Abstract

We update and improve the Gourinchas and Rey (2007a) dataset of the historical evolution of US external assets and liabilities at market value since 1952 to include the recent crisis period. We find strong evidence of a sizeable excess return of gross assets over gross liabilities. The center country of the International Monetary System enjoys an “exorbitant privilege” that significantly weakens its external constraint. In exchange for this “exorbitant privilege” we document that the US provides insurance to the rest of the world, especially in times of global stress. This “exorbitant duty” is the other side of the coin. During the 2007-2009 global financial crisis, payments from the US to the rest of the world amounted to 19 percent of US GDP. We present a stylized model that accounts for these facts.

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1 Introduction

The existence of a lasting “exorbitant privilege” -a higher return on US external assets than on its external liabilities- is an important and intriguing stylized fact in international economics. As shown in Gourinchas and Rey (2007a), this excess return can be divided into a composition effect resulting from an asymmetric structure of the external balance sheet of the United States -assets are riskier and less liquid than liabilities- and a return effect -an excess return within class of assets-. One direct consequence of the exorbitant privilege is to relax the external constraint of the U.S., allowing it to run larger trade and current account deficits without worsening its external position commensurately. Understanding the source of this exorbitant privilege is an important step in understanding the nature of the adjustment process for the U.S. and whether this is a sustainable phenomenon or not. From that perspective, the financial crisis provides a new and important empirical observation: the dramatic worsening of the net foreign asset position of the United States between the third quarter of 2007 and the first quarter of 2009. The precipitous fall of a magnitude amounting to 19% of GDP is both due to flows (with the US selling assets abroad more than foreigners selling US assets) and to a dramatic adjustment in valuations (the price of US holdings abroad contracting more than the rest of the world holdings in the US). This last development indicates a reversal of the usual “exorbitant” transfer from the rest of the world to the US: during a crisis, wealth flows from the US to the rest of the world. We call this phenomenon the “exorbitant duty” of the US: in times of global stress, the US effectively provides insurance to the rest of the world.

We argue that the “exorbitant duty” and the “exorbitant privilege” are two sides of the same coin. They reflect the structure of payments associated with an implicit insurance contract between the U.S. and the rest of the world. In his paper, (a) we provide the most up-to-date detailed evidence on the magnitude and composition of external returns in normal times. This evidence uses the latest available data as well as some recently unearthed historical surveys of cross border holdings. It also incorporates the developments
in the literature since Gourinchas and Rey (2007a). Our new results are largely in line with our earlier work; (b) we document the “exorbitant duty” i.e. the economic magnitude of the payments from the US to the rest of the world in the recent crisis. We show that this insurance mechanism was also there –although to a lower extent– during earlier episodes of global stress; (c) we provide a simple calibrated model that allows us to make sense of the structure of external returns. In the model, the US explicitly provides insurance to the ROW since it is assumed to have a greater risk tolerance. This captures a host of potential mechanisms by which the US economy may be able to better handle economic and financial risks. The model is able to reproduce the following features: (i) the US exhibits exorbitant privilege in normal times and exorbitant duty in times of global stress; (ii) the US runs trade deficits on average; (iii) the US portfolio is leveraged, hence there is both a composition and a return effect. However, the return effect requires that foreign government bonds experience larger default risk when global crisis occurs. The model does not account for everything we see in the data, however. In particular, while the model can generate the large collapse in net foreign assets of the US in crisis times, it cannot account for the large net foreign asset debtor position in good times. One possible interpretation, left to future work, is that under incomplete markets, foreign countries face excessive incentives to accumulate reserves due to a pecuniary externality (as in Caballero and Cowan (2008), or Lorenzoni (2008), or Aiyagari (1995)). This externality may push the center country into excessive debt and subject the international financial system to a Triffin (1960) type problem, where a decreased confidence in the centre country may lead to a run.

2 External balance sheet structure and returns

Financial globalization started in the 1980s and substantially accelerated in the 1990s, as evidenced by the massive surge in gross external assets and liabilities as a fraction of GDP. A recent burgeoning literature has extracted interesting stylized facts from cross country data on international investment positions (see Lane and Milesi-Ferretti (2001) for an early
contribution). Studying the composition of the balance sheet of countries is increasingly important to understand the dynamics of countries’ external adjustment. The traditional trade channel of adjustment, whereby current account deficits have to be made up for by future export surpluses has to be supplemented by a valuation channel, which takes into account capital gains and losses on the foreign asset position due to fluctuations in asset prices (Gourinchas and Rey (2007b)). An asymmetric structure of assets and liabilities, for example when assets and liabilities are in different currencies, leads to a very different adjustment process than a symmetric balance sheet. US external assets are mostly denominated in foreign currencies while US external liabilities are in dollars (Tille (2004), Lane and Shambaugh (2007)). It follows that a dollar depreciation gives rise to wealth transfers from the rest of the world to the United States. Similarly, earning excess returns on average on its external asset position allows a country to run larger current account deficits than it would otherwise, as the deterioration of the net international asset positions is muted by the capital gains.

Gourinchas and Rey (2007a) showed that the US earns an important average excess return on its net foreign asset position on the period 1952-2004. This finding fits well with the observation that in recent years, recent crisis excluded, the net international investment position of the United States has deteriorated at a speed significantly smaller than the current account deficit data would have suggested. Similar findings are reported in Obstfeld Rogoff (2005) or Meissner and Taylor (2006) on 1983-2003, Lane and Milesi-Ferretti (2005) on 1980-2004. In contrast, Curcuru et al. (2008) use a different methodology to compute returns and report no exorbitant privilege on the period 1994-2005. Forbes (2010) however reports 6.9% excess returns per year on 2002-2006 using Curcuru et al. (2008)’s methodology. More recently, Habib (2010) confirms the existence of excess returns of about 3% for the US on the period 1981-2008 and points out the singularity of the US in its ability to earn excess returns for long periods of time. None of the other countries of his broad panel has a similar “privilege”. Moreover Habib (2010)’s study points out that the bulk of the return differential comes from capital gains and not from differences in yields. Consistent with
Gourinchas and Rey (2007a), Habib (2010) also finds that a sizeable share of the excess returns does not come from a composition effect -external assets are less liquid and more risky than external liabilities and therefore earn a premium- but rather from a within asset class return differential. US bonds held by foreigners, for example, give a lower total real return than foreign bonds owned by US residents.

2.1 Data and methodology

This paper takes a fresh look at the historical evolution of the United States external position over the postwar period, including the recent crisis, by carefully constructing the US gross asset and liability positions since 1952 from underlying data and applying appropriate valuations to each components. The data construction methodology is described in Appendix A. Relative to our former work (Gourinchas and Rey (2007a)) we improve our existing dataset along several dimensions. We have disaggregated our data into government and corporate bonds on the bonds liability side and improved our measure of income flows for each type of assets. On the asset side we now also keep track of the dynamics of gold reserves. Importantly, we set initial net foreign asset positions using detailed Treasury surveys realized during the second world war: The 1943 Treasury Census of American-owned assets in foreign countries and the 1941 Treasury Census of foreign-owned assets in the US. Those surveys are detailed and reliable as they were of strategic importance for the United States while fighting against the Axis and for reparation payments after the war. The post-war estimates of the US net foreign asset position are based on these surveys on positions and measures of international capital flows. Since capital controls were in place during the Bretton Woods

1As explained in the foreword of the 1941 Survey: “On April 1940, when Germany invaded Denmark and Norway, the President of the United States issued an Executive order freezing the dollar assets of those two countries and their nationals. [...] Tens of thousands of banks, corporations and individuals in this country were required to file, on form TFR-300, reports giving detailed information with respect to foreign owned assets and the owners [...] Never before was as complete information available for analyses of the holdings of foreigners in this country.” The information contained in these surveys was of great strategic value to the United States. The 1941 Survey reports (p5) that “investigations to uncover enemy agents and enemy assets, especially after our entry into the war, were greatly facilitated by the TFR-300 information.” The 1943 Survey on American owned assets abroad “had its principal use in the war settlements and the postwar period generally, although it provided much greatly needed information during the latter part of the military phases of the war.”
period, the resulting estimates are quite precise as well. For the latter part of the sample we reconstruct the time series of the international investment position of the United States at market value and quarterly frequency from 1952:1 until 2010, benchmarking our series on the Bureau of Economic Analysis (BEA) official annual IIP positions. The data construction is described in details in the appendix. A key issue is the reconciliation of flow and position data often coming from different sources. The discrepancy between the two, labeled ‘other changes’ by the BEA, has been a residual item of significant size in recent years. A correct measure of the true returns on the net foreign asset position requires that this residual item be allocated between unrecorded capital gains, unrecorded financial flows, or mismeasured initial net asset position. Appendix A discusses formally how different measures of returns can be constructed under these different assumptions. Importantly, while the different assumptions have some impact on our calculated returns, they have no effect on our overall results: over long periods of times, the U.S. has experienced a high return on its net foreign assets, the ‘exorbitant privilege.’ Different results obtained in the literature seem to be mostly the result of a focus on relatively short time spans.

We provide a reconciliation spreadsheet of the external accounts of the US where all the accounting links between flows and valuations are very explicit. The spreadsheet is an interactive and transparent tool, allowing users to make the assumption they wish regarding measurement errors in the data. We believe this spreadsheet should be of major help to all researchers using US external accounts data and interested in their consistency.\textsuperscript{2}

\subsection{2.2 The “exorbitant privilege”}

The estimated excess returns are very robust to the assumptions one could make on methodology or errors in the data. We find that the excess total return of US gross external assets over its gross external liabilities is worth about 2\% per year between 1952 and 2009. During the Bretton Wood era, the very special role of the United States at the centre of the international monetary system was often lamented in French quarters. Besides finance minister

\textsuperscript{2}The spreadsheet “dynamic reallocation.xls” can be found at [TBA].
Giscard d’Estaing, who coined the term “exorbitant privilege” in 1965, economic advisor Jacques Rueff around the same time described the Dollar as a “boomerang currency”: the sizable external deficits of the US were not matched by commensurate gold losses, as creditor countries reinvested the dollar gained in their exports payments into the US economy. We adopt a somewhat narrower definition of the ‘exorbitant privilege’. In this paper, it refers to the excess return of US external assets on US external liabilities.

