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Advice:

Today:	lec 3	Linear Circuits II	
	lec 4	TUE	Jan 30
	lec 5	TH	Feb 1
	lec 6	TH	Feb 8

Get your labs handed in.
Read labs 2 & 3!

Concepts

Return to AC signals: steady state sinusoids:
For sinusoidal signals

$$i_R = \frac{V}{R} = \frac{V_o}{R}\cos\omega t \quad \textit{in phase with voltage}$$

$$i_c = C\frac{dV}{dt} = -\omega C V_o \sin\omega t$$
$$= \omega C V_o \cos(\omega t + 90^\circ)$$

⇒ current in capacitor <u>leads</u> the voltage by 90° but, still ∝ V_o

⇒ Rescue Ohm's Law by representing amplitude & phase of signal in the complex plane.

- Voltage & currents represented by the complex quantities $\underset{\sim}{V}$ & $\underset{\sim}{I}$ (amplitude & phase)
  $V_o \cos(\omega t + \phi) \to V_o e^{i\phi} e^{i\omega t}$

- Active voltages & I obtained by multiplying $\underset{\sim}{V}$, $\underset{\sim}{I}$ by $e^{i\omega t}$ & taking real part:
  $V(t) = \text{Re}\{\underline{V} e^{i\omega t}\}; \quad I(t) = \text{Re}(\underline{I} e^{i\omega t})$
  $\Rightarrow V(t) = \text{Re}\{V\}\cos\omega t - \text{Im}\{V\}\sin\omega t$
  $I(t) = \text{Re}\{I\}\cos\omega t - \text{Im}\{I\}\sin\omega t$

Forget differential equations: reduce to algebraic in a systematic way. $\left(\frac{d}{dt} \text{ pulls out } i\omega\right)$

N.B. $e^{i\phi} = \cos\phi + i\sin\phi \quad (NO\ \ j\ \ from\ me!)$

Add & subtract in rectangular form a + *i*b

Multiply & divide in polar form $z = |z|e^{i\omega}$

$$z_1 z_2 = |z_1||z_2|e^{i(\phi_1+\phi_2)}$$

$$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}e^{i(\phi_1-\phi_2)}$$

$\Rightarrow$ complex Ohm's Law:

$$V = IZ$$

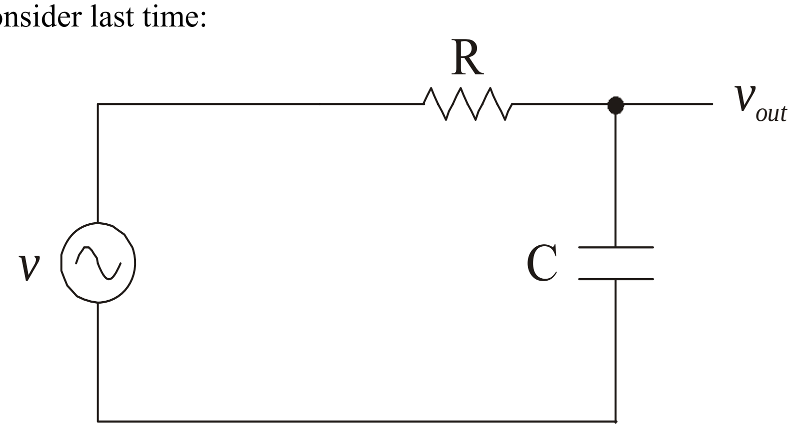
$$I = V/Z \quad , \quad Z\ ohms$$

If I = phase lead by 90°, (voltage?)

$I_C = i\omega CV$

And the ratio $V/I = \frac{1}{i\omega C} \equiv X_C \equiv Z_C$ is the reactance. ($\frac{V}{I} = R$ for a resistor reactance = resistance) (≡ complex resistance)

Consider last time:



voltage divider:

$$|v_{out}| = \left| \frac{\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}} \right| V_{in} = \left| \frac{1}{1 + i\omega RC} \right| V_{in}$$

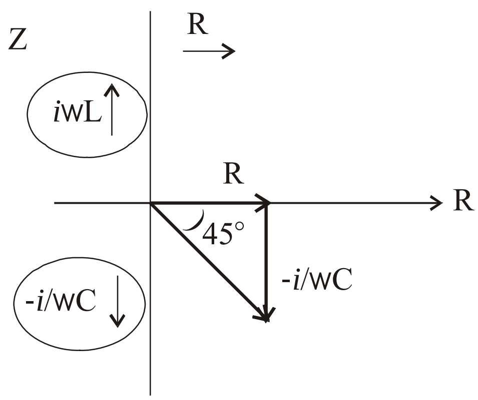
$$= \left[\frac{1}{(1 + i\omega RC)} \frac{}{(1 - i\omega RC)} \right]^{1/2} V_{in}$$

$$= \left[\frac{1}{1 + (\omega RC)^2} \right]^{1/2} V_{in}$$

$$= \frac{V_{in}}{\left[1 + (\omega RC)^2\right]^{1/2}}$$

transfer function: $H(i\omega) = \frac{|V_{out}|}{|V_{in}|} = \frac{1}{\left[1 + (\omega RC)^2\right]^{1/2}}$

