Patent Breadth, Patent Life, and the Pace of Technological Progress

Ted O'Donoghue
Department of Economics
Cornell University
414 Uris Hall
Ithaca, NY 14853

Suzanne Scotchmer
Department of Economics and Graduate School of Public Policy
University of California
Berkeley, CA 94720

Jacques-François Thisse
C.O.R.E.
Université Catholique de Louvain
Louvain, Belgium

In active investment climates where firms sequentially improve each other's products, a patent can terminate either because it expires or because a non-infringing innovation displaces its product in the market. We define the length of time until one of these happens as the effective patent life, *and show how it depends on patent breadth. We distinguish* lagging *breadth, which protects against imitation, from* leading *breadth, which protects against new improved products. We compare two types of patent policy with leading breadth: (1) patents are finite but very broad, so that the effective life of a patent coincides with its statutory life, and (2) patents are long but narrow, so that the effective life of a patent ends when a better product replaces it. The former policy improves the diffusion of new products, but the latter has lower R&D costs.*

We thank the France/Berkeley fund for financial support. For useful discussion we thank Paul-Emmanuel Piel and seminar participants at U.C. Berkeley, the NBER Workshop on Patent Policy, July 1995, the June 1996 Strasbourg conference on Innovation sponsored by NBER and CNRS, and the University of Minnesota.

Journal of Economics & Management Strategy, Volume 7, Number 1, Spring 1998, 1–32

1. Introduction

When technology grows cumulatively, there may be a large discrepancy between the social value of an innovation and the profit collected by the innovator. On one hand, the innovation may be very valuable because it has spillover benefits for future innovators. On the other hand, future innovators are a competitive threat. Each innovator fears that his profit flow will be terminated by invention of an even better product.

In such an environment the statutory life of a patent may be irrelevant. In this paper we introduce the notion of *effective patent life,* which is the expected time until a patented product is replaced in the market. We argue that effective patent life depends on patent breadth as well as on statutory patent life, since patent breadth determines which products can replace the patented product. We investigate the optimal design of patents, given that breadth helps determine effective life.

There is at least some evidence that effective patent lives are short. Mansfield (1984) reports from survey evidence that in some industries 60% of patents are effectively terminated within 4 years, which is considerably less than the statutory life of 17 years. This finding was corroborated by Levin et al. (1987), who reported (Table 9) that almost all patents are duplicated within five years. Further evidence of short effective patent lives comes from patent renewal data: Schankerman and Pakes (1986) conclude that European patents lose on the order of 20% of their value per year, and Pakes (1986) reports that only 7 percent of French patents and 11 percent of German patents are maintained until the patent expires [see also Pakes and Schankerman (1984)]. Lanjouw (1993) presents a more disaggregated model of how German patents become obsolete, and concludes that fewer than 50% are maintained more than ten years.

Of course, a discrepancy between effective and statutory patent lives is not inevitable: it vanishes if patents are very broad so that every subsequent innovation in a product line infringes on every unexpired patent in that product line. We are led to the following design question: Should patents be long-lived but narrow, so that they effectively expire at an endogenous time when a better product is made? Or should they be relatively broad but short-lived, so that the effective patent life coincides with the statutory patent life? It turns out that the two policies are not equivalent, even if both lead to the same rate of innovation. To sustain a given rate of innovation, the effective patent life in the first policy must be longer than the effective (statutory) patent life in the second policy, which exacerbates the inefficiencies due to market power.

To study this question, we must have a stylized understanding of patent law. We propose that there are at least two types of patent breadth: *lagging* breadth and *leading* breadth. We propose that lagging breadth protects against competition from products inferior to the patented product, and leading breadth protects against competition from products with higher quality. We show that lagging breadth alone may provide insufficient incentives for investment even when the statutory patent life is very long, and we show how leading breadth can extend effective patent life and stimulate R&D.

We realize that leading breadth seems prima facie untenable on the basis that it gives property rights on qualities (products) that the patentholder did not invent. We address this question at some length in the conclusion. For the moment we simply observe the implication of our analysis: Without some form of leading breadth, the effectiveness of the patent system to promote innovation is seriously impeded.

To isolate the questions of interest, we study a particularly simple investment environment where the rate of turnover in the market has an exogenous component, namely, the rate at which firms have *ideas* for improved products. An innovation requires both an idea and a decision to invest in it. The R&D literature has studied two types of research environments: those where opportunities for investment (i.e., ideas) are public knowledge, and those where opportunities for investment are private knowledge. When an investment opportunity is public knowledge, then firms may race for the patent. Since patent races have been well studied elsewhere [see Reinganum (1989), or O'Donoghue (1996) for patent races on quality ladders], we assume that ideas are private knowledge, so that a single firm has the opportunity to invest in each one. This is essentially the assumption of Nordhaus (1969) and the recent literature with two stages of innovation, e.g., Green and Scotchmer (1995). Each firm makes a single decision, which is whether or not to invest in a given investment opportunity.

As in all investigations of optimal patent policy, our conclusions involve a tradeoff between the rate of innovation and monopoly distortions. Broad patents may accelerate innovation, but since infringing improvements must be licensed, broad patents concentrate market power by consolidating quality improvements in the hands of one firm. Consolidation increases the quality gap between firms in the market, and leads to an inefficiency due to delayed diffusion. To show this, we use an axiomatic model of the output market based on the natural-oligopoly model of Gabszewicz and Thisse (1980) and Shaked and Sutton (1983).

Our main conclusions are:

- Patent breadth—in particular, leading breadth—can increase the rate of innovation by increasing the effective patent life, and without it, the rate of innovation may be seriously suboptimal.
- A specified rate of innovation can be achieved with either (1) a patent of infinite length and modest leading breadth, or (2) a patent with infinite leading breadth and modest length. The former is more efficient in minimizing R&D costs, but the latter is more efficient in minimizing the costs of delayed diffusion.

Although our notion of leading breadth is similar to the notion of patent breadth used in the two-stage models of Green and Scotchmer (1990, 1995), Scotchmer (1991, 1996), Chang (1995), Cheong (1994), Chou and Haller (1995), Matutes et al. (1996), Schmitz (1989), and Van Dijk (1996), we investigate a different issue. The two-stage models focus on how patent breadth operates via infringement and licensing as a vehicle to transfer profit from applications of a technology to its inventor. The time between innovations is assumed for convenience to be zero, and the statutory patent life determines the total profit while the breadth determines its division. [See in particular Green and Scotchmer (1995), Scotchmer (1991, 1996), Chou and Haller (1995).] With repeated innovation the division of profit between first and second innovators is not the focus, since every innovator will be in both positions. Instead the focus is how to increase total profit while minimizing monopoly distortions.

Tandon (1982), Gilbert and Shapiro (1990), Klemperer (1990), Gallini (1992), and Denicolo (1996) have addressed optimal patent breadth in the context of one-time innovation, where broader patents permit a shorter patent life. Broader patents increase static inefficiencies, but with a shorter life the inefficiencies terminate sooner. In contrast, when innovation is cumulative the patent breadth and patent life must work together to achieve an adequate *effective* patent life, which is an additional consideration.

In Section 2 we present a simplified model with homogeneous tastes, and expose the potential deficiencies of a patent system without leading breadth. In Section 3 we show how leading breadth can extend effective patent life and stimulate R&D, and we compare the two types of policy with leading breadth outlined above. In Section 4 we introduce oligopoly distortions by introducing heterogeneity in tastes and studying the natural-oligopoly model. This permits us to investigate which type of policy is least distortionary from the consumers' point of view. Section 5 investigates leading breadth in the natural-oligopoly model, and Section 6 concludes with a discussion of how the model described here reflects patent law.

2. A Model with Homogeneous Tastes

We first consider a simple output market with homogeneous tastes and no static inefficiency. This simple model allows us to illustrate why lagging breadth can lead to suboptimal investment, and how leading breadth can stimulate R&D. In later sections we introduce oligopoly distortions due to heterogeneous tastes.

2.1 Innovation

Innovation determines what qualities are technologically feasible at any given time (but patent law may restrict the qualities that are available to a given firm). We assume there is an infinite sequence of innovations described by $(\Delta_1, \Delta_2, \ldots, \Delta_i, \ldots)$, and qualities are given by $q_i = q_{i-1} + \Delta_i$, where Δ_i is the ith innovation (see Figure 1). After innovation i, any quality $q \leq q_i$ is technologically feasible.

