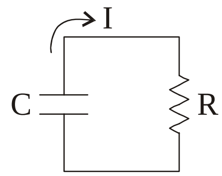


Vary I_{in}, measure V vs. $I_{in} \Rightarrow$ find R_{in} as $R_{in} = dV_{in}/dI_{in}$, the slope of the line.

Circuit Analysis RC Circuits

To introduce RC circuits:

consider



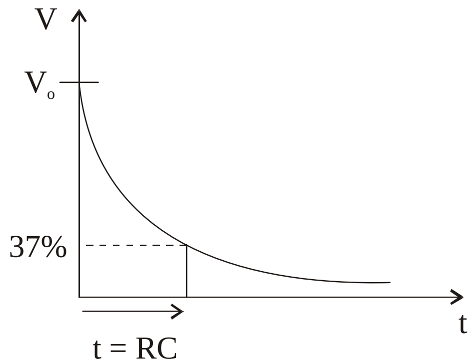
Current through the capacitor is:

$$Q = CV \quad \Rightarrow \quad \frac{dQ}{dt} = I = C\frac{dV}{dt}$$ [can't change voltage across C instantly!]

but $I = -\frac{V}{R}$ so $C\frac{dV}{dt} = -\frac{V}{R}$

Solution to this differential equation is $V = Ae^{-t/RC}$

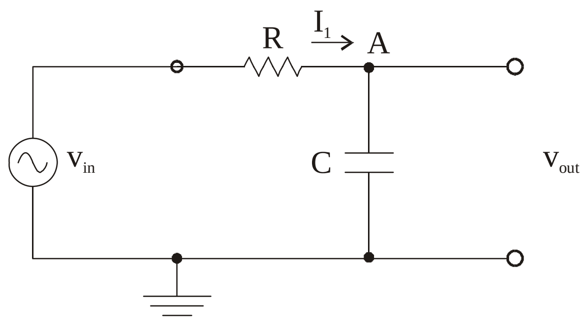
So a capacitor charged to V_o with R placed across:



discharge waveform, RC has units seconds, time constant of the circuit.

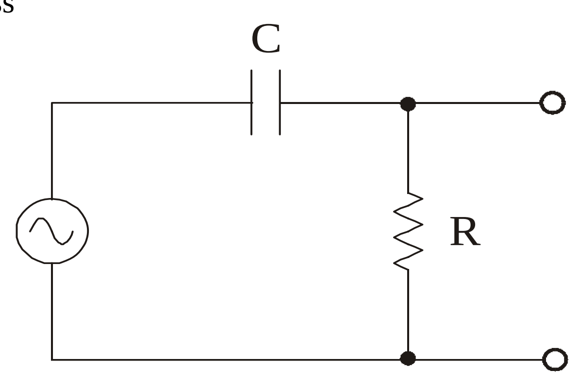
Circuit Analysis Example

Frequency Response – Low Pass Filter



Low Pass: C → short at high ω

Hi-Pass



v_{in} = V cos ωt

open circuited ⇒ Σ currents at node A = 0 ⇒ $Q = CV \quad \Rightarrow \quad \frac{dQ}{dt} = I = C\frac{dv}{dt}$

$$C\frac{d}{dt}v_{out} = \frac{V\cos\omega t - v_{out}}{R} \quad \Rightarrow \quad \frac{d}{dt}v_{out} + \frac{1}{RC}v_{out} = \frac{1}{RC}V\cos\omega t$$

linear differential equation with constant coefficient, driven by $\frac{V}{RC}\cos\omega t \;\Rightarrow$ v_{out} has the form $v_{out} \sim V_1\cos(\omega t+\phi)$

to solve, write: $V_1[\cos(\omega t+\phi)] = A\sin\omega t + B\cos\omega t$

so, $V_1(\cos\omega t\,\cos\phi - \sin\omega t\,\sin\phi) = A\sin\omega t + B\cos\omega t$

equate coefficients $\Rightarrow$

$$A = -V_1\sin\phi$$
$$B = V_1\cos\phi$$

take ratio $\Rightarrow$ $\tan\phi = -A/B$ & $V_1 = \left[A^2+B^2\right]^{1/2}$

going back to differential equation, put in

$$v_{out} = A\sin\omega t + B\cos\omega t = V_1\cos(\omega t+\phi) \qquad \text{with} \quad \begin{array}{l} A = -V_1\cos\phi \\ B = V_1\sin\phi \end{array}$$

so $\frac{dv_{out}}{dt} = \omega A\cos\omega t - \omega B\sin\omega t$ and our differential equation