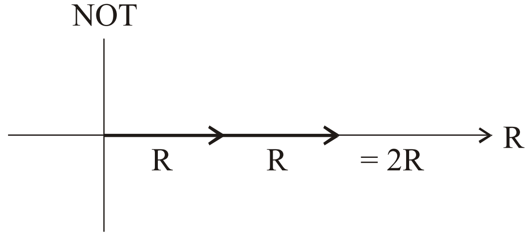
3db point: $\omega RC = 1, \quad Amplitude\ V_{out} = \frac{1}{\sqrt{2}} V_{in}$



$$Z_{total} = R - i/\omega C = R\sqrt{2}$$

$$|Z_{total}| = \left[R^2 + \frac{1}{\omega^2 C^2} \right]^{1/2} \neq R + \frac{1}{\omega C}$$

NOT



(6 db) down

Concepts

For an inductor, $v = L\, {di}/{dt}$

$$\frac{V}{I} = i\omega L \qquad inductor \Rightarrow short\ a\ low\ \omega$$

$$\frac{V}{I} = \frac{1}{i\omega C} \qquad capacitor \Rightarrow short\ at\ high\ \omega$$

$$\frac{V}{I} = R \qquad resistor$$

Impedance $Z \equiv \frac{V}{I}$ steady state sinusoids, between 2 points in the net, as before

In ohms, act like resistors.
reactance, resistance ≡ types of impedance

Further definition:

$$Admit\tan ce \equiv Y = \frac{1}{Z} = G + iB \qquad (Admit\tan ce) = \frac{1}{R}$$

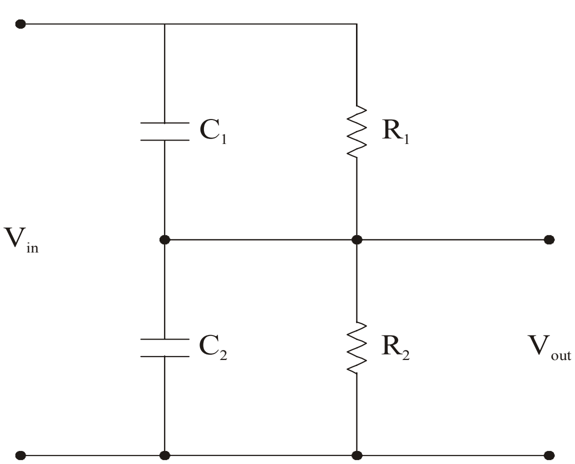
G is conductance
B is susceptance
Network theorems still hold.

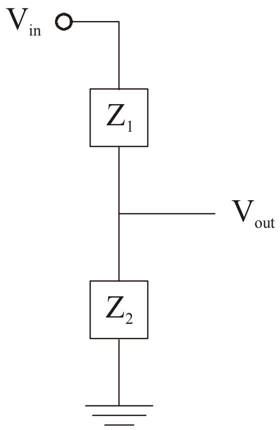
$$Power = \mathrm{Re}(IV^*) = \mathrm{Re}(V^*I)$$

NB: Power = 0 through pure reactive components
IV ~ sin*cos, ∫ over one cycle = 0

Concepts

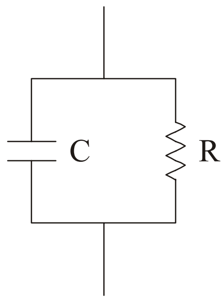
Apply to this problem:





$$V_{out} = \frac{Z_2}{Z_1 + Z_2} V_{in}$$

$Z_1, Z_2 \equiv$



$$\frac{1}{Z_{eq}} = \frac{1}{Z_C} + \frac{1}{Z_R} \Rightarrow Z_{eq} = \frac{Z_R\, Z_C}{Z_R + Z_C}$$

parallel combination $\Rightarrow$

$$Z_{eq} = \frac{R\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}} = \frac{R}{1 + iR\omega C}$$

$$\Rightarrow V_{out} = V_{in}\frac{Z_2}{Z_1 + Z_2} = \frac{\frac{R_2}{1 + iR_2\omega C_2}}{\frac{R_1}{1 + i\omega R_1 C_1} + \frac{R_2}{1 + i\omega R_2 C_2}} \cdot V_{in}$$