As a first benchmark, we allocate all mismeasured items in the evolution of the international investment position to mismeasured capital gains. As discussed in the appendix, this is the only assumption that leaves both measured positions and the recorded net exports unchanged, a reasonable assumption. Our results on external results are reported in Table 1, panel (a). We note that for the whole period 1952:1-2009:4 the excess returns of external assets \( r^a \) over liabilities \( r^l \) are very sizable at 2.69%. Since exchange rate movements are an important component of capital gains and losses (see Gourinchas and Rey (2007b)), we isolated the Bretton Woods and the Post Bretton Woods period. Interestingly, the magnitude of the “exorbitant privilege” has increased over time from about 1.3% between 1952:1-1972:4 to 3.47% during 1973:1-2009:4. One interpretation of that increased return is that the volatility of the leveraged US portfolio has increased during the fluctuating exchange rate period and that this increased volatility has gone hand in hand with an increase in excess returns. Indeed the volatility of external liabilities -almost exclusively in Dollars- is almost unchanged over the whole sample while the volatility of external assets, very low during the Bretton Woods era increased substantially after the collapse of the fixed exchange rate system.

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3 “The process works this way. When the U.S. has an unfavorable balance with another country (let us take as an example France), it settles up in dollars. The Frenchmen who receive these dollars sell them to the central bank, the Banque de France, taking their own national money, francs, in exchange. The Banque de France, in effect, creates these francs against the dollars. But then it turns around and invests the dollars back into the U.S. Thus the very same dollars expand the credit system of France, while still underpinning the credit system in the U.S. The country with a key currency is thus in the deceptively euphoric position of never having to pay off its international debts. The money it pays to foreign creditors comes right back home, like a boomerang. The functioning of the international monetary system is thus reduced to a childish game in which, after each round, the winners return their marbles to the losers. The discovery of that secret [namely, that no adjustment takes place] has a profound impact on the psychology of nations. This is the marvelous secret of the deficit without tears, which somehow gives some people the (false) impression that they can give without taking, lend without borrowing, and purchase without paying. This situation is the result of a collective error of historic proportions.” in Rueff (1971).
An alternative assumption would be to allocate all the mismeasurement to mismeasured flows, as in Curcuru et al. (2008). Following this path has a number of drawbacks. First, this would imply that US exports are growing more and more mismeasured over time. Second, it would also imply a mismeasurement of net exports on the order of 15% of exports on average in the recent period. This is not impossible, but sharply at odds with the Bureau of Census’ perception that the introduction of ARES, a new electronic system to record exports, at the end of the 1990s and its generalization after 2001 (98% coverage in 2002) has led to much more accurate exports data. A 15% measurement error year-on-year would largely dwarf the upper bound of the Census of 10% for export mismeasurement referring to data before 1998 (in fact reconciliation studies produce numbers which are more in the 3 to 7% range for data before 1998). It is even more unlikely that mismeasurement could be as high as 15% of exports after 2000. We also note that given the small shares of services in exports, it is not very plausible that even a serious mismeasurement in services would be large enough to account for the discrepancy. Nevertheless, one could decide to attribute all mismeasurement to mismeasured financial flows (see Table 1 panel (b)). Because the residual items reported by the BEA tend to be negative on the liability side (reducing external liabilities) and positive on the asset side (increasing external assets), excluding them from valuation will lower excess returns. In fact, Curcuru et al. (2008) argue that, once this adjustment is made, the excess return largely disappears for the US over 1994-2005. We find instead evidence of a sizable excess return of about 1.62% over the post Bretton Woods period. We conclude that the evidence on exorbitant privilege is largely robust to their correction.

Going back to the BEA’s Survey of Current Business narrative account for the change in net foreign asset position, there is convincing evidence that debt inflows may have been overstated, as redemptions may not always have been accounted for properly. In specification (c), we adopt a hybrid approach, allocating all the mismeasurement on debt assets and liabilities to mismeasured financial flows. We however allocate the remaining mismeasurements for portfolio equity, direct investment and other assets to valuation terms.

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4 Also, imports are traditionally well measured because of custom duties.
5 See Lane and Milesi-Ferretti (2009) for a thorough discussion.
This lowers slightly the excess return, from 2.69 percent to 2.44 percent per annum over the entire period, and from 3.47 percent to 3.11 percent for the post Bretton Wood era. We conclude that under a set of reasonable alternatives, the excess return of US external assets on external liabilities is large, between 1.6 and 3.5 percent per annum.

Further, we provide on our website a companion spreadsheet allowing a dynamic reallocation of mismeasured terms that vary by year and asset class. Our own experience is that it is difficult not to find an excess return on the net foreign asset position of the United States, whichever assumption one may use on the reallocation of the mismeasured items.

The country at the centre of the international monetary system acts as an international liquidity provider. As such its external balance sheet is particularly remarkable, featuring large gross liquid liabilities and investment in mostly long term risky assets. This is the traditional maturity transformation activity of a bank (Graph 1 and 2). Figure 1-2 report the breakdown of gross assets and liabilities into portfolio equity and debt, direct investment and other assets. Further, 1 breaks down US gross debt liabilities between corporate and government debt, while figure 2 breaks down other assets into gold and non-gold assets. The figure highlights how the composition of US assets and liabilities has changed over time, and documents the importance of liquid safe liabilities (government debt) and risky assets (portfolio equity and direct investment).

This composition effect explain part of the excess returns that the US earns on its external position. But, as already pointed out in Gourinchas and Rey (2007a) an important part of the excess returns comes from within-asset class returns differentials, which we document in table 3. We denote by \( \hat{r}^o \) the differential of returns within the “other assets” category, \( \hat{r}^d \) the differential of returns within the “debt” category, and similarly for direct investment (\( \hat{r}^{di} \)) and equity (\( \hat{r}^e \)). With the mixed allocation of ‘other changes’ (panel c), we see that the excess return has been particularly large in the equity and direct investment categories over the floating rate period (last row), with excess returns of nearly 5 percent per year on direct investment and 4.2 percent for equities. However, we still find a sizeable excess return even for the debt category, of 2.45 percent per year, although significantly reduced from the 4.7
percent in panel (a) obtained when allocating ‘other changes’ to valuation effects. Since, as argued above, a good case can be made that a sizeable fraction of the misallocation between flows and stocks for the debt asset category can be attributed to mismeasured financial flows, we view this estimate as a conservative measure of the excess return on US debt assets. Finally, we find little or negative excess returns on the ‘other assets’ category that includes bank loans and trade credit.

2.3 The “exorbitant duty”

Since at least the summer of 2007, financial markets have been in turmoil. The subprime crisis, followed by the near default or default of several investment banks, insurance companies and nation states has driven volatility to levels not seen in the last two decades. Inspection of the data on the net foreign asset position of the United States during the period of the recent crisis is quite revealing.

We observe a dramatic collapse of most international asset positions as a fraction of GDP. Figure 3 shows the steep declines of equity assets and liability positions as a percentage of GDP. The value of equity assets has declined by 19% of GDP between 2007:4 and 2009:1. A very similar picture emerges for FDI positions, and to a lesser extent for bank loans. US debt liabilities however increased massively as a proportion of GDP since at least 1999. There was a very small decrease in 2008:3 when Lehman Brothers collapsed and all markets froze, followed by a sharp increase. Importantly, the valuation of US Treasury Bills and bonds did not collapse during the crisis, like those of all the other assets. Graph 3 conveys clearly the contrast of safe external liabilities versus risky external assets, which is at the heart of our interpretation of the role of the United States in the centre of the international monetary system. Coupled with the appreciation of the dollar, the relative stability in the value of US bonds has led to a massive wealth transfer of the US towards the rest of the world. Graph 4 shows that between 2007:3 and 2009:1, the net foreign asset position of the United States has dropped by 19% of GDP. Such a precipitous fall of about 3% of GDP per quarter is unseen before in our data: The US has provided insurance to the world when the global crisis hit.
We argue that such an insurance provision in very bad states of the world is the “exorbitant duty” of the centre country. If the US provides insurance against global shocks, it follows that the rest of the world should pay an insurance premium to the US in normal times. Figure 5, shows liquid liabilities as a share of total liabilities and risky assets as a share of total assets. It provides a very striking illustration of the role of the US in providing safe assets to the word at times where the risky assets value tumble.

We therefore sketch an unconventional view of the role of the centre country in the international monetary system and give an alternative explanation to the determinants of the global currency role, compared to the literature. Traditional views rely on liquidity effects and the medium of exchange function of money, such as Krugman (1980), Rey (2001), Matsuyama et al. (1993), or more recently Devereux and Shi (2009), who ally medium of exchange and store of value in their model. In the traditional view of international currencies, size and /or trade links are important insofar as they render a currency more liquid. Stability of the currency is also a prerequisite to foster its international use. It is also often pointed out that the synergies between medium of exchange roles, store of value and unit of account explain why the Dollar is at the same time reserve currency, vehicle currency and pegging currency. From an empirical point of view, Eichengreen and Mathieson (2000) and Chinn and Frankel (2007) have provided an analysis of composition of world reserves. With a share of about 70% of observed total reserves, the US dollar has an uncontested lead. Political scientists have focused on military might and geopolitical power of the United States as underlying determinants of the international currency. In contrast, we focus on the insurance properties of the international currency.

2.3.1 Empirical evidence on the ‘exorbitant duty’

The Great Recession provided us with striking evidence of a massive wealth transfer from the US to the rest of the world during the crisis. Can we find systematic evidence of these transfers in other episodes of market turmoil? We relate empirically the net foreign asset position of the United States, and valuation gains and losses on this position to measures of
market volatility. More precisely, following Bloom (2009) our measure of market volatility is the VIX index on 1986-2010 supplemented by the volatility of the MSCI US stock market index on 1962-1986. Figure 6 shows suggestive evidence of the negative correlation between the net foreign asset position as a share of GDP and financial market volatility, consistent with our “insurance theory” of international currencies. In bad states of the world –such as the LTCM collapse, 9/11, around the tech bubble collapse and obviously the Lehman Brother default– the centre country transfers significant amounts of wealth to the rest of the world, while in good times, the rest of the world pays an insurance premium on US assets. We note that it does not matter whether the shock originates in the US or not as long as it is a global financial shock. As a matter of fact, large financial shocks originating in the US tend to become global shocks, against which the US then provides insurance.

In Table 4, we regress the net foreign asset position, and the valuation on the VIX index. The recent wealth transfer is very spectacular but we do find a negative correlation is present on the whole period 1962-2010. The correlation is stronger after 1990, that is financial globalization truly took hold.