$\frac{d}{dt}v_{out} + \frac{1}{RC}v_{out} = \frac{1}{RC}V\cos\omega t$ becomes the algebraic equation

$$\omega A\cos\omega t - \omega B\sin\omega t + \frac{1}{RC}(A\sin\omega t + B\cos\omega t) = \frac{1}{RC}V\cos\omega t$$

equate coefficients $\Rightarrow$

$$(\sin) \quad \frac{A}{RC} - \omega B = 0$$

$$(\cos) \quad \omega A + \frac{B}{RC} = \frac{V}{RC}$$

$$or \quad B = \frac{A}{\omega RC}$$

$$and \quad A\left(\omega + \frac{1}{\omega(RC)^2}\right) = \frac{V}{RC}$$

$$so \quad A = \frac{V}{\omega RC + \frac{1}{\omega RC}} \quad \& \quad B = \frac{A}{\omega RC} = \frac{V}{\omega RC}\frac{1}{\omega RC + \frac{1}{\omega RC}}$$

$$or \quad A = V\left[\frac{\omega^2}{(1+\omega^2 RC)^2} + \frac{\left(1/RC\right)^2}{1+(\omega RC)^2}\right]^{1/2}$$

so, from above,

$$V_1 = \sqrt{A^2 + B^2}$$

$$= A\sqrt{1 + \frac{1}{(\omega RC)^2}}$$

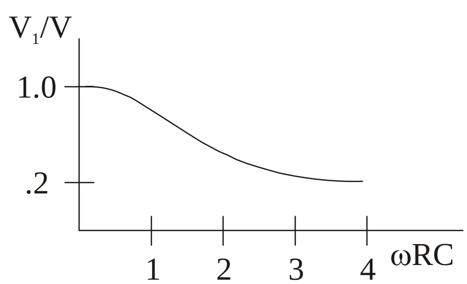
$$= \left(\frac{V}{\omega RC + \frac{1}{\omega RC}}\right)\sqrt{\frac{(\omega RC)^2 + 1}{(\omega RC)^2}}$$

$$= V\left(\frac{\omega RC}{(\omega RC)^2 + 1}\right)\left(\frac{\left[(\omega RC)^2 + 1\right]^{1/2}}{\omega RC}\right)$$

$$V_1 = \frac{V}{\left[1 + (\omega RC)^2\right]^{1/2}}$$

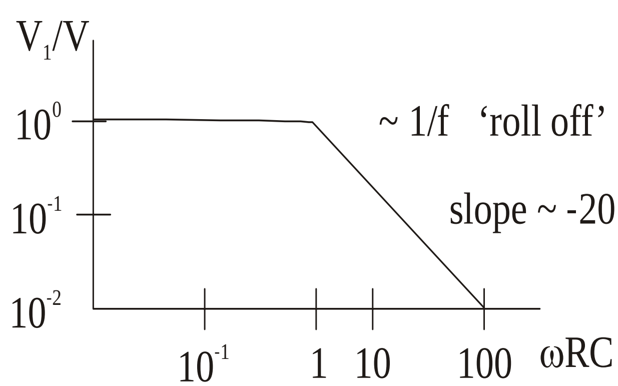
$$and \quad \phi = \tan^{-1}\left(-A/B\right) = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC)$$

Keep V fixed, look at v_{out} vs. ωRC –



⇒ transmits low ω well, high ω not at all ⇒ Low Pass Filter

Different frequencies give different amplitudes and phases for v_{out}.
Plot instead on a log-log scale (Bode Plot)

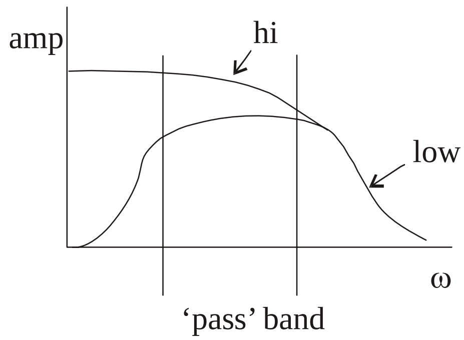


≈ set of straight line segments, where segments meet are corner or break frequencies $\left(= \frac{1}{RC}\right)$

As I said last time, useful unit: $Voltage(transfer)db \equiv db = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right)$

e.g. signal reduced by a factor of 10 has a gain of –20 db.

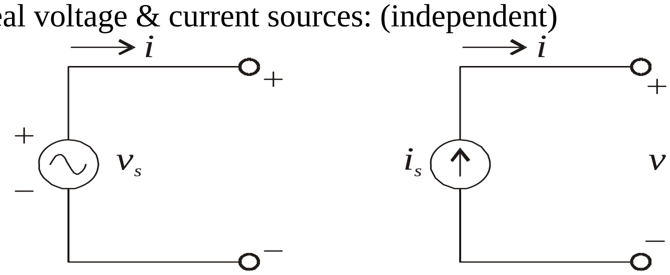
Put hi & low pass together to make filters: H&H 5.01-5.05, prob. 2.13.