divide by numerator

$$V_{out} = \frac{1}{1 + \frac{R_1}{R_2}\left(\frac{1 + i\omega R_2 C_2}{1 + i\omega R_1 C_1}\right)} \cdot V_{in}$$

$$\omega \to 0 \Rightarrow \frac{R_2}{R_1 + R_2} V_{in}$$

$$\omega \to \infty \Rightarrow \frac{V_{in}}{1 + \frac{i\omega R_1 R_2 C_2}{i\omega R_1 R_2 C_1}} = \frac{C_1}{C_1 + C_2} \cdot V_{in}$$

$$\omega \ independence: \quad \frac{1 + i\omega R_2 C_2}{1 + i\omega R_1 C_1} = K$$

$$(or \ take \ derivative = 0)$$

solution: $\omega R_2 C_2 = \omega R_1 C_1$
ratio a constant if $R_2 C_2 = R_1 C_1$ by inspection

Concepts

Fourier Analysis

Note that any repetitive signal can be decomposed into an (infinite) series of harmonic sinusoids.

e.g. $F(t) = \sum_{n=0}^{\infty} A_n \sin(n\omega t) + B_n \cos(n\omega t)$

$$A_n = \frac{2}{T}\int_0^T F(t)\sin(n\omega t)\,dt$$

with $$B_n = \frac{2}{T}\int_0^T F(t)\cos(n\omega t)\,dt$$

& $$B_o = \frac{1}{T}\int_0^T F(t)\,dt$$

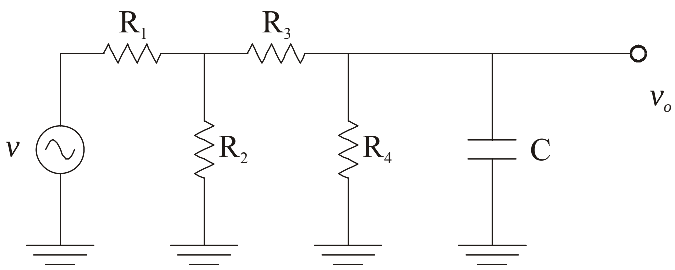
where T is the period, $\omega = 2\pi / T$

You have seen this in your courses already:
(my) reference – Arfken, Mathematical Methods for Physicists.

Concepts

Single Time Constant Circuits

⇒ Circuits that can be reduced to or composed of one capacitor & one resistor.
⇒ Important issue is figuring out the time constant.



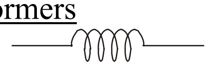
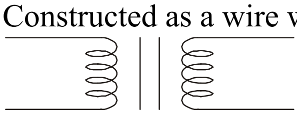
Quick method to find equivalent circuit:

1. reduce excitation to zero
    a. short voltage source
    b. open current source
2. if circuit has one capacitor (or inductor): 'grab' 2 terminals of C, find R_{eq}
   e.g. in example:
   $R_{eq} = R_4 // (R_3 + (R_2 // R_1))$
   $\tau = R_{eq} C$
3. invert for one R, many C or L

Read about step & pulse response of STC – H&H 1.14; S&S App F

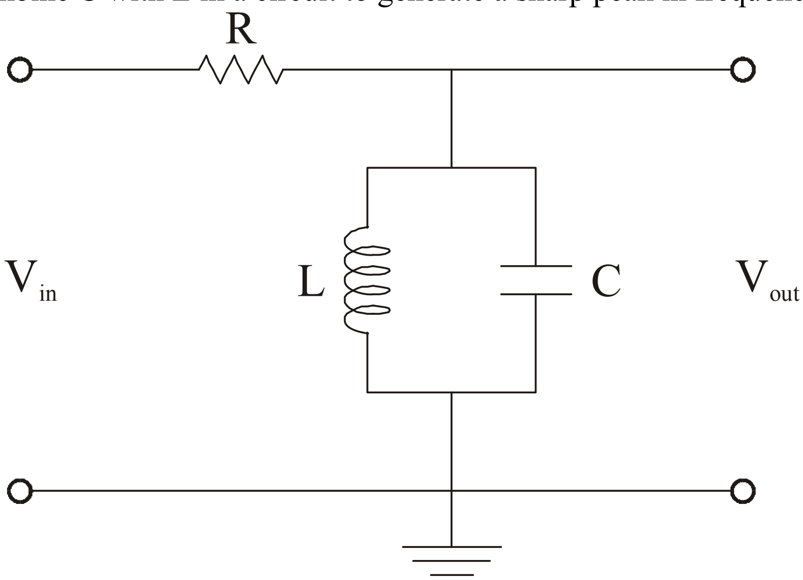
<u>Circuit Analysis</u>

<u>Inductors and Transformers</u>

- Inductors L 
  rate of change of I in L depends on applied voltage $V = L\frac{dI}{dt}$
  L is inductance in Henries
- Power is not dissipated as heat (IV) but stored in the magnetic field of L $Z_L = i\omega L$
- Constructed as a wire wound around magnetic material to make a transformer:
  