3 Theory

We take the following stylized facts from the above empirical evidence:

1. There are excess returns in normal and stress times (‘exorbitant privilege’)

2. The US plays the role of an insurance provider to the rest of the world

3. This insurance is particularly relevant in times of global stress (‘exorbitant duty’).

4. The US is able to run persistent trade deficits.

In this section, we show that facts 1-4 are consistent with a simple model of insurance provision where the US exhibits smaller risk aversion than the rest of the world. Here, we take this lower risk aversion as given and show that the related equilibrium exhibits many
of the characteristics that we observe in the data. One possible interpretation, although we
don’t want to push it too much, is that the US has access to better technology to deal with
risk, a technology that it is able to ‘export’ to the rest of the world. The planner’s allocation
takes into account that the U.S. has access to this technology and optimally allocates more
risk to the US. This interpretation is isomorphic to the simple model, and is only one of many
possible interpretations. Left for future research is to investigate to what extent perceived
risk aversion in the U.S. is indeed lower than in the rest of the world.

The model also features rare events as in Barro (2006) and non-traded goods as in Hassan
(2009). The first feature allows us to look at the impact of left skewness in the distribution
of global output on the distribution of equilibrium returns. To the extent that the home
country offers insurance to the rest of the world, that insurance will be more valuable when
large negative shocks can happen. The second feature allows us to compare the return on
domestic and foreign real risk free assets, and explore whether the model can deliver the
pattern of excess returns that we document in section 2.

3.1 Motivation with a simple example

In this section, we present a stylized model to illustrate how differences in risk aversion affect
equilibrium allocations. Consider a world economy consisting of two countries, Home and
Foreign, with equal population size equal to $1/2$. Following the usual convention, foreign
variables are denoted with an asterisk $^*$. Time is discrete. In each period $t$, Home is endowed
with a stochastic amount of a single tradable good $y_t/2$. Home consumption decisions are
made by a representative household with additively separable preferences over consumption
sequences of the form $\sum_{t=0}^{\infty} \beta^t u(c_t)$ where $\beta < 1$ is the discount factor, and $u(c)$ exhibits
constant relative risk aversion (CRRA): $u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$ when $\sigma \neq 1$ and $u(c) = \log(c)$ when
$\sigma = 1$. Foreign receives an endowment $y_t^*/2$. Foreign consumption decisions are also made by
a representative household with CRRA preferences, but we assume that foreign households
are more risk averse, that is: $\sigma^* \geq \sigma$. Markets are complete so that households in each
country can trade state-contingent claims over all the relevant states of nature and global
output $\bar{y} = 0.5 \left( y + y^* \right)$ is i.i.d with mean $E\bar{y} = Ey = Ey^*$.\footnote{It is important that global output exhibit no trend growth. Otherwise, the less risk averse agent dominates the market asymptotically. See Cvitanic et al. (2009).}

The equilibrium allocation can easily be derived. Setting the ratio of the marginal utility of the home and foreign households to a constant and substituting into the resource constraint, one obtains, in an ex-ante symmetric equilibrium:\footnote{To obtain this equilibrium condition, observe that in the symmetric equilibrium without risk, i.e. $\bar{y} = E\bar{y}$, the equilibrium would be $c = c^* = E\bar{y}$. This pins down the weights of the equivalent planner’s problem.}

$$\frac{1}{2} \frac{c}{E\bar{y}} + \frac{1}{2} \left( \frac{c}{E\bar{y}} \right)^{\sigma/\sigma^*} = \frac{\bar{y}}{E\bar{y}}. \quad (1)$$

Figure 7 plots the equilibrium consumption function $c(\bar{y})$ that solve equation (1), together with foreign consumption $c^*(\bar{y})$. The properties of these consumption rules are well-known: $c(\bar{y})$ is strictly convex, $c^*(\bar{y})$ strictly concave when $\sigma \neq \sigma^*$. When global output is low ($\bar{y} < E\bar{y}$), home consumption falls more than foreign consumption: $c(\bar{y}) < \bar{y} < c^*(\bar{y})$ as Home provides insurance to Foreign. The reverse obtains in good times. As a result, Home consumption is more volatile than Foreign. It is also easy to show that this consumption rule can be locally decentralized with Home holding a leveraged portfolio $\sigma^*/(\sigma + \sigma^*) > 1/2$ of the world equity and borrowing in the risk free asset.\footnote{To see this, observe that a log-linearization of domestic consumption around its mean yields $\frac{1}{2} \hat{c} = \frac{\sigma^*}{\sigma + \sigma^*} \hat{\bar{y}}$. This can be achieved locally with aggregate domestic holdings of a claim to global output equal to $\frac{\sigma^*}{\sigma + \sigma^*} > \frac{1}{2}$.} Thus, the international investment position of Home resembles that of the United States, long in equities and short in riskless assets. Second, the net foreign asset position of Home worsens in bad times, since it earns a lower return on gross assets (equities) than it pays on gross liabilities (riskless debt). The deterioration in net foreign assets is necessary to reduce domestic wealth and induce Home consumption to fall more than Home output, improving Home’s trade balance. This is consistent with the improvement in the trade balance and worsening in the net foreign asset position of the U.S. in times of global stress. Third, consider the domestic autarky risk-free interest rate $R_{aut}^t$. Since under autarky consumption equals output, it satisfies $\beta R_{aut}^t E_t \left[ (y_{t+1}/y_t)^{-\sigma} \right] = 1$. Assume that domestic output is log-linearly distributed: $\ln y_{t+1} = \ln E\bar{y} + \epsilon_{t+1}$ where $\epsilon_{t+1}$ is i.i.d normal $N(-\sigma^2/2, \sigma^2)$. Then, the unconditional
autarky risk-free rate satisfies:

\[ E \ln R_{t}^{\text{aut}} = - \ln \beta - \frac{\sigma^2}{2} \sigma^2 \epsilon. \]

The second term reflects the effect of the precautionary saving motive on equilibrium rates: as the variance of shocks or risk aversion increases, so does the demand for the safe asset, pushing down equilibrium risk free returns. Similar calculations for the foreign autarky rate imply:

\[ E \ln R_{t}^{\text{aut}*} - E \ln R_{t}^{\text{aut}} = \frac{\sigma^2 - \sigma^2}{2} \sigma^2 \epsilon < 0, \]

since \( \sigma < \sigma^* \). The lower autarky risk-free rates abroad reflects the stronger precautionary saving motive in the foreign country. With a lower autarky rate in Foreign than Home, financial integration implies that Home will run a trade deficit on average: \( E [y - c (\bar{y})] < 0. \)

Again, this feature of the data accords well with the broad empirical evidence for the U.S. Differences in risk aversion play a similar role here as differences in the supply of assets in Caballero et al. (2008) or differences in the degree of domestic risks sharing in Mendoza et al. (2009) and generate ‘global imbalances.’

How should we interpret differences in risk aversion between home and foreign households? Beyond a direct interpretation as differences in risk appetite, which we don’t find particularly plausible, other interpretations are possible. For instance, suppose that Home has identical risk preferences as Foreign. However, Home has access to a technology that ‘transforms’ a given level of expenditures \( e \) into a consumption stream \( c \) that is then consumed by domestic households: \( c = T (e) \). It is easy to check that the equilibrium allocation of expenditures is identical to the previous case, with \( e \) in place of \( c \) in equation (1), if \( T (e) = e^{(1-\sigma)/(1-\sigma^*)} \). More generally, any concave transformation \( T (e) \) will have the effect of increasing the apparent risk appetite of domestic households relative to their foreign counterparts. While Home households appear less risk averse, they enjoy in fact a consump-

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9This can also be directly verified by noting that \( E [y - c (\bar{y})] = E \bar{y} - E [c (\bar{y})] \), and \( E \bar{y} = c (E \bar{y}) < E [c (\bar{y})] \) since \( c(.) \) is a strictly convex function.
tion allocation that is even less volatile than foreign households (compare $T(e)$ and $e^*$ on figure 7). The equilibrium allocation recognizes that Home households have access to a risk-reducing technology and optimally leverages Home equilibrium expenditures. One possible interpretation of this risk reducing technology is that it reflects the interplay between domestic financial development and financial frictions. For instance, in a more elaborate model, financial development at home may reduce the importance of liquidity constraints, increasing the perceived risk appetite of home households (See Gertler and Kiyotaki (2009)). It is beyond this paper to provide a full justification for observed differences in risk appetite. We simply take them as given when characterizing equilibrium returns and allocations and leave the question of their origin open for future research.

While the simple model above captures the essence of the mechanism we want to study, it is too stylized for a detailed exploration of equilibrium returns and trade flows. In a one-good setting, it is not possible to explore differences in risk-free returns between the home and foreign country in equilibrium. To do so, we introduce non-traded goods, in a manner similar to Hassan (2009). In that paper, differences in size generate systematic differences in risk free returns. The intuition for Hassan’s result is simple and intuitive: since shocks to larger countries matter more to the global investor, insurance, in the form of the risk-free bond of the large economy, is also more valuable. In our model, differences in risk appetite introduce another reason why holding the risk-free asset of the less risk averse economy may be more valuable. As in Hassan (2009), we keep markets complete so as to obtain an easy characterization of equilibrium allocations and asset prices. By introducing both size and risk aversion as sources of heterogeneity, we can compare their relative role in generating excess returns. Second, we follow Barro (2006) and introduce rare disasters. These disasters generate left skewness in the distribution of global output, allowing us to clearly identify the impact of global stress on equilibrium returns and allocations. Because the foreign country

---

10Since $\sigma < \sigma^*$, it is immediate that $T(e)$ is more concave than $e^*(e)$.

11The technology $T(.)$ alters the resource constraint of the economy, which is why the solution is not the symmetric allocation of the planner under identical preferences. The implicit assumption is that the risk altering technology is only applied to the expenditure allocation of the home country, and not to global output, otherwise the equilibrium would be $c = c^* = T(\bar{y})$. 

15
is more risk averse, insurance is especially valuable when disasters are possible.\footnote{Unlike Guo (2007), our disaster shocks are global and symmetric, affecting both traded and non-traded output in both countries.}

3.2 A Model of Global Disasters and Insurance

This section introduces a model of risk sharing with heterogeneity in risk aversion and size. The model extends the simple example studied in the previous section along the following dimensions: (a) there are traded and non-traded goods, so that home and foreign real risk-free bonds offer potentially different returns; (b) the economy is subjected to rare disasters as in Barro (2006). These disasters are symmetric, i.e. they affect output in all countries and all sectors identically; (c) countries can differ in size. We are interested in characterizing the equilibrium pattern of risk sharing, equilibrium returns.