We assume there is a Poisson process with parameter λ that determines the rate at which firms collectively receive ideas for improvements, with each idea received by a single random firm. We assume that there are a large number of firms, so that a firm is unlikely to be its own successor. In fact, for simplicity we assume that a firm's probability of being its own successor is zero; the next firm to get an idea will be a different firm.[1]

An *idea* is a pair (Δ, c), where Δ is the quality improvement facilitated by investment and c is the cost. If the firm pays the investment cost, the idea becomes an *innovation,* namely an increment to the previous maximum quality. If the firm does not pay the investment cost, the idea is lost. For simplicity, we assume that every idea has the same investment cost c, but that the quality improvement Δ is distributed (conditional on having an idea) according to a stationary distribution F with support in $\mathbb{R}_+$ and density f. Our assumption of constant investment costs is a proxy to mean that costs are distributed independently of Δ.

1. This assumption is for analytical ease. In the basic model of this section and the next, this assumption will be irrelevant: Whether a firm invests in a given idea is independent of whether that firm had the previous idea. In the natural-oligopoly model of Sections 4 and 5, market incumbents are less likely to invest than outside firms; however, the qualitative nature of our results for outside firms (i.e., leading breadth can increase the incentive to invest) also holds for market incumbents.

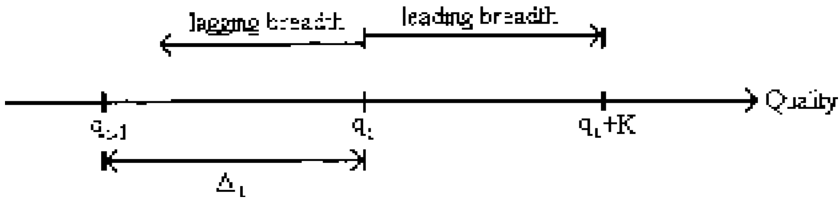

FIGURE 1.

2.2 Patent Instruments

The premise, of course, is that the government cannot tell firms which ideas to invest in. Instead, the government has a limited set of policy tools that can create incentives to invest.

We consider three policy tools concerning patents: patent length, lagging breadth, and leading breadth. The statutory patent life is the number of years, T, until the product can be marketed by competitors. To define lagging and leading breadth, suppose that a quality q_i is patented. Leading breadth is a number $K > 0$ such that any firm producing a quality in the interval $(q_i, q_i + K)$ (qualities higher than the patented quality) would *infringe* the patent, which means for our purposes that such a quality cannot be produced without a license. By lagging breadth, we mean a protected zone of qualities lower than the innovator's quality with the same infringement interpretation. See Figure 1. We assume that all innovations are patentable, whether they infringe another patent or not.[2]

2.3 Output Market

In this section and the next we assume that all agents purchase one unit of the good, and have the same willingness to pay for quality, i.e., their preferences are $q - p$, where q is the quality and p is the price. We refer to this model as *homogeneous tastes.* Firms compete on price, so if a firm produces quality q_i and the highest quality produced by a rival is q_{i-1}, then the price will be $p = q_i - q_{i-1}$. Hence, if the ith innovator can protect the entire quality increment Δ_i facilitated by his innovation,

2. A fourth policy instrument is whether an innovation is patentable, whether or not it infringes. See O'Donoghue (1997) for a discussion of this additional policy instrument in the context of an infinite stream of innovation, and Scotchmer and Green (1990), Scotchmer (1996), and Luski and Wettstein (1995) for discussions in the context of two-stage innovation.

then he can charge price Δ_i. We assume that the market has size one, so that the flow of profit is equal to the price.

The social value of an innovation Δ is Δ/r (where r is the discount rate), since it becomes the base for all future quality improvements, and its incremental value lasts forever. Let Δ^{opt} be the minimum idea such that all ideas $\Delta \geq \Delta^{\text{opt}}$ should optimally become innovations. Then Δ^{opt} satisfies $\Delta^{\text{opt}}/r = c$.

The *rate of innovation* when all ideas $\Delta \geq \underline{\Delta}$ become innovations is defined by

$$\Phi(\underline{\Delta}) \equiv \lambda \int_{\underline{\Delta}}^{\infty} \Delta \, dF. \tag{1}$$

The rate of innovation is the arrival rate of ideas, λ, multiplied by the expected innovation size conditional on having an idea. The socially optimal rate of innovation is $\Phi(\Delta^{\text{opt}})$.

2.4 The Limits of Lagging Breadth

Perhaps the most natural type of patent breadth is lagging breadth, since it protects against imitation of a product that has already been invented. In contrast, leading breadth grants property rights on qualities that have *not* been invented by the patentee. We now point out that lagging breadth alone provides insufficient incentives for R&D.

To make the most favorable case for lagging breadth, we assume that it protects the entire quality gap between the patented quality and the previously patented quality. Hence, flow revenue for the ith market incumbent is Δ_i, and lasts until the next innovation. Since the ith innovation creates flow welfare Δ_i, it follows that each innovator appropriates the entire flow of social benefit he created, but only during his incumbency.

To make the case for lagging breadth even more favorable, we assume that the statutory patent life is infinite. However, the flow of profit ends when the $(i + 1)$st innovation occurs. If firms invest in ideas $\Delta \geq \underline{\Delta}$, the arrival rate of innovations is

$$\Gamma(\underline{\Delta}) \equiv \lambda[1 - F(\underline{\Delta})], \tag{2}$$

and the net discounted profit for the ith innovator is $\Delta_i/[r + \Gamma(\underline{\Delta})] - c$.[3]

3. If the flow of profit Δ lasts for length t, the discounted profit is $\Delta(1 - e^{-rt})/r$. If the duration t is distributed according to a Poisson process with arrival rate Γ (which is $\lambda[1 - F(\underline{\Delta})]$ above), it has probability density $\Gamma e^{-\Gamma t}$, $t \in [0, \infty)$. Hence we have $\int_0^\infty [\Delta(1 - e^{-rt})/r] \, \Gamma e^{-\Gamma t} dt = \Delta/(r + \Gamma)$.

The profitability of an idea Δ to a given firm depends on the investment strategy of future firms, since their investment strategy determines the effective patent life.

An *equilibrium without leading breadth* is Δ^* such that $\Delta^*/[r + \Gamma(\Delta^*)] = c$, and all ideas $\Delta \geq \Delta^*$ become innovations.

The equilibrium rate of innovation is $\Phi(\Delta^*)$, where Φ is defined in equation (1).

Proposition 1 (Without Leading Breadth, Firms Underinvest in R&D): *The equilibrium rate of innovation is suboptimal, that is,* $\Phi(\Delta^*) < \Phi(\Delta^{\mathrm{opt}})$.

Proof. The result follows because Δ^* satisfies

$$\frac{\Delta^*}{r + \lambda[1 - F(\Delta^*)]} = c,$$

and Δ^{opt} satisfies $\Delta^{\mathrm{opt}}/r = c$; hence $\Delta^* > \Delta^{\mathrm{opt}}$.

The intuition for Proposition 1 is that discussed in the introduction: Even though patent life is infinite and each innovator can fully appropriate the flow benefits of his innovation during his market incumbency, the patent effectively terminates when another firm invents a better product. To emphasize this intuition, we define *effective patent life* as the expected length of time for which an innovator remains the incumbent. Without leading breadth (but with infinite statutory patent life), the effective patent life is $1/\Gamma(\Delta^*)$.[4] In the next section, we ask how leading breadth can extend effective patent life and stimulate R&D.

3. The Role of Leading Breadth

Our contribution in this section is to show how leading breadth can stimulate R&D by increasing effective patent life. The intuition is as follows: With leading breadth $K > \Delta^*$, the effective patent life can be longer than $1/\Gamma(\Delta^*)$, because any innovation $\Delta \in (\Delta^*, K)$ infringes the previous patent and therefore does not terminate that patent. The objective is to elicit investment in infringing innovations $\Delta < \Delta^*$, thus increasing the rate of innovation. A longer effective patent life makes an infringing innovation more profitable by extending the length of time it can be licensed to the market incumbent.

4. If the length of market incumbency, t, is distributed according to a Poisson process with arrival rate Γ, then the expectation of t is $1/\Gamma$.