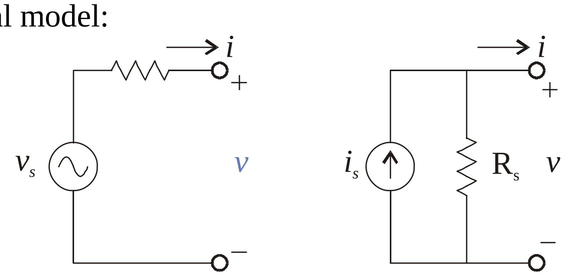
II. More Concepts on Networks

Networks

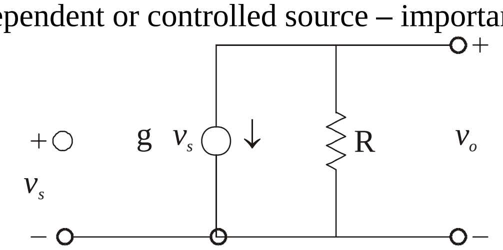
ideal voltage & current sources: (independent)



real model:



dependent or controlled source – important for modeling



Remember in using KVL – loop equations – there is a positive drop in the direction of current through a resistor and there is a positive drop in the direction '+' to '–' terminal of a battery, independent of the current direction. The effective resistance between any 2 points in the net is given by measuring the current I drawn by an external voltage source V, $R = V/I$, ideal voltage sources are shorted, ideal current sources are opened, and dependent sources are left ON.

Network Theorems

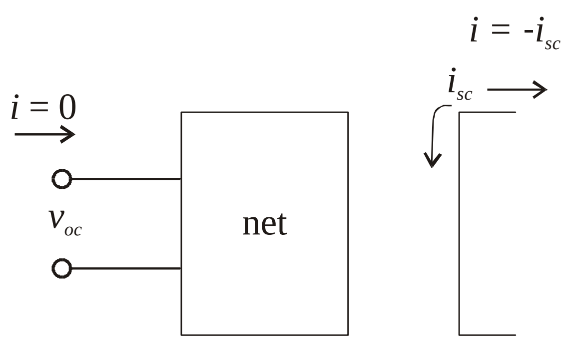
1) Superposition
The response of a linear network containing several independent sources is found by considering each generator separately and then adding the individual responses. (i.e. rest are shut off as above)

2) Thévenin's Theorem
Any linear network may, with respect to a pair of terminals, be replaced by a voltage generator V_{th} (equal to the open-circuit voltage) in series with the resistance R_{th} seen between these terminals. As above, to find R_{th}, all independent voltage sources are short-circuited, and all independent current sources are opened.

3) Norton's Theorem
Any linear network may, with respect to a pair of terminals, be replaced by a current generator (equal to the short-circuit current) in parallel with the resistance seen between the 2 terminals.
(two generators on page 5 are equal if $v_s = v$, $i_s = i$)
NB: $R = R_n = R_{th}$; $I_n = V_{th}/R$

4) Open Circuit Voltage – Short Circuit Current Theorems
If

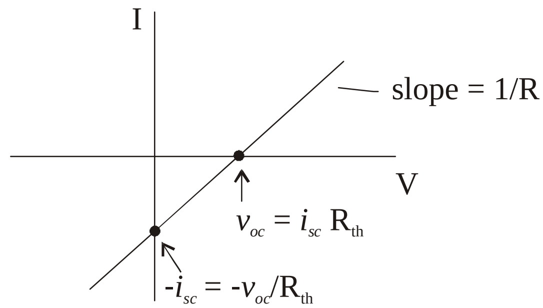
v_{oc} = open circuit voltage
i_{sc} = short circuit current
R_{th} = resistance between terminals

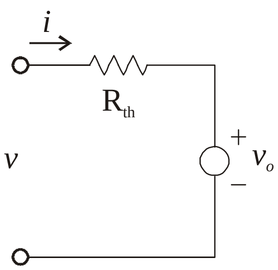


Then

$$v_{oc} = i_{sc} R_{th}\ ; \quad R_{th} = v_{oc}/i_{sc}$$

2 points determine the line:

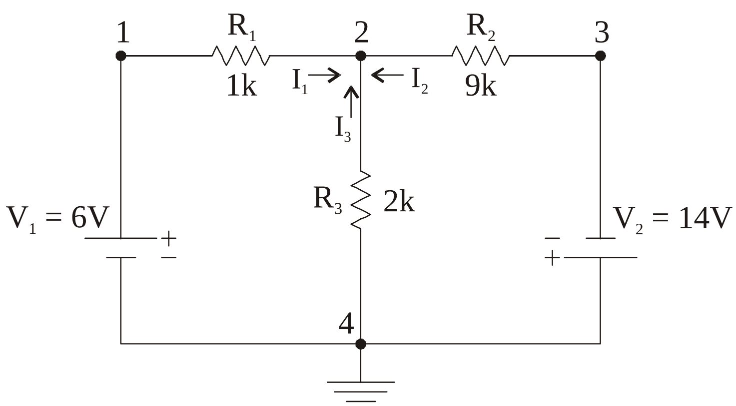




$$i = \frac{v}{R_{th}} - \frac{v_o}{R_{th}}$$

Using the Concepts

Simple Example:



What's the voltage across R_3?