3.2.1 Model Setup

The world economy consists of two countries, Home and Foreign. The world is populated by a continuum of households of constant mass equal to 1. A share $\alpha$ of the world population is located in the home country and a share $1 - \alpha$ in the foreign country. Time is discrete, $t = 1, 2, \ldots$. Each period, each household $\omega$ receives a stochastic endowment of a traded good $y^T_t (\omega)$ and of a country-specific non-traded good $y^N_t (\omega)$. We denote $y^T_t$ and $y^N_t$ (resp. $y^{*T}_t$ and $y^{*N}_t$) the average endowment of the Home (resp. Foreign) traded and non-traded goods in period $t$.\footnote{That is, $y^T_t = \int_{\omega \in \Omega_H} y^T_t (\omega) \, d\mu (\omega)$ where $\mu$ denotes the measure of households over $\omega$ and $\Omega_H$ is the set of domestic households. $y^{*T}_t$, $y^{*N}_t$ and $y^{*N}_t$ are defined similarly.} Under our preference assumptions, we will only need to keep track of these country-level average endowments. Each country admits a representative household representation with additively separable preferences defined over aggregate consumption sequences $\{c_s\}_{s=t}^\infty$:

$$U_t = E_t \sum_{s=t}^\infty \beta^{s-t} u (c_s),$$

$$U_t = E_t \sum_{s=t}^\infty \beta^{s-t} u (c_s),$$
where the per-period utility function $u(c)$ exhibits constant relative risk aversion (CRRA): $u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$ when the coefficient of relative risk aversion $\sigma \neq 1$ and $u(c) = \log(c)$ when $\sigma = 1$. Foreign preferences are defined identically, except that the foreign representative household is potentially more risk averse: $\sigma^* \geq \sigma$. This difference in risk aversion is constant and permanent. The consumption aggregate $c$ is defined identically in both countries as a constant elasticity index of traded and non-traded consumption:

$$c = \left[ \gamma^{1/\theta} (c^T)^{\theta-1} + (1 - \gamma)^{1/\theta} (c^N)^{\theta-1} \right]^{\theta/(\theta-1)},$$

where $\theta > 0$ is the elasticity of substitution between traded and non-traded goods, and $\gamma \in (0, 1)$ denotes the steady state share of traded consumption expenditures. A similar definition applies to the foreign consumption aggregate. Taking the traded good as the numeraire and denoting $q$ the price of the domestic non-tradable good (in terms of the traded good), the domestic price index is the Fisher-ideal deflator of domestic aggregate consumption:

$$P = \left[ \gamma + (1 - \gamma) q^{1-\theta} \right]^{1/(1-\theta)},$$

with a similar definition for the foreign price index in terms of the price of foreign non-traded goods $q^*$. The resource constraints are given by

$$\alpha c^T + (1 - \alpha) c^T = \bar{y}^T; \ c^N = y^N; \ c^N = y^N,$$

where $\bar{y}^T$ is the global supply of the traded good: $\bar{y}^T \equiv \alpha y^T + (1 - \alpha) y^T$.

We assume that markets are complete internationally, so that a full menu of state-contingent claims denominated in the traded good can be exchanged between Home and Foreign. As usual, the complete market allocation solves a standard planning problem of maximizing a weighted sum of discounted utilities
\[
\max_{\{c_t^T, c_t^T, c_t^N, c_t^N\}} \mu \alpha E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) + (1 - \mu)(1 - \alpha) E_0 \sum_{t=0}^{\infty} \beta^t u^*(c_t^*)
\]

subject to the resource constraints (2), where \( \mu \in [0, 1] \) represents the weight given by the planner to Home households.

The first-order condition of the planner’s problem impose that the marginal utility of tradable consumption be proportional across states and countries:

\[
c_t^{(1/\theta - \sigma)} (c_t^T)^{-1/\theta} \kappa^{-1/\theta} = c_t^{*(1/\theta - \sigma^*)} (c_t^T)^{-1/\theta}, \tag{3}
\]

where \( \kappa = (\mu / (1 - \mu))^{-\theta} \) is a constant. According to the risk sharing condition (3), shocks to the endowment of non-traded goods will shift the marginal utility of traded good consumption when preferences are non-separable, i.e. when \( \sigma \neq 1/\theta \). When \( \sigma > 1/\theta \), traded and non-traded goods are gross substitutes: a decline in the endowment of non-traded good increases the marginal utility of traded good consumption. Conversely, when \( \sigma < 1/\theta \), the traded and non-traded goods are gross complements: a decline in the endowment of the non-traded good reduces the marginal utility of traded good consumption.

Given an equilibrium allocation, the price of the non-traded good can be obtained as the ratio of marginal utilities for traded and non traded goods:

\[
q_t = \left( \frac{\gamma y_t^N}{(1 - \gamma) c_t^T} \right)^{-1/\theta}. \tag{4}
\]

From (3), the –common– stochastic discount factor is given by

\[
M_{t,t+1} = \beta \left( \frac{c_{t+1}}{c_t} \right)^{1-\sigma} (\frac{c_t^T}{c_t^T})^{-1/\theta}, \tag{5}
\]

and satisfies

\[
E_t [M_{t,t+1}R_{t+1}] = 1, \tag{6}
\]

for any traded asset with gross return \( R_{t+1} \) in terms of the traded good.
3.2.2 Characterization

The set of risk sharing conditions (3), together with the resource constraints (2), provide a set of necessary and sufficient conditions to characterize the equilibrium allocation. One can simplify the analysis of the equilibrium allocation by defining 

\[ x = \kappa c^{\theta - 1} / c^{* \theta - 1} \]

The risk sharing condition (3) become \( c^{* T} = x c^T \) and the resource constraint yields \( c^T = \tilde{y}^T / [\alpha + (1 - \alpha) x] \). \( x \) controls the equilibrium allocation of the global endowment of traded goods between domestic and foreign households. When \( x = 1 \), consumptions per capita are equated and \( c^T = c^{* T} = \tilde{y}^T \). When \( x > 1 \), the foreign country obtains a larger share of the traded good: \( c^{* T} > \tilde{y}^T > c^T \).

Substituting the previous expression into the definition of the domestic and foreign consumption index, \( x \) needs to satisfy

\[
\left( \frac{x}{\kappa} \right)^\frac{\theta - 1}{\theta} = \frac{\left( \gamma^{1/\theta} \left( \tilde{y}^T / [\alpha + (1 - \alpha) x] \right) \right)^\frac{\theta - 1}{\theta} + (1 - \gamma)^{1/\theta} \left( y^N \right)^\frac{\theta - 1}{\theta}}{\left( \gamma^{1/\theta} \left( x \tilde{y}^T / [\alpha + (1 - \alpha) x] \right) \right)^\frac{\theta - 1}{\theta} + (1 - \gamma)^{1/\theta} \left( y^* N \right)^\frac{\theta - 1}{\theta}}}{\sigma^\theta - 1}.
\] (7)

This expression highlights how \( x \) varies with the realizations of both traded and non-traded goods endowments. Consider the case where \( \sigma > 1/\theta \). A decline in \( y^N \) raises Home’s marginal utility of traded good consumption. Risk sharing requires that Home consumes relatively more of the traded good, a decrease in \( x \). A similar effect occurs when \( y^* N \) increases (since \( \sigma^* > 1/\theta \)). A fall in the global endowment of tradables \( \tilde{y}^T \) impacts relatively more the more risk averse country. As a result, risk sharing requires that \( x \) increases, allocating more traded consumption to Foreign.

It is immediate from (7) that as long as endowments follow a stationary process, so does \( x \). It follows that the equilibrium distributions of home and foreign consumption is also stationary in that case.\(^{14}\) Formally, (7) admits a solution \( x = x(y; \kappa) \) where \( y = (y^T, y^* T, y^N, y^* N)' \) is the vector of endowments. In general, \( x \) is determined only implicitly. In two special cases, we obtain an analytical solution. First, when \( \sigma = \sigma^* = 1/\theta \), one can

\(^{14}\)By contrast, if endowments are growing over time, it is easy to check that \( x \) converges to 0 : the less risk averse households dominates aggregate consumption asymptotically. See Cvitanic et al. (2009).
check that the solution is \( x = \kappa = 1 \). The consumption of traded goods in each country is equal to the global endowment of traded goods, and the stochastic discount factor simplifies to the usual formula \( M_{t,t+1} = \beta \left( \frac{\bar{y}_{t+1}}{y_t} \right)^{-\sigma} \). Second, when \( \theta = 1 \) and \( \alpha = 1 \) (large country limit), we obtain \( x^{1+\gamma(\sigma^*-\gamma)} = \kappa \bar{y}^\gamma(\sigma-\sigma^*) (y^N(\sigma-1)/y^*N(\sigma^*-1))^{1-\gamma} \). This expression illustrates that the allocation of traded goods between Home and Foreign depends upon the global endowment of traded good \( \bar{y}^T \) only to the extent that risk appetite differs across countries \((\sigma \neq \sigma^*)\).