We consider two types of patents with leading breadth: (1) patents with finite leading breadth and infinite patent life, and (2) patents with infinite leading breadth and finite patent life. Under the first type of policy, the effective patent life is determined endogenously by when a sufficiently better product is invented. Infinite patent life is a proxy for a long, finite patent life, where an innovator's monopoly power is more likely to end because a better product is invented than because the patent expires. Under the second type of policy, the effective patent life is the statutory patent life T.

Formally, a *patent policy with leading breadth* is $(K, T) \in \mathbb{R}_+ \times \mathbb{R}_+$. The previous section without leading breadth investigated the policy $(0, \infty)$. We now consider policies (K, ∞), $0 < K < \infty$, and (∞, T), $0 < T < \infty$.

Infringing innovations will be licensed to the market incumbent, and they increase the incumbent's market power by increasing the gap between the quality he sells and the previous patented quality. If a market incumbent with patented quality q_i and market gap (price) Δ_i licenses an innovation Δ, the new quality gap (price) will be $\Delta_i + \Delta$, and the quality he sells will be $q_i + \Delta$.

Leading breadth elicits investment in infringing innovations that will be licensed. Both policies (K, ∞) and (∞, T) create larger market gaps than would occur without leading breadth, but the gap does not grow forever. Under the policy (K, ∞), licensed innovations eventually stop accumulating when a noninfringing improved product appears in the market. Under the policy (∞, T), licensed innovations eventually stop accumulating because older patents expire.

The incentives for R&D depend on how profit is shared in licensing negotiations. Without specifying a bargaining game, we make three assumptions about how profit is shared:

Assumption 1: *When a firm receives an idea for an infringing improvement, it can bargain with an infringed patentholder without giving away the idea.*

An alternative assumption would be that bargaining takes place after the innovator has invested c in developing the improvement. Then the bargaining surplus would exclude costs c, which are sunk. *Ex post* bargaining would discourage some innovations, hence reduce investment, but would not change the nature of our conclusions. Our assumption makes the arguments simpler.

Assumption 2: *The incremental profit facilitated by an infringing innovation is divided in a licensing agreement such that the owner's share is nonincreasing in the number of infringed patents.*

This assumption holds for familiar bargaining solutions such as Nash bargaining, and means that the bargaining power of the infringing innovator does not increase if he faces more opponents.

Assumption 3: *The parties to a licensing agreement divide the incremental profit so that each one receives a nonnegative share.*

3.1 Finite Leading Breadth

We now consider the patent policy (K, ∞).

Under a policy (K, ∞), an innovator with a noninfringing innovation will remain the market incumbent until the next noninfringing innovation, and may license infringing innovations in the interim. Innovations $\Delta \geq K$ are noninfringing. If $K < \Delta^*$, then leading breadth has no effect. Assuming that $K \geq \Delta^*$, the arrival rate of noninfringing innovations is $\Gamma(K) \equiv \lambda[1 - F(K)]$. The effective patent life is thus $1/\Gamma(K)$, and the effective patent life increases with K.

The effect of leading breadth on the rate of innovation turns on which *infringing* ideas will become innovations, since those are the ideas $\Delta < \Delta^*$ that would otherwise not become innovations. Assuming $K > \Delta^*$, it is immediate that all ideas $\Delta \geq K$ will become innovations. These ideas are noninfringing, and if they are profitable without leading breadth, then they are even more profitable when leading breadth is granted, since leading breadth increases the effective patent life.

An infringing innovation Δ contributes increment Δ to the market incumbent's flow of profit. The profit surplus created by licensing is therefore $\Delta/[r + \Gamma(K)] - c$, and Assumption 1 implies that investment occurs if this surplus is nonnegative. (See footnote 3 above for the calculation.)

For $K \geq \Delta^*$, an *equilibrium under policy* (K, ∞) is $\Delta^K(K)$ such that $\Delta^K(K)/[r + \Gamma(K)] = c$ and all ideas $\Delta \geq \Delta^K(K)$ become innovations.

The equilibrium rate of innovation is $\Phi(\Delta^K(K))$, where Φ is defined in equation (1).

Proposition 2 (Under Policies (K, ∞), the Limit Rate of Innovation as K Becomes Large is Optimal): *The equilibrium rate of innovation $\Phi(\Delta^K(K))$ is increasing in K, and $\lim_{K\to\infty} \Phi(\Delta^K(K)) = \Phi(\Delta^{\text{opt}})$.*

Proof. $\Phi(\cdot)$ and $\Delta^K(\cdot)$ are both decreasing, so $\Phi(\Delta^K(K))$ is increasing in K. Using the definition of equilibrium, as $K \to \infty$, $\Delta^K(K) \to \Delta^{\text{opt}}$. The result follows.

3.2 Infinite Leading Breadth

Now suppose the patent policy is (∞, T). Under a policy (∞, T), every future improved product infringes until expiration of the patent. Hence, the effective patent life is identical to the statutory patent life T.

To understand how licensing operates in this case, consider an innovation at time t. If patents have infinite leading breadth and patent length T, the innovation at time t infringes all patents issued between $t - T$ and t, and the innovator must reach licensing agreements with their owners. Further, any innovation made between t and $t + T$ will infringe the patent issued at time t, and licenses will be negotiated with those innovators. Every patent goes through the following life cycle: In early life it infringes prior patents, in midlife it is both infringing and infringed, and in late life subsequent patents infringe it.

We are not specific about which firm is the seller at any given time, e.g., the seller might be the latest innovator or the owner of the oldest unexpired patent. The division of profit can be negotiated independently of who is the actual seller. The parties to the negotiation are the prospective innovator and the owners of unexpired patents. As in the previous section, the incentive to invest in an idea Δ depends only on the surplus profit that the innovation would create, and not on the specific licensing fees.

The profit surplus has two components: (i) the direct addition to output market profits, and (ii) the incremental claims on subsequent innovations that are patented between t and $t + T$. These are the first and second terms of equation (3) below. An innovation Δ contributes increment Δ to market profits for the entire life of its patent. Hence, part (i) has discounted value $\Delta(1 - e^{-rT})/r$.

Part (ii), which is $L(\cdot)$ below, will depend on the history of innovation, since profit on subsequent infringing innovations will be shared in a way that reflects the number of infringed patents. We let H be the set of possible patenting histories. A specific history $h \in H$ describes the dates and sizes of all previous quality improvements. The dates of all previous patents and the patent length T determine the number of infringed patents at any time t. The number $L(T, h)$ describes expected claims on innovations invented between t and $t + T$ facilitated by the patent at time t when the history is h; that is, claims that could not otherwise be made on future innovations. Assumption 2 implies that $L(T, h) > 0$ for all $T > 0$, $h \in H$.

The profit surplus of an innovation Δ is therefore

$$\Delta \frac{1 - e^{-rT}}{r} + L(T, h) - c. \tag{3}$$

An *equilibrium under policy* (∞, T) is a set of values $\{\Delta^T(T, h), h \in H\}$ such that for each $h \in H$, $\Delta^T(T, h)(1 - e^{-rT})/r + L(T, h) = c$ and all ideas $\Delta \geq \Delta^T(T, h)$ become innovations.

The rate of innovation under policy (∞, T) is defined by

$$\lambda \int_H \left(\int_{\Delta^T(T,h)}^{\infty} \Delta \, dF \right) dG(h; T) \equiv \int_H \Phi(\Delta^T(T, h)) \, dG(h; T). \tag{4}$$

The equilibrium induces a probability distribution on histories, described by elements of H. That is, at a random time t, there is a probability distribution on the dates of previous innovations, which we shall refer to by a cumulative distribution function $G(\cdot; T)$.

Proposition 3 (Under Policies (∞, T), the Limit Rate of Innovation as T Becomes Large Is No Smaller than Optimal): *Suppose that for each $h \in H$, $L(T, h)$ converges to a nonnegative limit as T becomes large. Then for each $h \in H$, $\lim_{T\to\infty} \Phi(\Delta^T(T, h)) \geq \Phi(\Delta^{\text{opt}})$.*

Proof. This follows from (3) because $L(T, h) > 0$ and hence $\lim_{T\to\infty} \Delta^T(T, h) \leq \Delta^{\text{opt}}$.