  two coupled coils [primary, secondary]
  AC applied to primary ⇒ <u>constant power</u> through transformer ⇒ step down or up
  voltage multiplication ∝ turns ratio
  current multiplication ∝ $\frac{1}{turns\ ratio}$
  N.B. <u>Turning off</u> circuits with inductors ⇒ *boom* they don't like it so, ramp down slowly, e.g. an accelerator.

<u>Resonant Circuits</u>

Combine C with L in a circuit to generate a sharp peak in frequency characteristics.



Reactance of the LC combination at frequency ω is

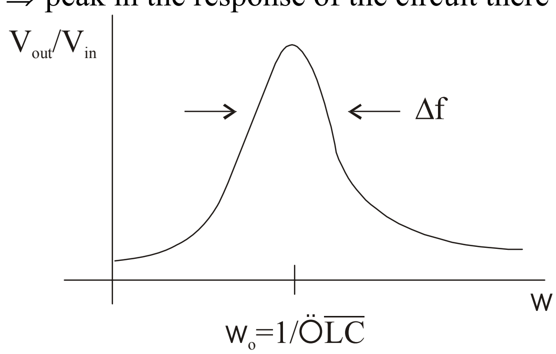
$$\frac{1}{Z_{LC}} = \frac{1}{Z_L} + \frac{1}{Z_C} = \frac{1}{i\omega L} - \frac{\omega C}{i}$$

$$= i\left(\omega C - \frac{1}{\omega L}\right)$$

$$or \ \ Z_{LC} = \frac{i}{\left[\left(1/\omega L\right) - \omega C\right]}$$

At $\omega_o = \frac{1}{\sqrt{LC}}$, then $Z_{LC} = \infty \quad (open)$

$\Rightarrow$ peak in the response of the circuit there



$$Quality \ \ factor \ \ Q \equiv \frac{resonant \ \ frequency \ \omega_o}{\Delta f \ \equiv width \ at \ the \ -3db \ \ points}$$

<u>Devices, Instruments, and Tools</u>

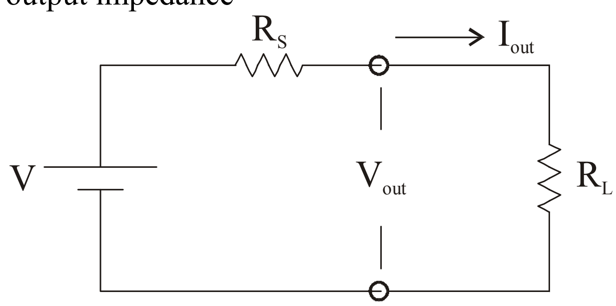
1. Spectrum Analyzer
2. Network Analyzer

Measuring Techniques

Filter ω response: sweep ω, measure ${V_{in}}/{V_{out}}$

straightforward for linear circuits, $\omega_{in} = \omega_{out}$, only phase and amplitude change

For a real voltage source, we said we found i_s by varying R_L to measure I-V characteristic ⇒ output impedance



What are the best values of R_L to use for a good measurement?

Note $V_{out} = I_{out} R_L = \frac{V}{(R_S + R_L)} R_L$

How does V_{out} change with changes in R_L when we make the measurement?

$$\frac{dV_{out}}{dR_L} = \frac{(R_S + R_L)V - VR_L}{(R_S + R_L)^2}$$

$$= \frac{R_S V}{(R_S + R_L)^2}$$

for $R_L \gg R_S$, ${dV_{out}}/{dR_L} \sim 0$, no sensitivity, ($V_{out} = V$)

for $R_L \ll R_S$, ${dV_{out}}/{dR_L} \sim {V}/{R_S}$, no sensitivity & ratio can't be separated (short = I_{th} = ${V}/{R_S}$) so short + open gives R_S

We are not using an ammeter ⇒

use $R_L \sim R_S$ – R_L near R_S gives best sensitivity

Reading for next week, note:

Tools

New tool in lab 3 – PSPICE, circuit simulation package.
We'd be using that to model circuit performance.
Discussion in Appendix C of Sedra/Smith.

- Many, many other simulations, modeling, and design packages used by 'real people', but SPICE is earliest version & an early standard.
- For complex circuits, absolutely crucial.
  We'll only get a flavor…