In the general case, we can write

\[
M_{t,t+1} = M(y_t, y_{t+1}; \kappa) = \beta \left( \frac{\gamma^{1/\theta} \left( \frac{\bar{y}_{t+1}}{\alpha + (1 - \alpha) x_{t+1}} \right)^{\frac{1-\sigma}{\theta}} + (1 - \gamma)^{1/\theta} (y_{t+1}^N)^{\frac{1-\sigma}{\theta}}}{\gamma^{1/\theta} \left( \frac{\bar{y}_t}{\alpha + (1 - \alpha) x_t} \right)^{\frac{1-\sigma}{\theta}} + (1 - \gamma)^{1/\theta} (y_t^N)^{\frac{1-\sigma}{\theta}}} \right) \cdot \left( \frac{\bar{y}_{t+1}}{\bar{y}_t} \right)^{-1/\theta} \left( \frac{\bar{y}_{t+1}}{\bar{y}_t} \right)^{-1/\theta} \left( \frac{\bar{y}_{t+1}}{\bar{y}_t} \right)^{-1/\theta}.
\]

This expression illustrates Hassan (2009)’s main point: as \( \alpha \) increases, the stochastic discount factor increasingly reflects the endowments shocks of the larger economy. In the large country limit \((\alpha = 1)\),

\[
\lim_{\alpha \to 1} M_{t,t+1} = \beta \left( \frac{\gamma^{1/\theta} \left( \frac{\bar{y}_{t+1}}{\alpha + (1 - \alpha) x_{t+1}} \right)^{\frac{1-\sigma}{\theta}} + (1 - \gamma)^{1/\theta} (y_{t+1}^N)^{\frac{1-\sigma}{\theta}}}{\gamma^{1/\theta} \left( \frac{\bar{y}_t}{\alpha + (1 - \alpha) x_t} \right)^{\frac{1-\sigma}{\theta}} + (1 - \gamma)^{1/\theta} (y_t^N)^{\frac{1-\sigma}{\theta}}} \right) \cdot \left( \frac{\bar{y}_{t+1}}{\bar{y}_t} \right)^{-1/\theta},
\]

and the stochastic discount factor responds exclusively to Home’s endowment shocks.\(^1\)

Finally, the relative weight \( \kappa \) needs to be such that the planner’s allocation coincides with the competitive equilibrium in which households in both countries have no initial debt and own the claims to their domestic traded and non-traded endowments. Formally, the domestic endowment

\[^1\text{The latter condition obtains by symmetry when } \sigma = \sigma^*.\]

\[^2\text{In the limit of } \alpha = 1, \bar{y}^T = y^T.\]
The intertemporal budget constraint in the competitive equilibrium is

\[ \sum_{t=0}^{\infty} E_0 [M_{0,t} (P_t c_t - (y^T_t + q_t y^N_t))] = 0, \quad (9) \]

where \( M_{0,t} \) is the stochastic discount factor between period 0 and period \( t \), defined recursively as \( M_{0,t} = M_{0,t-1}M_{t-1,t} \) and \( M_{0,0} = 1 \). The restriction (9) can equivalently be rewritten as \( NA_0 = 0 \), where the net foreign asset in period \( t \), \( NA_t \), is defined as \( NA_t = -\sum_{s=t}^{\infty} E_t [M_{t,s} (y^T_t - c^T_s)] \).\(^{17}\) The term inside the parenthesis represents the domestic trade surplus in period \( t \). Hence the net foreign asset position always equal the opposite of the present value of future trade surpluses, valued using the equilibrium pricing kernel \( M_{t,s} \). If the endowment process is Markov, so that \( E [y_{t+1}|y_t, y_{t-1}, \ldots y_{t-k}, \ldots] = E [y_{t+1}|y_t] \), we can write \( NA = NA (y; \kappa) \) and solve for \( \kappa \) such that \( NA (y_0; \kappa) = 0 \).

### 3.2.3 Business cycles and disasters

To illustrate the impact of heterogeneity in risk aversion and size in times of global stress, we assume the following process for traded and non-traded domestic output

\[
\begin{align*}
\ln y^T_t &= \ln \gamma + \epsilon^T_t + v_t, \quad (10a) \\
\ln y^N_t &= \ln (1 - \gamma) + \epsilon^N_t + v_t, \quad (10b)
\end{align*}
\]

and

\[
\begin{align*}
\ln y^{*T}_t &= \ln \gamma + \epsilon^{*T}_t + v_t, \quad (11a) \\
\ln y^{*N}_t &= \ln (1 - \gamma) + \epsilon^{*N}_t + v_t, \quad (11b)
\end{align*}
\]

for traded and non-traded foreign output.

\(^{17}\)To see that \( NA_t \) is indeed the net foreign asset position, notice that it is equal to the difference between the domestic wealth of the representative household, defined as the market value of current and future domestic consumption expenditures \( W_t = \sum_{s=t}^{\infty} E_t [M_{t,s} P_t c_t] \), and the market value of a claim to current and future domestic endowments \( V_t = \sum_{s=t}^{\infty} E_t [M_{t,s} (y^T_t + q_t y^N_t)] \).
The random terms $\epsilon^T, \epsilon^N$ and $\epsilon^*T, \epsilon^*N$ are uncorrelated, i.i.d normally distributed shocks with mean $-\sigma^2/2$ and variance $\sigma^2$. These terms capture regular business cycle fluctuations in output. Fluctuations in output trigger a precautionary saving motive whose strength varies across countries when $\sigma \neq \sigma^*$.

The random term $v_t$ captures low-probability disasters, as in Barro (2006). As in that paper, disasters are independent from $\epsilon$ shocks. Unlike Barro (2006), we assume that the output process is stationary in levels: disasters are –eventually– followed by recoveries. This assumption is made mostly for tractability since it ensures that the consumption process remains stationary, even when home and foreign households have different risk appetite. However, this assumption has also substantive merits. Nakamura et al. (2010) found that roughly half of the fall in consumption during disasters is subsequently reversed, indicating partial recovery. Given the curvature of the utility function it remains true that disasters matter much more than recoveries for equilibrium asset returns. We model $v_t$ as a two-state Markov process, with values $v_d$ and $v_n$ and transition probabilities $P(v_d|v_n) = p_d$ and $P(v_n|v_d) = p_n$. $v_d$ and $v_n$ satisfy

$$
\begin{align*}
    v_n &= -\ln (\bar{p}_d (1 - b) + 1 - \bar{p}_d) \\
    v_d &= -\ln (\bar{p}_d + (1 - \bar{p}_d) / (1 - b)) ,
\end{align*}
$$

(12a) (12b)

where $\bar{p}_d$ is the unconditional probability of disaster, and $b \in [0, 1]$. This representation of the disaster process ensures that output drops by a factor $(1 - b)$ when a disaster occurs, a number that has been estimated in the literature, and that $Ey^T = Ey^*T = \gamma$, and $Ey^N = Ey^*N = 1 - \gamma$ regardless of $b$. In other words, by varying $b$, we are changing the left skewness of the output process, keeping expected output constant. $p_d$ and $p_n$ represent respectively the conditional probability of a disaster (in good times) and the probability of a recovery (from a disaster).

Our specification implies that rare events are global: when a disaster occurs, output
jumps down in the same proportion in all sectors and countries. It would be straightforward to extend the analysis to the case of country-specific disasters. The empirical evidence discussed in Nakamura et al. (2010) supports the notion that some disasters are local and others global. In the context of the model, it is immediate that patterns of risk sharing resembling what we observe in the data would emerge if disasters are either more severe or more frequent in Foreign. But this hardly seems a reasonable assumption considering that the recent crisis originated in the U.S., not in the rest of the world. Instead, our approach explores the extent to which Home is able to provide insurance in times of global stress. Under equations (10)-(12), the stochastic process for endowments follows a stationary Markov process and we can solve for $x(y; \kappa)$ and $\kappa$.

### 3.2.4 Approximate analytical results without disasters

As a starting point, consider the case without disaster shocks ($b = p_d = 1 - p_n = 0$). For small $\epsilon$ shocks around the steady state, we can obtain approximate the solution to the planner’s problem and characterize analytically the properties of the allocation.

**Allocation, and Asset Returns.** For small $\epsilon$ shocks, the return on any traded asset $R_{t+1}$, in terms of tradable, satisfies approximately

$$
\ln E_t R_{t+1} \approx - \ln E_t M_{t,t+1} - cov_t (\hat{m}_{t,t+1}, \hat{r}_{t+1}),
$$

where $\hat{z}$ denotes the log-deviation of variable $Z_t$ from its steady state $\bar{Z}$. It follows that the difference in log-expected return between two assets $R^i_{t+1}$ and $R^j_{t+1}$ satisfies

$$
\ln E_t R^j_{t+1} - \ln E_t R^i_{t+1} \approx cov_t (\hat{m}_{t,t+1}, \hat{r}^i_{t+1}) - cov_t (\hat{m}_{t,t+1}, \hat{r}^j_{t+1}).
$$

(13)

As Hassan (2009) noted, while the returns $R^i$ and $R^j$ are measured in terms of tradable, the excess return is invariant to the choice of numeraire. Assuming that a country’s risk-free

---

19 See Guo (2007).
government bond is simply an asset that pays that country’s price index, and applying (13), the expected excess return between domestic and foreign real risk-free interest rates is

\[ \ln E_t R^f_{t+1} - \ln E_t R^d_{t+1} = \text{cov}_t (\hat{m}_{t,t+1}, \hat{p}_{t+1}) - \text{cov}_t (\hat{m}_{t,t+1}, \hat{p}^*_t) \]

(14)

\[ = (1 - \gamma) \text{cov}_t (\hat{m}_{t,t+1}, \hat{q}_{t+1} - \hat{q}^*_t), \]

where the second line makes use of the log-approximation \( \hat{p} = (1 - \gamma) \hat{q} \). This expression makes clear that domestic real interest rates will –on average– be lower when the ratio of domestic to foreign non traded price is positively correlated with the stochastic discount factor, that is when times of relative scarcity \( \hat{m} > 0 \) are also times when domestic non-traded prices are high \( \hat{q} > \hat{q}^* \).