3.3 Comparison of the Two Policies with Leading Breadth

We have shown in a model with homogeneous tastes that both types of policy with leading breadth can extend effective patent life and stimulate R&D. However, they are not equivalent, as they may have different R&D costs. In addition, different effective patent lives may be required to support the same rate of innovation. The latter will be important in the next section, where we introduce heterogeneous tastes and oligopoly distortions.

Proposition 4 (Comparison of the Policies with Leading Breadth): *Suppose two policies (K, ∞) and (∞, T) induce the same rate of innovation. Then:*

1. *Policy (∞, T) has a shorter effective patent life, or $T < 1/\Gamma(K)$.*
2. *Policy (K, ∞) has lower total R&D costs.*

Proof. See Appendix A.

Proposition 4(1) states that to achieve a specified innovation rate, the effective patent life under a policy (K, ∞) must be longer than under a policy (∞, T). Alternatively stated, if the effective patent life under (K, ∞) were the same as the statutory patent life T, then the policy (K, ∞) would induce a smaller rate of innovation. This might seem paradoxical, since policies with the same patent lives should generate more or less the same total profit. Hence the difference in the amount of investment must be traceable to how the profit is distributed.

In the policy (∞, T), patents eventually expire, so that a prospective innovation causes future innovations to be infringing in periods when they would otherwise be noninfringing. By causing them to be infringing, the patent creates claims against them that were not available to the previous patentholders. The profit of these infringing innovations is shared with the infringed patentholder through licensing, represented by the licensing surplus $L(\cdot)$.

Compare with the alternative policy (K, ∞). The marginal idea $\Delta^K(K)$ that determines the innovation rate is always an infringing idea. An infringing idea does not create claims against future innovations that are not already owned by the previous patentholder, since the previous patent lasts forever. The surplus on infringing ideas that is shared through licensing accrues to the owners of *noninfringing* ideas, who collect a large share of the total profit. But it is the willingness to invest in infringing ideas, not noninfringing ideas, that determines the rate of innovation, and the infringing ideas receive a small share of total profit. In contrast, the policy (∞, T) makes no such clear distinction between infringing and noninfringing ideas.

Although the policy (∞, T) can sustain a specified rate of innovation with a shorter patent life, Proposition 4(2) states that it may have the offsetting disadvantage of higher R&D costs. The intuition for Proposition 4(2) is as follows. The minimum acceptable idea under the policy (∞, T) is stochastic (it depends on the history h), so sometimes ideas with low Δ may be undertaken, whereas other times higher-valued ideas are forgone. Costs can obviously be cut by investing once in 2Δ rather than twice in Δ, and this is why the common minimum value $\Delta^K(K)$ leads to lower total costs, conditional on a fixed innovation rate.

If all quality innovations reach all consumers instantly and demand is inelastic, policy (K, ∞) is clearly better than policy (∞, T): (K, ∞) has lower R&D costs, and the longer effective patent life is irrelevant, since there are no consumer losses due to market distortions. But with static inefficiencies, a longer effective patent life reduces consumer welfare. That is the topic of the next two sections.

4. Oligopoly Distortions

The purpose of the previous section was to isolate the role of leading breadth in extending effective patent life and stimulating R&D. We therefore studied a model with no market distortions once the innovations had occurred. The only inefficiencies were that the rate of innovation could be too low, and R&D costs could be unnecessarily high. We showed that R&D costs would be higher in the policy (∞, T) than in (K, ∞).

We now show that market distortions can change our conclusions in two ways: Lagging breadth alone might *over*reward R&D so there is too much innovation, and the market distortions might be more onerous in the policy (K, ∞) than in (∞, T), and thus overturn its cost advantage.

4.1 The Output Market

To develop these points we introduce heterogeneity in tastes into the above model. This gives us the natural-oligopoly model of Gabszewicz and Thisse (1980) and Shaked and Sutton (1983). In order not to be distracted by details of the output market, we first take an axiomatic approach to how the market operates, and then point out in Appendix C how the axioms follow from the natural-oligopoly model. An inefficiency arises because consumers have heterogeneous tastes for quality, and as a result do not all consume the highest-quality product, even though that would be efficient. Instead, the consumers with lower willingness to pay for quality consume a lower-quality product. Quality improvements are diffused to the whole market only after a delay.

Our axioms pertain to the output market once the qualities of the potential entrants are fixed by innovation and patents:

1. Competition in the output market allows exactly two firms to earn positive profit at any point in time, a *market leader* with a higher-quality product denoted q_H, and a *market follower* with a lower-quality product denoted q_L.
2. Each consumer consumes one unit of the product at each point in time, but the quality purchased improves as innovation occurs. Consumers have preferences $\theta q - p$, where $\theta \in [\theta_0, \theta^0]$. They thus differ in their willingness to pay for product quality, and in the outcome of market competition, they separate into two groups that do not change over time. A group of size n_L with lower willingness to pay for quality always purchases from the current market follower, and a group of size n_H with higher willingness to pay for quality always purchases from the current market leader.

3. Define the *quality gap* as $\Delta \equiv q_H - q_L$. The two firms in the market each earn profit that is a linear function of the quality gap. We let $\pi_H\Delta$ and $\pi_L\Delta$ refer respectively to the equilibrium profits of the firms with higher quality and lower quality, where π_L and π_H are constants with $\pi_H > \pi_L$.
4. Let B represent the sum of all consumers' willingness to pay for a unit increase in quality, and let ℓ represent the portion of that willingness to pay attributable to the n_L consumers who purchase the lower-quality product. Then the flow welfare when the market qualities are q_H, q_L is $Bq_H - \ell\Delta$.
5. The flow of profit $\pi_H\Delta$ that accrues to the higher-quality firm is no greater than the summed willingness to pay of its customers, $(B - \ell)\Delta$. Hence, $B - \ell > \pi_H$.

Axiom 1 is the main conclusion of the natural-oligopoly model. When consumers differ in their willingness to pay for quality, there can be more than one firm in the market, but there is room for only a finite number of firms. With appropriate specification of parameters, as discussed in Appendix C, there is room for exactly two firms. Axiom 2 reflects the division of consumers among firms in the natural-oligopoly model. The profit functions in Axiom 3 are the outcome of price competition in the natural-oligopoly model when there is a flow cost to market participation. Axiom 4 reflects that each consumer's utility is linear in the quality consumed.

4.2 Lagging Breadth in the Natural-Oligopoly Model

We begin our analysis of the natural-oligopoly model by reinvestigating what happens with complete lagging breadth and infinite patent life, but no leading breadth.

In the natural-oligopoly model, the ith innovator earns payoffs in two periods: as market leader just subsequent to the innovation, when it collects flow profit $\pi_H\Delta_i$, and as market follower after the next innovation, when it collects flow profit $\pi_L\Delta_{i+1}$. After subsequent innovations, the firm is displaced from the market. Hence, if future firms invest in ideas $\Delta \geq \underline{\Delta}$, the net discounted profit for the ith innovator is

$$\Pi(\Delta, \underline{\Delta}) \equiv \frac{\pi_H\Delta}{r + \Gamma(\underline{\Delta})} + \frac{\pi_L\Delta^e(\underline{\Delta})}{r + \Gamma(\underline{\Delta})}\left(\frac{\Gamma(\underline{\Delta})}{r + \Gamma(\underline{\Delta})}\right) - c, \tag{5}$$

and $\Gamma(\cdot)$ is the arrival rate of innovations and is defined by $\Gamma(\underline{\Delta}) = \lambda[1 - F(\underline{\Delta})]$ [as in equation (2)]. The expectation function Δ^e is defined by

$$\Delta^e(\underline{\Delta}) \equiv \frac{1}{1 - F(\underline{\Delta})} \int_{\underline{\Delta}}^{\infty} \Delta \, dF.$$

The first term of (5) is the discounted flow of profit as market leader, which lasts until the innovator is displaced as market leader by the next innovation (see footnote 3). The second term is the discounted flow of profit after the innovator becomes the market follower. During the period as follower, the innovator's profit is determined not by his own quality increment Δ_i, but by the quality increment of his successor, which has expected value $\Delta^e(\underline{\Delta})$. Discounted from when it begins, the flow profit $\pi_L \Delta^e(\underline{\Delta})$ has discounted expected value $\pi_L \Delta^e(\underline{\Delta})/[r + \Gamma(\underline{\Delta})]$, which reflects that the flow of profit ends when a third innovation occurs. $\pi_L \Delta^e(\underline{\Delta})/[r + \Gamma(\underline{\Delta})]$ is multiplied by $\Gamma(\underline{\Delta})/[r + \Gamma(\underline{\Delta})]$ because the flow of profit begins when the innovator is displaced as market leader by the next innovation.[5]

It is convenient in this section to make explicit in the definition of equilibrium that the minimum profitable idea Δ^* depends on λ. An *equilibrium without leading breadth* is

$$\tilde{\Delta}^*(\lambda) = \begin{cases} \Delta' & \text{if } \Pi(\Delta', \Delta') = 0, \\ 0 & \text{if } \Pi(0, 0) \geq 0, \end{cases}$$

where it is understood that $\Pi(\cdot, \cdot)$ depends on λ. All ideas $\Delta \geq \tilde{\Delta}^*(\lambda)$ become innovations.