Short node 2 to 4, then, using superposition:

$$i_{sc} = \frac{V_1}{R_1} - \frac{V_2}{R_2}$$

$$= 6\,mA - \frac{14}{9}\,mA$$

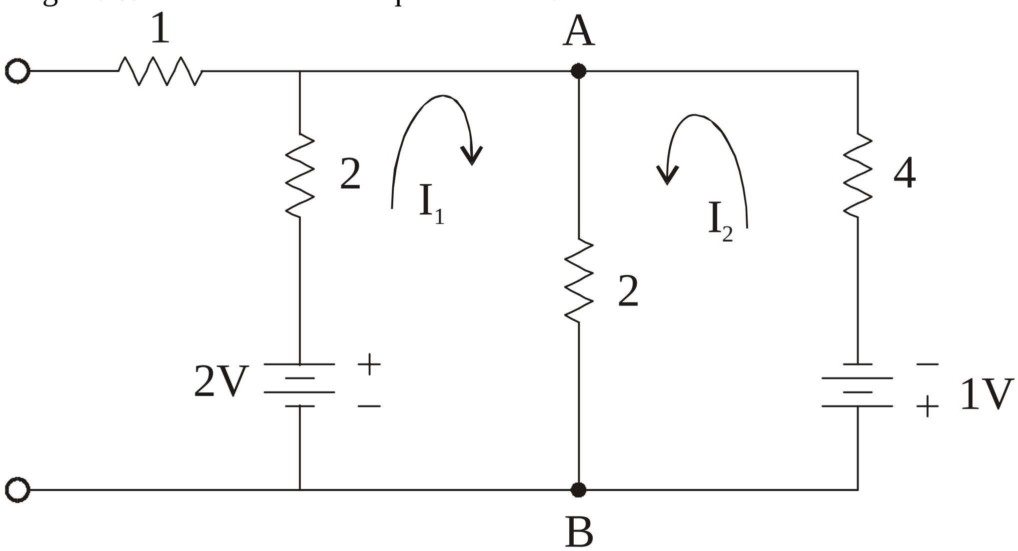
$$\sim (6 - 1.6)\,mA = 4.4\,mA$$

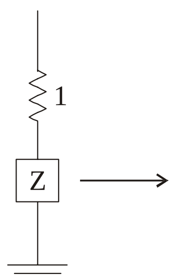
Turn off voltage sources, 2-4 resistance corresponds to $R_{1,2,3}$ all in parallel ⇒

$$\frac{1}{R_{th}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{9} = 1.6\left(\frac{1}{\Omega}\right) = 1.6\, mA/V$$

$$\Rightarrow \quad V_{24} = i_{sc} \cdot R_{th} = \frac{4.4\, mA \quad V}{1.6\, mA} = 2.8V$$

Lab Page 2: What is Thévenin's equivalent for:





find equation for box Z

voltage across A-B is

$$\text{for } A-B:\ i_{sc} = I_1 - I_2$$

$$= \frac{2V}{2\Omega} - \frac{1V}{4\Omega}$$

$$= \frac{3}{4}\frac{V}{\Omega} = \frac{3}{4} Amp$$

short voltage sources, A-B resistance is 2, 2, 4, in 11 ⇒

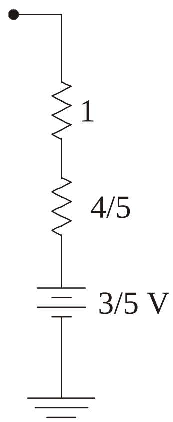
$$\frac{1}{r_{eq}} = \frac{1}{2} + \frac{1}{2} + \frac{1}{4}$$

$$r_{eq} = 4/5\,\Omega \quad so \quad V_{AB} = i_{sc} \cdot R_{eq}$$

$$= \frac{3}{4}\frac{V}{\Omega} \cdot \frac{4}{5}\Omega$$

$$= \frac{3}{5}V$$

so



so for whole box,

$$V_{oc} = 3/5\,V$$

$$r_{eq} = 9/5\,\Omega$$

so