As discussed in the empirical section, one definition of the ‘exorbitant privige’ is that the U.S. experiences a lower risk free return: \( \ln E_t R^f_{t+1} - \ln E_t R^d_{t+1} > 0 \). Can our model of risk-sharing reproduce this feature? Log-linearizing equations (4), (7) and (8), appendix B derives the following equilibrium allocations in deviation from steady state:

\[ \hat{x}_t = \frac{\gamma \theta (\sigma - \sigma^*) \hat{y}^T_t + (1 - \gamma) ((\sigma \theta - 1) \hat{y}^N_t - (\sigma^* \theta - 1) \hat{y}^N_t)}{1 + \gamma (\sigma \theta - 1) (1 - \alpha) + \gamma (\sigma^* \theta - 1) \alpha}, \]

\[ \hat{c}_t^T = \hat{c}_t^T - (1 - \alpha) \hat{x}_t; \hat{c}_t = \gamma \hat{c}_t^T + (1 - \gamma) \hat{y}^N_t, \]

\[ \hat{q}_t = \frac{1}{\theta} (\hat{c}_t^T - \hat{y}^N_t); \hat{p}_t = (1 - \gamma) \hat{q}_t \]

\[ \hat{m}_{t,t+1} = \frac{1}{\theta} (\hat{c}_{t+1} - \hat{c}_t) - \frac{1}{\theta} \left( \hat{c}_{t+1}^T - \hat{c}_t^T \right). \]

After simple but tedious algebra, one can show that expected excess return on risk free

\[ \text{To see this, observe that, in terms of tradables, } R^f_{t+1} = P_{t+1}/E_t [M_{t,t+1} P_{t+1}] \text{ with a similar definition for } R^d_{t+1}. \]

Although each real interest rate is risk free in terms of its own consumption price index, they are not riskless in terms of the tradable good, or any common numeraire.
bonds can be expressed as

$$\ln E_t R^t_{t+1} - \ln E_t R^f_{t+1} = \frac{(1 - \gamma)}{\left[1 + (\sigma \theta - 1) \gamma (1 - \alpha) + (\sigma^* \theta - 1) \gamma \alpha\right]^2} \frac{\sigma^2 \epsilon}{\theta} \tag{15}$$

$$\left[-\gamma (\sigma^* - \sigma) (1 + (\sigma \theta - 1) \gamma) (1 + (\sigma^* \theta - 1) \gamma) / (1 - \gamma) + \alpha (\sigma \theta - 1) (1 + (\sigma^* \theta - 1) \gamma) (\sigma + (\sigma^* - \sigma) \gamma \alpha) - (1 - \alpha) (\sigma^* \theta - 1) (1 + (\sigma \theta - 1) \gamma) (\sigma^* - (\sigma^* - \sigma) \gamma (1 - \alpha))]\right].$$

The first term inside the brackets reflects the covariance between the marginal utility of wealth and the relative price of non-traded goods, conditional on shocks to traded output. It is proportional to the difference in risk aversion ($\sigma^* - \sigma$) and is always negative: when the global output of traded goods declines ($\hat{y}_T^T < 0$), domestic consumption of traded goods fall more than foreign consumption ($\hat{x} > 0$) because foreign is more risk averse. Consequently, the price of domestic non-traded goods falls more than the price of foreign non-traded goods: $\hat{q} < \hat{q}^*$ (non-traded goods are relatively more abundant at home than abroad). The domestic price index declines more than the foreign one, hence foreign real bonds provide a comparatively better hedge. This effect is scaled by the size of the traded sector $\gamma$, disappears when home and foreign have the same risk appetite ($\sigma = \sigma^*$) and is independent of size $\alpha$. The second term in brackets reflects the effect of shocks to domestic non-traded output $y^N$. A decline in the endowment of domestic non-traded goods ($\hat{y}^N < 0$) increases the domestic marginal utility of traded goods when the goods are gross substitutes ($\sigma \theta > 1$). Home traded consumption increases ($\hat{x} < 0$), so the relative price of domestic non-traded goods increases: $\hat{q} > \hat{q}^*$ (domestic non-traded are relatively scarcer at home) and the domestic real bond provides a good hedge. The last term represents the effect of shocks to the endowment of foreign non-traded. By a similar argument, the foreign bond provides a good hedge against shocks to the foreign non-traded good. The last two terms are proportional to the size of the home and foreign country ($\alpha$ and $(1 - \alpha)$). This extends the results from Hassan (2009).
to the case where countries have different risk appetites.\textsuperscript{21}

How large are these expected excess returns? To answer this question, we need to calibrate the following parameters of the model: $\gamma, \theta, \sigma, \sigma^*, \sigma_\epsilon$ and $\alpha$. Our approach is to adopt fairly standard values for $\gamma, \theta, \sigma$ and $\sigma_\epsilon$ and to vary $\alpha$ and $\sigma^*$. $\gamma$ measures the share of traded goods in consumption expenditures around the steady state. We assume a low value $\gamma = 0.25$, consistent with the indirect evidence on high trade costs (see Obstfeld and Rogoff (2005) for a discussion). $\theta$ measures the elasticity of substitution between traded and non-traded goods. Estimates in the literature are fairly low, between 0.5 and 1.3.\textsuperscript{22} We adopt a value $\theta = 1$, towards the higher end of that range. Reasonable values for the coefficient of relative risk aversion $\sigma$ vary between 2 and 5. We choose $\sigma = 3$ as a benchmark and will vary $\sigma^* \geq \sigma$. Finally, the model requires an estimate of the volatility of output around its steady state. As argued above, the assumption of stationarity is mostly maintained to ensure stationarity of consumption allocations. Consequently, we need input the standard deviation of log output deviations from a Hodrick-Prescott filter. Using annual data, a common value is $\sigma = 0.02$. [To be expanded].

The top row of Figure 8 reports the expected riskfree excess return in two configurations. The left panel sets $\sigma^* = 4$ and varies $\alpha$ between 0.5 and 1. The excess return increases with $\alpha$ from -0.04 percent to 0.08 percent. The right panel sets $\alpha = 0.75$ and varies $\sigma^*$ between $\sigma = 3$ and 5. We find that the excess returns decrease with $\sigma^*$, from 0.045 percent to 0.005 percent, as the foreign bond increasingly becomes a better hedge. In either simulation, the size of the excess returns is really tiny, reaching at most 0.08 percent per annum. We conclude that, while the model is theoretically capable of generating excess returns, these are quite small compared to the empirical evidence. One issue, of course, is that the model cannot generate significant risk premia - be it the equity risk premium, or in the case that

\textsuperscript{21}When $\sigma = \sigma^*$, (15) collapses to Hassan (2009)'s formula

$$\ln E_t R^F_{t+1} - \ln E_t R^F_{t+1} = \frac{\sigma (\sigma - 1/\theta) (1 - \gamma)^2}{1 + (\sigma_\theta - 1) \gamma} \frac{(2\alpha - 1) \sigma_\epsilon^2}{\sigma_\epsilon}$$

\textsuperscript{22}See the discussion in Obstfeld and Rogoff (2005).
concerns us, a risk premium between home and foreign risk-free bonds.

**Portfolios** Appendix B also shows how to construct a local portfolio that supports the efficient consumption allocation. Such a portfolio can be obtained by trading claims to next period’s global endowment –traded and non-traded– as well as the domestic and foreign risk-free bonds.\(^{23}\) That these three assets are sufficient to support locally the efficient allocation is immediate since there are only three relevant sources of risk in our set-up: \(\bar{y}^T\), \(y^N\) and \(y^{*N}\) and the returns on these three assets satisfies a spanning condition.\(^{24}\)

The bottom row of figure 8 reports the equilibrium holdings of these three assets for the same set of simulations described above. As expected, we observe that Home holds a leveraged portfolio when \(\sigma < \sigma^*\). Domestic holdings of the global equity always exceed unity.\(^{25}\) This long equity portfolio is financed by issuing riskfree bonds. While the overall domestic bond position is necessarily short, domestic households concentrate the leverage in the foreign riskfree bond, while holding a long position in their own bond. This feature of the model is quite different from the data, where the U.S. is holding a short position in its own riskfree bond. As the home country becomes larger or the foreign country less risk averse, leverage diminishes.

In summary, variation in size or risk appetite are insufficient per se to generate excess returns of the size we observe in the data. Moreover, the pattern of leverage does not resemble the structure of the external position of the U.S.

### 3.2.5 Quantitative results with disasters

We now extend the model by adding disaster risk. As is well known, disaster risk has the potential to increase risk premia. To the extent that domestic and foreign government bonds have different risk exposure from the point of view of the marginal investor, this may magnify

\(^{23}\)With a slight abuse of language, we call the claim to next period’s endowment ‘global equity.’ A global equity also includes a claim on endowments beyond period \(t + 1\).

\(^{24}\)Of course, one could indifferently use any set of three assets whose payoffs span the space defined by the original shocks. Our focus on global equity and risk-free bonds is natural and in the spirit of the Lucas (1982) original decentralization result.

\(^{25}\)Recall that these holdings \(S\) are per household. Aggregate domestic holdings of equity are equal to \(\alpha S\).
expected excess returns, potentially accounting for the ‘exorbitant privilege’. Furthermore, when risk appetite differs across countries, the occurrence of a symmetric disaster triggers a reallocation of resources and associated valuation adjustments that resembles what happened in 2007-2008 between the U.S. and the rest of the world. In other words, the model can also account for the ‘exorbitant duty’.

We begin with a calibration of the remaining parameters of the models. First, we set the discount factor $\beta$ so as to generate the same price-earning ratio as in a model with output growth, or $\beta = 0.923$ in our benchmark calibration. Next, we set $\sigma^* = 4$ and $\alpha = 0.75$. The large value of $\alpha$ reflects the relative importance of the U.S. economy for global outcomes and captures traditional arguments in favor of the U.S. as the issuer of the reserve currency. Second, we calibrate the process for disaster events. We set the conditional probability of a disaster occurring $p_d$ at 1.17 percent per year, as in Barro (2006). We also set the probability of recoveries $p_n$ at 2 percent. This implies that disasters are rather persistent affairs and yields an unconditional probability of disasters slightly larger than 45 percent. Next, we calibrate the size of the disaster by setting $b = 0.42$. Barro (2006) estimates a similar parameter between 0.15 and 0.65. Our estimate is around the middle of that range.

Lastly, as in Barro (2006), we allow for the possibility that government T-bills experience a partial default when disaster occurs. That is, we assume that real government bonds pay the local consumer price index in normal times, but only a fraction of the promised payment in periods of global stress. An important parameter is the expected recovery rate $r_i$ on government bonds from country $i$. The face value in normal times (in terms of traded goods) of a government bond that pays $P^i$ is then (in terms of tradables):

$$P_{t}^{bi} = (1 - p_d) \mathbb{E}_t \left[ M_{t,t+1} P_{t+1}^i | n \right] + p_d r^i \mathbb{E}_t \left[ M_{t,t+1} P_{t+1}^i | d \right],$$

26In a model without shocks (disasters or otherwise) and no growth, the PE ratio is $1 / (\beta^{-1} - 1)$. With a growth rate of $g$, the PE ratio becomes $1 / (\beta^{-1} - 1 + (\sigma - 1) g)$. Consequently, we set $\beta^{-1} = 1.03 + (\sigma - 1) 0.025$.