We first point out that the natural-oligopoly model creates a "foot-in-the-door effect": Since firms earn profit as market follower in addition to their profits as market leader, there is an added incentive to invest. The profit an innovator will earn as market follower will depend on the size of the *next* innovation, which could be large. As a result, firms may invest in small ideas simply as a means to secure a market position. The following example shows that the minimum profitable idea can even be $\Delta = 0$, which is clearly inefficient, since $\Delta = 0$ provides no consumer value.

Example (Foot-in-the-Door Effect): Suppose that $\Delta^e(0) = 1$, $c = 1$, $\lambda = 0.5$ (so ideas occur on average every 2 years), $r = 0.1$ and $\pi_L = 1$.[6] Then $\pi(0, 0) > 0$, so firms will invest in all ideas.

5. If the value $v \equiv \pi_L \Delta^e(\underline{\Delta})/(r + \Gamma(\underline{\Delta}))$ is received at time t, the discounted value is ve^{-rt}. As in footnote 3, t has probability density Γe^{-rT}, $t \in [0, \infty)$, so $\int_0^\infty (ve^{-rt})\Gamma e^{-\Gamma t}\, dt = v\Gamma/(r + \Gamma)$.

6. $\pi_L = 1$ is consistent with $(\theta_0, \theta^0) = (3, 9)$ in Appendix C.

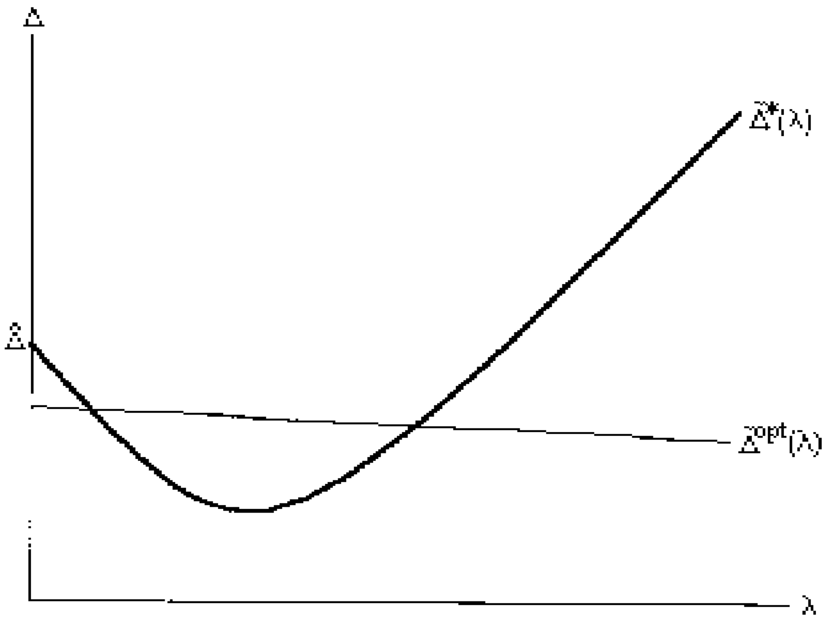

FIGURE 2.

In this example, firms invest in ideas that yield zero flow profit as market leader because the expected future profits as market follower are sufficient to cover R&D costs. The foot-in-the-door effect implies that firms might overinvest in R&D even without leading breadth.[7] But when λ is high so that market incumbency is short, firms must make high profit as both market leader and market follower in order to cover their costs. As a result, for high λ, investment will be suboptimal despite the foot-in-the-door effect.

Figure 2 summarizes how the equilibrium $\tilde{\Delta}^*(\cdot)$ depends on λ. The parameter $\hat{\Delta}$ is defined by $\pi_H \hat{\Delta}/r - c = 0$ and represents the equilibrium cutoff if an innovator remained market leader forever. The following lemma is proved in Appendix A.

Lemma 1: $\tilde{\Delta}^*(0) = \hat{\Delta}$, *and* $\tilde{\Delta}^*(\cdot)$ *is either nondecreasing or U-shaped.*

To compare equilibrium with optimal investment we must char-

7. A separate reason for excess investment in R&D is that patent races speed up investment. See Reinganum (1989) for a summary of the patent-race literature. A related idea is what the endogenous-growth literature calls the "business stealing effect" [see Aghion and Howitt (1992), Grossman and Helpman (1991), and Romer (1994)].

acterize optimal investment. The following welfare expression $W^*(\underline{\Delta})$ is derived in Appendix B:

$$W^*(\underline{\Delta}) \equiv \frac{\Gamma(\underline{\Delta})}{r}\left[\frac{B}{r}\Delta^e(\underline{\Delta}) - \frac{\ell}{r+\Gamma(\underline{\Delta})}\Delta^e(\underline{\Delta}) - c\right]. \tag{6}$$

This expression assumes for simplicity that the initial market gap is 0, but we point out in Appendix B that our propositions remain intact if the initial market gap is positive. A simple (but slightly misleading) intuition for the expression is as follows.

The bracketed factor is the expected value of each innovation. There are $\Gamma(\underline{\Delta})$ innovations per unit time, and we divide by r to allow for all the innovations in the infinite future. The first bracketed term represents the expected social benefit of a quality increment if it is received by all consumers immediately. The increment to welfare flow $B\Delta^e(\underline{\Delta})$ is divided by r to reflect that it lasts forever, since the quality increment becomes a foundation for future quality increments. From this value we subtract the loss due to the delay in diffusing the quality increment to consumers with low willingness to pay. Using the reasoning in footnote 3, the second term captures the expected discounted value of losing flow welfare $\ell\Delta^e(\underline{\Delta})$ until the next innovation. Finally we subtract off the cost of the innovation.

Let $\tilde{\Delta}^{\text{opt}}(\lambda)$ represent the maximizer of W^* (which depends on λ). That is, $\tilde{\Delta}^{\text{opt}}(\lambda)$ is the minimum idea that should optimally become an innovation, recognizing that each innovation reaches the low-willingness-to-pay consumers only after the subsequent innovation. Our objective in calculating this second-best type of optimum is to compare the second-best optimum with equilibrium. The following is easily verified:

Lemma 2: *$\tilde{\Delta}^{\text{opt}}(0) < \hat{\Delta}$, and $\tilde{\Delta}^{\text{opt}}(\cdot)$ is decreasing.*

In Figure 2, the foot-in-the-door effect appears where $\Delta^*(\lambda) < \Delta^{\text{opt}}(\lambda)$. It can only occur for low hit rates λ of ideas. If the hit rate of ideas is high, then investment without leading breadth is always deficient, just as in the simple model with homogeneous tastes. This is the content of the following proposition:

Proposition 5 (In the Natural-Oligopoly Model, If Ideas Are Frequent Then Firms Underinvest without Leading Breadth): *Suppose tastes are heterogeneous. There exists λ^* such that $\tilde{\Delta}^*(\lambda) > \tilde{\Delta}^{\text{opt}}(\lambda)$ for all $\lambda \geq \lambda^*$.*

Proof. $\tilde{\Delta}^*(\cdot)$ is continuous and $\lim_{\lambda\to\infty} \tilde{\Delta}^*(\lambda) = \infty$. Given this, the result follows directly from Lemmas 1 and 2.