27This number has no impact on allocations (consumption and trade balances) and a minimal impact on excess returns in normal times. It influences expected returns during a disaster since a higher $p_n$ implies a larger chance of a recovery and incipient high return.
and the expected return on the government bond is

\[
\ln E_t P_{t+1}^{bi} = \ln \left[ (1 - p_d) E_t \left[ P_{t+1}^{i} \mid n \right] + p_d r^i E_t \left[ P_{t+1}^{i} \mid d \right] \right] - \ln P_t^{bi}.
\]

We interpret \( r^i \) as capturing in a simple way the ‘fiscal capacity’ of country \( i \), i.e. the capacity for the government of that country to honor its debt obligations through taxation of the domestic economy. Barro (2006) documents that the real return on T-bills in many countries was low during rare events, either because of outright default or expropriation, or—a more common scenario—because of the real depreciation of nominal claims through high inflation. \( r \), therefore, represents another important parameter, conceptually separate but not entirely unrelated to size or risk appetite. For instance, \( r \) captures implicitly a host of political economy factors that determine a country’s ‘willingness to pay’ as opposed to its ‘ability to pay.’ as measured by its size Most of the literature on sovereign debt emphasizes the important of a country’s ‘willingness to pay’ in understanding episodes of sovereign default. We allow for differences in recovery rates across countries. Specifically, we assume that Home can enforce repayment (\( r = 1 \)) while Foreign may suffer from partial implicit or explicit default (\( r^* < 1 \)).

To solve the model, we discretize the state space and solve for the optimal consumption allocation in each state such that there are initially no net external positions (\( NA(x_0; \kappa) = 0 \)). We then construct the stochastic discount factor \( M_{t,t+1} \) and use this SDF to price government real bonds and global equities. We then construct the local portfolio that replicates the return on domestic financial wealth through holdings of a global equity claim and domestic real bond government bond. Finally, we construct gross external assets and liabilities using

\[
NA^i = A^i - L^i = W^i - V^i \quad \text{where } W^i \text{ is the value of claim on current and future domestic expenditures } \{ P_s^{i(C_s)} \}_{s=t}^\infty, \text{ and } V^i \text{ is the value of a claim to domestic output } \{ y_s^{iT} + q_s^{i} y_s^{iN} \}_{s=t}^\infty.
\]

Details are provided in appendix B.

Table 5 reports the results under different scenarios. In each column, we report proper-

---

28 These two assets are sufficient to replicate wealth returns. In practice, it is natural to consider Home’s government debt since Home will hold a leveraged portfolio in equilibrium.
ties of the equilibrium allocation under a set of parameters. Column (1)-(4) report results assuming that there are no global shocks. Column (1) corresponds to Hassan (2009). The sole difference between Home and Foreign is in size ($\alpha = 0.75$). The excess return between domestic and foreign government bonds, 0.04 percent, is precisely that predicted in that paper. Note however, that the model cannot account for the pattern of external assets and liabilities we observe in the data (no trade deficits and no net foreign asset position). Moving across the rows, we introduce first differences in risk aversion (column (2)), increase the size of the home country (column (3)), and reduce the elasticity of substitution between traded and non-traded goods (column (4)). We define the equity premium, $\ln E_t R^e_{t+1} - \ln E_t R^h_{t+1}$, as the excess return on a claim to global endowment (traded and non-traded) relative to the domestic government bond. In specifications (1)-(4), the equity premium is small, between 10 and 17 bp per year. This is not surprising, since the model without disaster risk delivers an equity risk premium approximately equal to $\sigma \sigma^2 = 3 (0.02)^2 = 0.0012$. As Foreign risk aversion increases (column (2)), the expected excess return on foreign government bonds decreases, as in figure 8. Home now holds a leveraged portfolio, borrowing from Foreign and investing in global equities, but the size of that portfolio is small, only about 7.5 percent of Home’s output. Further, Home is not running a significant deficit or surplus. Finally, while there is still some small excess return on government bonds, there is no excess return on gross external assets over gross external liabilities. Columns (3) and (4) document that this result is robust to reasonable variations in parameters. Increasing the size of the domestic economy or reducing the elasticity of substitution between traded and nontraded have minimal impact on the results.\(^{29}\)

Columns (5)-(7) reports results when we introduce disaster shocks, as specified above. In all three specifications, the equity premium jumps to a more reasonable value, between 4.08 and 4.52 percent in normal times and a price earning ratio between 15 and 16 in normal times, collapsing to 10.5 in times of stress. Columns (5) and (6) assume that there is no partial default on foreign government obligations ($r^* = r = 1$) while column (5) maintains

\(^{29}\)Column (3) reports the trade balance, net foreign assets and net debt liabilities of Foreign, since under the large country assumption ($\alpha = 1$), home trade balance and net foreign asset position are necessary zero.
\( \sigma^* = \sigma = 3 \), in order to tease out the pure effect of economic size in presence of disaster risk. Not surprisingly, given that disasters are symmetric, the excess return on government bonds remains very small, at 0.04 percent, while the excess return on the net foreign asset position is zero. We conclude from this that economic size per se (as measured by \( \alpha \)) matter very little for the structure of returns.

Column (6) introduces differences in risk appetite. The results are dramatically different. First, with differences in risk appetite, Home will provide insurance against global risks to Foreign. This is reflected in the pattern of trade deficits/surpluses of the Home country. In normal times, it is running a trade deficit, of about 0.72 percent of output. When a disaster occurs, however, this trade deficit becomes a trade surplus of 1.38 percent of output.\(^{30}\) This pattern of trade deficits has a counterpart in the domestic net foreign asset position.\(^{31}\) In normal times, Home has a small net foreign asset position, varying between +/-0.8 percent of output. Once a disaster strikes, however, Home’s net foreign asset position worsens considerably, on average by 14.5 percent of output. With reasonable parameters, the model can therefore reproduce the size of the net wealth transfer from Home to Foreign during times of global stress, the ‘exorbitant duty.’

How can Home stabilize it’s net foreign asset position in normal times despite repeated trade deficits, varying between 0 and 1.6 percent of output? The answer is that Home’s net foreign position benefits from valuation gains in normal times that offset trade deficits. In terms of portfolios, Home now holds a very leveraged portfolio, with net debt liabilities equal to roughly 55 percent of output, that are reinvested in global equities. This portfolio delivers small positive excess returns in good times –enough to offset the trade deficits of 0.8 percent of output– and significant losses in bad times. The collapse in net foreign asset positions reflects the collapse in the value of global equity, from 16 times output to 10 when a disaster strikes. Hence, the model also delivers the ‘exorbitant privilege’ that we documented earlier. It might seem surprising, from that point of view that Table 5 reports excess returns

\(^{30}\)Unconditionally, since disasters are less likely than normal times, Home runs a small trade deficit of 0.18 percent of output.

\(^{31}\)Recall that \( NA_t = -NX_T + E_t [M_{t+1} NA_{t+1}] \).
on gross assets equal to only 0.15 percent. This calls for two remarks. First, this small excess return is substantially leveraged since the return on the net foreign asset position is
\[ R^{NA} = R^L + \mu_A (R^A - R^L) \]
where \( \mu_A = A / (A - L) \) can be a very large number. Hence, the return, as a fraction of output can be substantially larger. Second, the trade deficit in normal times remains small, -0.8 percent of output, so the valuation gain needs to be of the same order to stabilize the external debt. Finally, while the model does deliver excess returns on Home’s external position, it does not produce a lower yield on Home’s real government bonds. The government bond excess return is -1.87 percent in normal times. To understand this result, observe that when a disaster strikes global endowment fall proportionately in all sectors and countries. Since Home insures Foreign, however, Home’s consumption of traded goods falls relatively more than Foreign’s. Hence Home runs a smaller trade deficit, or trade surplus. This implies that the domestic non-traded goods are relatively more abundant and their price falls more, relative to Foreign. The net result is that the domestic real bond is not a particularly good hedge against global shocks since the domestic price index will fall more than foreign. The higher return is required to induce Foreign to hold domestic real government debt. This result changes in column (7) when we allow for partial default on foreign real bonds. We set an average recovery rate \( r^* = 0.75 \), which is consistent with the numbers reported in Barro (2006).\(^{32}\) The allocation is unchanged by this assumption, since we are only modifying the payoff structure of one asset (foreign real bonds) but keeping the overall market structure unchanged. Therefore, the pattern of trade deficits in good times, trade surpluses in bad times, remains the same, as does the structure of excess returns on gross assets and liabilities and net debt holdings.\(^{33}\) The only difference is the excess return between real domestic and foreign government bonds. We now find small positive excess returns on real bonds, of 0.34 percent. Foreign real bonds are now less desirable since their payoff is likely to be reduced in times of global stress.

\(^{32}\)Barro considers a 40 percent probability of partial default during a disaster in which case the loss is of a size similar to the loss of output. This yields a slightly higher recovery rate of 0.83.

\(^{33}\)This last result is immediate since in equilibrium only Home issues debt.
4 Conclusion

To be written.
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<tr>
<td>( r^a - r^l )</td>
<td>2.69%</td>
<td>1.30%</td>
<td>3.47%</td>
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<td>( r^a )</td>
<td>5.84%</td>
<td>5.04%</td>
<td>6.30%</td>
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<td>( r^l )</td>
<td>3.16%</td>
<td>3.74%</td>
<td>2.83%</td>
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(a) : Valuations

| \( r^a - r^l \) | 1.49%         | 1.25%         | 1.62%         |
| \( r^a \)       | 4.91%         | 4.71%         | 5.02%         |
| \( r^l \)       | 3.42%         | 3.46%         | 3.40%         |

(b) : Financial Flows

| \( r^a - r^l \) | 2.44%         | 1.28%         | 3.11%         |
| \( r^a \)       | 5.76%         | 4.96%         | 6.21%         |
| \( r^l \)       | 3.31%         | 3.68%         | 3.11%         |

(c) : Mixed

Table 1: Annualized total real returns on external assets and liabilities. In Panel (a) all "Other changes" are allocated to valuations; in Panel (b) "Other changes" are allocated to financial flows; in Panel (c) "Other changes" are allocated to valuations except for debt liabilities (corporate and government) and debt assets for which "Other changes" are allocated to financial flows. \( r^a \) refers to the total return on gross assets, \( r^l \) to the total return on gross liabilities. Returns are quarterly (annualized).

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<tr>
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<tr>
<td>( r^a - r^l )</td>
<td>3.22%</td>
<td>2.18%</td>
<td>3.57%</td>
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<td>( r^a )</td>
<td>4.11%</td>
<td>2.41%</td>
<td>4.82%</td>
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<tr>
<td>( r^l )</td>
<td>3.14%</td>
<td>3.18%</td>
<td>3.24%</td>
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Table 2: Standard Deviation of Quarterly Returns. The table reports the quarterly standard deviation of total returns on gross external assets and liabilities. \( r^a \) refers to the total return on gross assets, \( r^l \) to the total return on gross liabilities. "Other changes" are allocated to valuations.
Table 3: Excess Returns by Asset Class. In Panel (a) "Other changes" are allocated to valuations; in Panel (b) "Other changes" are allocated to financial flows; in Panel (c) "Other changes" are allocated to valuations except for debt liabilities (corporate and government) and debt assets for which "Other changes" are allocated to financial flows. \( r^o \) refers to the annualized quarterly excess return on 'other assets'; \( r^d \) to 'debt'; \( r^{di} \) to direct investment and \( r^e \) to equities.

<table>
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<tr>
<th></th>
<th>( r^o )</th>
<th>( r^d )</th>
<th>( r^{di} )</th>
<th>( r^e )</th>
</tr>
</thead>
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<tr>
<td>1952:1-2009:4</td>
<td>-0.63%</td>
<td>4.71%</td>
<td>4.00%</td>
<td>4.11%</td>
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<tr>
<td>1952:1-1972:4</td>
<td>-2.02%</td>
<td>4.79%</td>
<td>2.24%</td>
<td>3.96%</td>
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<tr>
<td>1973:1-2009:4</td>
<td>0.16%</td>
<td>4.67%</td>
<td>4.99%</td>
<td>4.19%</td>
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<table>
<thead>
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<th>( b ) : financial flows</th>
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</thead>
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<tr>
<td>1952:1-2009:4</td>
<td>-1.37%</td>
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<tr>
<td>1952:1-1972:4</td>
<td>-2.12%</td>
</tr>
<tr>
<td>1973:1-2009:4</td>
<td>-0.94%</td>
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<table>
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<th>( c ) : mixed</th>
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</thead>
<tbody>
<tr>
<td>1952:1-2009:4</td>
<td>-0.63%</td>
</tr>
<tr>
<td>1952:1-1972:4</td>
<td>-2.02%</td>
</tr>
<tr>
<td>1973:1-2009:4</td>
<td>0.16%</td>
</tr>
</tbody>
</table>

Table 4: Exorbitant Duty over Time. The table reports the results from an OLS regression of the U.S. net foreign asset position relative to GDP (nagdp) on the VIX index extended before 1986 with the volatility of the MSCI-ex US index. vagdp refers to the valuation component (relative to GDP) defined as \( VA_t = NA_t - NA_{t-1} - FX_t \) where \( FA_t \) represents the net financial flows in period \( t \).

<table>
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<tr>
<td>( vix )</td>
<td>-0.60**</td>
<td>-0.05**</td>
</tr>
<tr>
<td></td>
<td>(.11)</td>
<td>(.02)</td>
</tr>
<tr>
<td>( c )</td>
<td>-1.75</td>
<td>1.28**</td>
</tr>
<tr>
<td></td>
<td>(2.1)</td>
<td>(.36)</td>
</tr>
<tr>
<td>( N )</td>
<td>190</td>
<td>190</td>
</tr>
<tr>
<td>Adj. ( R^2 )</td>
<td>0.14</td>
<td>0.04</td>
</tr>
<tr>
<td>Parameters</td>
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<td>(2)</td>
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<td>-------------------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>0.75</td>
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<tr>
<td>$\theta$</td>
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<td>1</td>
</tr>
<tr>
<td>$\sigma^*$</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$b$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r^*$</td>
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| Equity Premium    | normal | 0.12 | 0.13 | 0.17 | 0.09 | 4.08 | 4.52 | 4.52 |
| Price Earning Ratio | normal | 12.02 | 12.02 | 12.02 | 12.02 | 15.64 | 16.00 | 16.00 |
| T-bill excess return | normal | 0.04 | 0.03 | 0.08 | 0.00 | 0.04 | -1.87 | 0.34 |
| External Excess Return | normal | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.15 |
| Trade Balance     | normal | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.72 | -0.72 |
| Net Foreign Assets | normal | 0.00 | 0.00 | -0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| Net Debt Liabilities | normal | 0.00 | 7.54 | -31.55 | 12.12 | 0.17 | 55.09 | 55.09 |

Table 5: Excess Returns and External Imbalances. Equity premium is defined as $\ln E_t \left[ \bar{R}_{t+1} \right] - \ln E_t \left[ R_{t+1}^f \right]$ where $\bar{R}_{t+1}$ is the gross return on a claim to current and future total endowment (global equity) and $R_{t+1}^f$ is the gross return on domestic real government bonds. T-bill excess return is defined as $\ln E_t \left[ R_{t+1}^{f*} \right] - \ln E_t \left[ R_{t+1}^f \right]$ where $R_{t+1}^{f*}$ is the gross return on real foreign government bonds. All returns are measured in terms of tradable goods. The domestic trade balance-output ratio is defined as $(y^T - c^T) / (y^T + qy^N)$. The net foreign asset position is defined as $NA_t = W_t - V_t$ where $W_t$ is the value of a claim to current and future domestic consumption: $W_t = P_t c_t + E_t [M_{t,t+1} W_{t+1}]$ and $V_t$ is the value of a claim to current and future domestic endowment, $V_t = y_t^T + qy_t^N + E_t [M_{t,t+1} V_{t+1}]$. Net debt liabilities are obtained by projecting the return on domestic wealth $R^w$ onto the return on global equity $\bar{R}$ and the return on domestic government debt $R^f$ to reconstruct the portfolio of domestic wealth.
Figure 1: Gross Liabilities of the United States, percent of GDP
Figure 2: Gross Assets of the United States, percent of GDP
Figure 3: U.S. External Debt and Equity, percent of GDP

Figure 4: U.S. Net Foreign Asset Position, percent of GDP
Figure 5: Liquid liabilities (percent of total liabilities) and risky assets (percent of total assets)

Figure 6: VIX and Net Foreign Asset Position (percent of US GDP)
Figure 7: Risk sharing with heterogeneous risk aversion. The figure is drawn under the following assumptions: $E\bar{y} = 1$, $\sigma = 2$, $\sigma^* = 5$. 
Figure 8: Riskfree excess return and optimal portfolio without disaster risk. See text for parameters.
References


Appendix

A Data

To illustrate our methodology, consider the following stock-flow equation, describing the law of motion for a given class of assets $i$. Assets $i$ include all the broad categories of assets classified as in the balance of payments: portfolio debt (with a distinction between corporate and government bonds whenever the data allow us to do so), direct investment, portfolio equity investments, and other assets (bank loans and trade credit)

$$PX_{i,t+1} = PX_i^t + FX_{i,t+1}^t + VX_{i,t+1}^t + OC_{i,t+1}^t. \quad (16)$$

In writing equation (16), we adopt the representation of the BEA: $PX_i^t$ represents the position given by the BEA at the end of period $t$ for assets $i$, $FX_{i,t}^t$ the financial flow during period $t$, $VX_i^t$ the explicit valuation gain that can be attributed to currency and asset-price movements, while $OC_i^t$ is a residual item for ‘other changes’. For a given class of asset $i$, we can compute an explicit total return $R_{t+1}^{ex,i}$ as

$$\left( R_{t+1}^{ex,i} - 1 \right) PX_i^t = I_{t+1}^i + VX_{t+1}^i,$$

where $I_{t+1}^i$ is the distributed yield, as measured by net income receipts for asset $i$. Summing over all asset classes, measured (or explicit) total returns on the net foreign asset positions are given by

$$\left( R_{t+1}^{ex} - 1 \right) NFA_t = I_{t+1} + VAL_{t+1}$$

where $I_{t+1}$ is the net income balance (including interest income, distributed dividends and direct investment earnings) and $VAL_{t+1}$ is the sum across all assets of the net valuation changes reported by the BEA (currency and asset prices).

The final step is to go back to balance of payment accounting to insure consistency of the data. Substituting financial flows using the fundamental Balance of Payment equation gives us the international investment position at the end of period $t+1$, $NA_{t+1}$ as:

$$NFA_{t+1} = R_{t+1}^{ex} NFA_t + NX_{t+1} + SD_{t+1} + OC_{t+1}$$

where $SD_t$ is the statistical discrepancy between trade and financial flows reported in the balance of payments, $NX_t$ is the trade balance. Other changes $OC_t$ can represent either mis-measured valuations (as is assumed in Gourinchas and Rey (2007a)), mismeasured financial valuations in the BEA’s IIP reconciliation table 3, other changes represent “changes in coverage due to year-to-year changes in the composition of reporting panels, primarily for bank and nonbank estimates, and to the incorporation of survey results. Also includes capital gains and losses of direct investment affiliates and changes in positions that cannot be allocated to financial flows, price changes, or exchange-rate changes.”

According to the Balance of Payments manual, direct investment income in the current account includes distributed earnings as well as the share of reinvested earnings. So there is an entry in the current account for reinvested earnings and an offsetting entry in the financial account.

For more details, see Appendix A. In these derivations, we ignore the capital account as well as unilateral transfers. Both components are small components of the US balance of payments.
flows (as in Curcuru et al. (2008) and Forbes (2010)), mismeasured initial positions, or any combination thereof.  

If we allocate $OC_{t+1}$ to mismeasured valuations, we get a new estimate of total (implicit) returns $R_{t+1}$ such that:

$$\begin{align*}
NFA_{t+1} &= R_{t+1}NFA_t + NX_{t+1} + SD_{t+1} \\
(R_{t+1} - 1) NFA_t &= I_{t+1} + VAL_{t+1} + OC_{t+1}
\end{align*}$$

If ‘other changes’ reflect mismeasured financial flows, the return on the net foreign position is unchanged, but the Balance of Payments identity requires that net exports $NX_t$ are mismeasured by a commensurate amount:

$$\begin{align*}
NFA_{t+1} &= Rex_{t+1}NFA_t + NX'_{t+1} + SD_{t+1} \\
NX'_{t+1} &= NX_t + OC_{t+1}
\end{align*}$$

Finally, if other changes represent mismeasured initial positions, both initial position and returns are mismeasured, with:

$$\begin{align*}
NFA_{t+1} &= R'^{ex}_{t+1}NFA'_t + NX_{t+1} + SD_{t+1} \\
NFA'_t &= NFA_t + \frac{OC_{t+1}}{(I_{t+1} + VAL_{t+1})/NAt + 1} \\
(R'^{ex}_{t+1} - 1) NFA'_t &= I_{t+1} + VAL_{t+1}
\end{align*}$$

### B Characterization of the model

To be added.

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37Lane and Milesi-Ferretti (2009) propose an allocation of these ‘other changes’ based on best judgement.

38See appendix A for a formal derivation.