Thus if the hit rate of ideas is high, lagging breadth is inadequate in the natural-oligopoly model despite the foot-in-the-door effect. We now investigate whether leading breadth can solve this problem by prolonging effective patent life as in the model with homogeneous tastes.

5. Leading Breadth in the Natural-Oligopoly Model

The premise of this section is that ideas are frequent (λ is high) and firms underinvest in R&D without leading breadth. To stimulate R&D, we reconsider the two policies (K, ∞) and (∞, T) considered in Section 3. We first describe the equilibria with these policies, pointing out that leading breadth plays the same role in the natural-oligopoly model as in the model with homogeneous tastes. However, the natural-oligopoly model introduces a static inefficiency.

In what follows, we suppress the argument λ from $\tilde{\Delta}^*$, assuming λ is large enough to warrant leading breadth.

5.1 Finite Leading Breadth

Under the policy (K, ∞), an innovator with a noninfringing innovation becomes the market leader, and will maintain this position until the next noninfringing innovation. In addition, the innovator may license infringing innovations while market leader. Unlike in the model with homogeneous tastes, the next noninfringing innovation does not terminate the innovator's flow of profit. Instead the innovator becomes market follower, and earns a second flow of profit until the subsequent noninfringing innovation.

As before, we define the *effective patent life* as the expected time between noninfringing innovations, namely $1/\Gamma(K)$.[8]

As with homogeneous tastes, we focus on $K \geq \tilde{\Delta}^*$, since otherwise K has no effect. As before, the effect on the rate of innovation turns on how K affects the profitability of *infringing* ideas $\Delta < K$. All ideas $\Delta \geq K$ will become innovations, since K extends effective patent life relative to no leading breadth, and thus increases the profitability of noninfringing innovations.

8. The length of time that a noninfringing innovation earns profit under the policy (K, ∞) is actually longer than this, since the innovator also earns profit as market follower. However, this is not true of infringing innovations. It is investment in infringing innovations that determines the rate of innovation.

An infringing innovation Δ contributes an increment $\pi_H\Delta$ to the market leader's flow of profit until the next noninfringing innovation. However, an infringing innovation does not add any anticipated profit from becoming market follower. The market leader will eventually become market follower whether or not he licenses the infringing improvement. In addition, his profit when market follower depends on the size of the noninfringing innovation that displaces him, and not on the quality of the product he sells, hence not on the accumulation of licensed improvements. Thus, the profit surplus created by licensing is

$$S^K(\Delta, K) \equiv \frac{\pi_H \Delta}{r + \Gamma(K)} - c,$$

and investment occurs if this surplus is nonnegative.

In the natural-oligopoly model, an *equilibrium under policy* (K, ∞) is

$$\tilde{\Delta}^K(K) = \begin{cases} K & \text{if} \quad S^K(K, K) \leq 0, \\ \Delta' & \text{if} \quad S^K(K, K) > 0 \text{ and } S^K(\Delta', K) = 0 \end{cases}$$

such that all ideas $\Delta \geq \tilde{\Delta}^K(K)$ become innovations. Since S^K is increasing in its first argument, the equilibrium $\tilde{\Delta}^K(K)$ is unique.

The basic intuition for how the policy (K, ∞) can extend effective patent life and stimulate R&D is identical to that in Section 3, and we do not repeat the argument. Our main discovery regarding the policy (K, ∞) in the natural-oligopoly model is that a small amount of leading breadth can retard R&D relative to no leading breadth at all ($K = 0$). This is perhaps counterintuitive, but can be explained as follows.

Without leading breadth, the marginal innovation is noninfringing, and the innovator stands to profit both as market leader and as market follower. The foot-in-the-door effect (which results from the profit as market follower) may elicit investment. In contrast, with leading breadth the marginal innovation infringes the prior patent. The foot-in-the-door effect is absent, as reflected in the fact that when $K = \tilde{\Delta}^*$, $S^K(\Delta, K) < \Pi(\Delta, \tilde{\Delta}^*)$. The marginal innovation is less profitable when it infringes the prior patent and must be licensed.

Proposition 6 (Small Leading Breadth Can Retard R&D): *Suppose tastes are heterogeneous.*

1. $\tilde{\Delta}^K(K) = \tilde{\Delta}^*$ *for all* $K \in [0, \tilde{\Delta}^*]$.
2. *There exists* $K^* > \tilde{\Delta}^*$ *such that the rate of innovation and social welfare are strictly larger for* $K = 0$ *(no leading breadth) than for any* $K \in (\tilde{\Delta}^*, K^*)$.

Proof. 1: If $K < \tilde{\Delta}^*$, then leading breadth does not affect any ideas that firms would invest in.

2: Recalling Figure 2, consider two cases: $\hat{\Delta} \geq \tilde{\Delta}^*$ and $\hat{\Delta} < \tilde{\Delta}^*$. In the first case, take any $K^* > \tilde{\Delta}^*$, since $\tilde{\Delta}^K(K) \geq \hat{\Delta}$ for all K. In the second case, we have $S^K(\tilde{\Delta}^*, \tilde{\Delta}^*) < \Pi(\tilde{\Delta}^*, \tilde{\Delta}^*) = 0$. Since S^K is increasing in its second argument, there exists a unique $K^* > \tilde{\Delta}^*$ such that $S^K(\tilde{\Delta}^*, K^*) = \Pi(\tilde{\Delta}^*, \tilde{\Delta}^*)$, which implies $\tilde{\Delta}^K(K^*) = \tilde{\Delta}^*$. For any $K \in (\tilde{\Delta}^*, K^*)$, $S^K(\tilde{\Delta}^*, K) < 0$, and $\tilde{\Delta}^K(K) > \tilde{\Delta}^*$, since S^K is increasing in its first argument. Hence, the rate of innovation is smaller for any $K \in (\tilde{\Delta}^*, K^*)$ than for $K = 0$.

Social welfare is smaller for any $K \in (\tilde{\Delta}^*, K^*)$ than for $K = 0$ because not only is there less innovation, but in addition low-θ consumers receive innovations with a longer lag than without leading breadth.

5.2 Infinite Leading Breadth

Suppose the patent policy is (∞, T), so the effective patent life is identical to the statutory patent life T. The analysis of this policy is essentially identical to the case of homogeneous tastes in Section 3, and we will omit much of it.

Under policy (∞, T), there will be a market follower who makes positive profits; however, market-follower profit is not a consideration when a firm decides whether to invest in an idea. Unlike under the policy (K, ∞), a patentholder will not necessarily become market follower when he ceases to be market leader. Under the policy (K, ∞), the previous market leader's patent is still valid when his product is supplanted in the market. Since no other firm can market his product, he becomes market follower. Under the policy (∞, T), the market follower produces the highest-quality product whose patent has expired. But since the patent has expired, there is no reason to believe that its inventor will market it. We therefore assume that a random firm is the market follower.[9]

9. If each patentholder enjoyed a period as market follower, our main conclusion regarding relative effective patent lives would be strengthened. Since every innovation then becomes even more profitable under the policy (∞, T), the same rate of innovation can be sustained with an even shorter patent life T. The market for the low-quality product should be competitive, since the relevant patent has expired. In our formulation, this market is competitive in the sense of free entry—given the flow cost of market participation (see Appendix C), entry would not be profitable. If there were no cost of market participation, then the price of the low-quality product would be driven to zero. In that case, market-leader profits are still linear in the quality gap, so the qualitative results are unchanged. However, the magnitude of market-leader profits is smaller, undermining the effectiveness of policy (∞, T). For policy (K, ∞), this issue does not arise, since the market follower owns a valid patent.

The profit surplus is therefore the same in the natural-oligopoly model as in the homogeneous-tastes model, except that the increment to the market leader's flow profit is $\pi_H \Delta$ instead of Δ. Hence, investment occurs if the following surplus is nonnegative:

$$S^T(\Delta, T, h) \equiv \pi_H \Delta \left(\frac{1 - e^{-rT}}{r}\right) + L(T, h) - c.$$

In the natural-oligopoly model, an *equilibrium under policy* (∞, T) is a set of values $\{\tilde{\Delta}^T(T, h), h \in H\}$, such that

$$\tilde{\Delta}^T(T, h) = \begin{cases} \Delta' & \text{if} \quad S^T(\Delta', T, h) = 0 \text{ and } \Delta' > 0, \\ 0 & \text{if} \quad S^T(0, T, h) \geq 0, \end{cases}$$

and, conditional on a history h, all ideas $\Delta \geq \tilde{\Delta}^T(T, h)$ become innovations. Since S^T is increasing in its first argument, the equilibrium values $\{\tilde{\Delta}^T(T, h), h \in H\}$ are unique.

The intuition for how policy (∞, T) can stimulate R&D in the natural-oligopoly model is identical to that in the model with homogeneous tastes.

5.3 Delayed Diffusion

In the model with homogeneous tastes we showed that both policies (K, ∞) and (∞, T) can stimulate R&D, but conditional on having the same rate of innovation, policy (K, ∞) has lower total R&D costs. This result is driven by the fact that under (K, ∞) the equilibrium cutoff is constant, while under (∞, T) it is stochastic. Clearly, this result will hold in the natural-oligopoly model as well. The inefficiency introduced by heterogeneous tastes is on the consumer side, and it comes in the form of delayed diffusion.

In the natural-oligopoly model, not all consumers consume the highest-quality product. Although every innovation eventually reaches every consumer, there is a lag before it reaches the consumers with relatively low willingness to pay for quality. Under policy (K, ∞) the expected lag is the expected time between noninfringing innovations, and under policy (∞, T) the lag is the statutory patent life T. Under either policy, a longer lag increases the social cost of delayed diffusion.

In Section 3, we showed that to induce the same rate of innovation, policy (K, ∞) must have a longer effective patent life than policy (∞, T). With homogeneous tastes, effective patent life is irrelevant for social welfare. In the natural-oligopoly model, however, the longer effective patent life of policy (K, ∞) leads to a higher cost of delayed diffusion.

To compare the cost of delayed diffusion in the two policies, we need a welfare expression for each. The welfare expressions defined below, W^K for the policy (K, ∞) and W^T for the policy (∞, T), represent the expected social benefits (excluding costs) of the infinite streams of innovations:

$$W^K(K) = \frac{\Gamma(\tilde{\Delta}^K(K))}{r}\left(\frac{B}{r}\Delta^e(\tilde{\Delta}^K(K)) - \frac{\ell}{r+\Gamma(K)}\Delta^e(\tilde{\Delta}^K(K))\right)$$

$$\text{or} \quad W^K(K) = \frac{\lambda}{r}\left(\frac{B}{r} - \ell\frac{1}{r+\Gamma(K)}\right)\int_{\tilde{\Delta}^K(K)}^{\infty} \Delta\, dF; \tag{7}$$

$$W^T(T) = \int_H \frac{\Gamma(\tilde{\Delta}^T(T,h))}{r}\left[\frac{B}{r}\Delta^e(\tilde{\Delta}^T(T,h)) - \ell\left(\frac{1-e^{-rT}}{r}\right)\Delta^e(\tilde{\Delta}^T(T,h))\right] dG(h;T)$$

$$\text{or} \quad W^T(T) = \frac{\lambda}{r}\left[\frac{B}{r} - \ell\left(\frac{1-e^{-rT}}{r}\right)\right]\int_H\left(\int_{\tilde{\Delta}^T(T,h)}^{\infty}\Delta\, dF\right) dG(h;T). \tag{8}$$

In each case the social benefit of an innovation, which would be $(B/r)\Delta$ if all consumers received it immediately, is reduced by the expected delay until it reaches the consumers with lower willingness to pay. The reasoning is analogous to that behind W^* in Section 4. (See also our comments in Appendix B.)

Proposition 7 [Conditional on the Rate of Innovation, Policy (∞, T) Has Lower Costs of Delayed Diffusion than Policy (K, ∞)]: *Suppose tastes are heterogeneous, and that two policies (K, ∞) and (∞, T) have the same rate of innovation. Then $W^K(K) < W^T(T)$.*

Proof. The rates of innovation are the same if and only if

$$\int_{\tilde{\Delta}^K(K)}^{\infty}\Delta\, dF = \int_H\left(\int_{\tilde{\Delta}^T(T,h)}^{\infty}\Delta\, dF\right) dG(h;T). \tag{9}$$

Equation (9) and the equilibrium conditions imply $(1 - e^{-rT})/r < 1/[r + \Gamma(K)]$. The proof for the natural oligopoly is identical to the proof for homogeneous tastes in Proposition 4(1). In addition, equation (9), in conjunction with equations (7) and (8), implies $W^T(T) > W^K(K)$ if and only if $(1 - e^{-rT})/r < 1/[r + \Gamma(K)]$. The result follows.

The intuition for Proposition 7 is straightforward. Given that the effective patent life required to sustain a given rate of innovation is greater in (K, ∞) than in (∞, T), the delay is longer before low-willingness-to-pay consumers receive quality improvements, so it should be no surprise that the costs of delayed diffusion are greater.

6. Discussion

Our main observation in this paper is that the profitability of R&D depends on the effective patent life, and that effective patent life is determined not only by statutory patent life but also by patent breadth.

Unless there is some sustained advantage to entering the market, such as occurs in the natural-oligopoly model (the foot-in-the-door effect), lagging breadth alone will not provide sufficient incentives for R&D. We compared two remedies with leading breadth: (1) infinite patent life with finite leading breadth and (2) finite patent life with infinite leading breadth. In the first policy, effective patent lives are endogenous to the leading breadth and the hit rate of ideas. In the second policy, effective patent lives coincide with statutory patent lives, and both are immune to the hit rate of ideas. We showed that the first policy is superior in reducing R&D costs (conditional on the rate of innovation), while the second policy has a shorter effective patent life, and therefore reduces the market distortions.

We conclude with a few remarks on patent doctrine in order to put our observations into perspective. The fundamental protection of a patent is that other firms are barred from using the patented technology without the patentholder's consent. This is as straightforward as the law can be, but many controversies arise regarding the technologies that can be covered by the patent. The US patent statute does not refer to patent breadth, except implicitly in how claims are limited by the enabling disclosure, and in the requirements of novelty and nonobviousness. These requirements mean collectively that the claimed technologies must differ substantially from "prior art." The disclosure requirement is a test of whether the patentee actually invented the technologies claimed.

For the context of cumulative innovation it seems plausible that an innovator "invented" all the qualities between the previous state of the art and his new improved state of the art, and therefore (in our terms) complete lagging breadth seems justifiable under the law. However, there are at least two ways to think about leading breadth. Perhaps the most straightforward is to say that every subsequent improvement *uses* the previous technologies, and hence infringes for the duration of the previous patents. This interpretation would support the policy with infinite leading breadth and finite patent life.

But another interpretation is to say that since previous patent-holders have not invented the superior qualities, it is unreasonable to give them effective patent protection against all such products; hence leading breadth is by its nature untenable. In our model, such an interpretation might support the policy with long patent life and finite patent breadth, where effective patent lives are terminated by noninfringing innovations. A small amount of leading breadth might be justified under the "doctrine of equivalents." [See, for example, Merges and Nelson (1990).]

The patent recommendation in this paper is that leading breadth should be granted when the hit rate of ideas is high; that is, there is an exogenous force toward rapid turnover in the market. Of course one must ask what the patent authorities must observe in order to implement such a policy. The main consideration is how quickly an innovator would lose his market position in the absence of such protection. However, such considerations are not part of the patent statute, and one could even see how the patent authorities might reach the opposite conclusion: An active investment climate can be seen as prima facie evidence that ideas are "obvious" and therefore not protected at all. We point this out in order to emphasize that the effectiveness of patent law in supporting research is seriously impeded by the fact that it does not refer to costs or market structure in how patent protection is circumscribed.[10]

Appendix A: Proofs of Propositions

Proof of Proposition 4. The rates of innovation are the same if and only if

$$\int_{\Delta^K(K)}^{\infty} \Delta \, dF = \int_H \left(\int_{\Delta^T(T,\,h)}^{\infty} \Delta \, dF \right) dG(h; T) \tag{10}$$

1: Using the equilibrium conditions, for each $h \in H$,

$$\frac{\Delta^K(K)}{r + \Gamma(K)} = c = L(T, h) + \Delta^T(T, h) \left(\frac{1 - e^{-rT}}{r} \right) > \Delta^T(T, h) \left(\frac{1 - e^{-rT}}{r} \right). \tag{11}$$

Using (10), there exists h such that $\Delta^K(K) \leq \Delta^T(T, h)$, so using (11),

$$\frac{1 - e^{-rT}}{r} < \frac{1}{r + \Gamma(K)} \cdot \tag{12}$$

10. For a broader discussion of the deficiencies of patent law, see Scotchmer (1991, 1996).

The average effective patent life in the policy (K, ∞) is $1/\Gamma(K)$. For all $T \geq 0$, $e^{rT} \geq 1 + rT$; hence if $T > 1/\Gamma(K)$, $e^{rT} \geq [r + \Gamma(K)]/\Gamma(K)$, which contradicts (12).

2: Total R&D costs are proportional to the total number of innovations. Thus it is enough to show that

$$1 - F(\Delta^K(K)) \leq \int_H [1 - F(\Delta^T(T, h))]\, dG(h; T). \tag{13}$$

Let $\hat{H} \subset H$ be the set $\{h \in H \mid \Delta^T(T, h) > \Delta^K(K)\}$. From (10) it follows that

$$\int_{\hat{H}} \left[\int_{\Delta^K(K)}^{\Delta^T(T,h)} \Delta\, dF \right] dG(h; T) = \int_{H \setminus \hat{H}} \left[\int_{\Delta^T(T,h)}^{\Delta^K(K)} \Delta\, dF \right] dG(h; T).$$

We also have

$$\begin{aligned} &\Delta^K(K) \int_{\hat{H}} [F(\Delta^T(T, h)) - F(\Delta^K(K))]\, dG(h; T) \\ &\quad \leq \int_{\hat{H}} \left[\int_{\Delta^K(K)}^{\Delta^T(T,h)} \Delta\, dF \right] dG(h; T) = \int_{H \setminus \hat{H}} \left[\int_{\Delta^T(T,h)}^{\Delta^K(K)} \Delta\, dF \right] dG(h; T) \\ &\quad \leq \Delta^K(K) \int_{H \setminus \hat{H}} [F(\Delta^K(K)) - F(\Delta^T(T, h))]\, dG(h; T). \end{aligned}$$

This implies $\int_H [F(\Delta^T(T, h))]\, dG(h; T) \leq F(\Delta^K(K))$, from which (13) follows.

Proof of Lemma 1. If $\lambda = 0$, then $\Pi(\Delta, \underline{\Delta}) = \pi_H \Delta / r - c$ for all $\underline{\Delta}$, so $\tilde{\Delta}^*(0) = \hat{\Delta}$. $\tilde{\Delta}^*(\cdot)$ is differentiable, and $\tilde{\Delta}^*(\lambda) \to \infty$ as $\lambda \to \infty$.

For any $\tilde{\Delta}^*(\lambda) > 0$, we have

$$\frac{d\tilde{\Delta}^*}{d\lambda} = \frac{\dfrac{-\partial \Pi(\tilde{\Delta}^*(\lambda), \tilde{\Delta}^*(\lambda))}{\partial \lambda}}{\dfrac{\partial \Pi(\tilde{\Delta}^*(\lambda), \tilde{\Delta}^*(\lambda))}{\partial \tilde{\Delta}^*}}.$$

It is straightforward to show that the denominator is positive. The numerator is

$$\begin{aligned} &\frac{-\partial \Pi(\tilde{\Delta}^*(\lambda), \tilde{\Delta}^*(\lambda))}{\partial \lambda} \\ &\quad = \frac{1 - F(\tilde{\Delta}^*(\lambda))}{[r + \Gamma(\tilde{\Delta}^*(\lambda))]^2} \left(\pi_H \tilde{\Delta}^*(\lambda) + \frac{\Gamma(\tilde{\Delta}^*(\lambda)) - r}{r + \Gamma(\tilde{\Delta}^*(\lambda))} \pi_L \Delta^e(\tilde{\Delta}^*(\lambda)) \right). \end{aligned}$$

Let $\tilde{\Delta}_i^*$ be the equilibrium cutoff when $\lambda = \lambda_i$. It is enough to show that if $\tilde{\Delta}_1^* = \tilde{\Delta}_2^*$ and $\lambda_1 < \lambda_2$, it cannot occur that $d\tilde{\Delta}^*/d\lambda\,|_{\lambda_1} \geq 0$ and $d\tilde{\Delta}^*/d\lambda\,|_{\lambda_2} \leq 0$ because then

$$\pi_H \tilde{\Delta}_1^* + \frac{\Gamma(\tilde{\Delta}_1^*) - r}{\Gamma(\tilde{\Delta}_1^*) + r}\pi_L \Delta^e(\tilde{\Delta}_1^*) > 0 > \pi_H \tilde{\Delta}_2^* + \frac{\Gamma(\tilde{\Delta}_2^*) - r}{\Gamma(\tilde{\Delta}_2^*) + r}\pi_L \Delta^e(\tilde{\Delta}_2^*).$$

But since $\tilde{\Delta}_1^* = \tilde{\Delta}_2^*$, since $(\Gamma - r)/(\Gamma + r)$ is increasing in λ, and since $\Gamma(\tilde{\Delta}_1^*) < \Gamma(\tilde{\Delta}_2^*)$, this is a contradiction.

Appendix B: The Welfare Expressions

We now derive the social welfare expression (6) for the case that no innovations infringe previous patents (i.e., there is no leading breadth).

The ith innovation Δ_i affects social welfare in three ways: it costs c, it contributes increment Δ_i to the quality consumed by the consumers with high willingness to pay, and it contributes increment Δ_{i-1} to the consumers with low willingness to pay.

The expression below represents social welfare when the cutoff idea for all generations is $\underline{\Delta}$ and the initial quality gap is Δ_0. The first term shows the value of the first innovation. Since the value in large parentheses accrues at a random time with Poisson hit rate $\Gamma(\cdot)$, we multiply by the factor $\Gamma(\cdot)/[r + \Gamma(\cdot)]$ according to the reasoning in footnote 5. The value has the three components above. The cost is $-c$, the expected benefit to high-willingness-to-pay consumers is $[(B - \ell)/r]\,\Delta^e(\underline{\Delta})$, and the value transferred to the low-willingness-to-pay consumers is $(\ell/r)\,\Delta_0$. These values are divided by r to reflect the fact that each increment becomes the foundation for future increments, and hence the value lasts forever.

The second term reflects the value of the second innovation. We square the factor $\Gamma(\cdot)/[r + \Gamma(\cdot)]$ to reflect that the value in large parentheses accrues after two innovations instead of one. The value in parentheses differs from the first term in that the quality increment transferred to the low-willingness-to-pay consumers is the expected value of the first innovation, $\Delta^e(\underline{\Delta})$.

The subsequent terms continue in the obvious way to give us the following expression for social welfare:

$$\frac{\Gamma(\underline{\Delta})}{r + \Gamma(\underline{\Delta})}\left(-c + \frac{B - \ell}{r}\Delta^e(\underline{\Delta}) + \frac{\ell}{r}\Delta_0\right)$$
$$+ \left(\frac{\Gamma(\underline{\Delta})}{r + \Gamma(\underline{\Delta})}\right)^2\left(-c + \frac{B - \ell}{r}\Delta^e(\underline{\Delta}) + \frac{\ell}{r}\Delta^e(\underline{\Delta})\right)$$
$$+ \left(\frac{\Gamma(\underline{\Delta})}{r + \Gamma(\underline{\Delta})}\right)^3\left(-c + \frac{B - \ell}{r}\Delta^e(\underline{\Delta}) + \frac{\ell}{r}\Delta^e(\underline{\Delta})\right)$$
$$+ \cdots.$$

Rearranging this expression, social welfare is

$$\frac{\Gamma(\underline{\Delta})}{r+\Gamma(\underline{\Delta})}\frac{\ell}{r}\Delta_0+\frac{\Gamma(\underline{\Delta})}{r}\frac{B-\ell}{r}\Delta^e(\underline{\Delta})-\frac{\Gamma(\underline{\Delta})}{r}\frac{\ell\,\Gamma(\underline{\Delta})}{r+\Gamma(\underline{\Delta})}\Delta^e(\underline{\Delta})-\frac{\Gamma(\underline{\Delta})}{r}c.$$

When $\Delta_0 = 0$, social welfare is $W^*(\underline{\Delta})$ as given by equation (6). Thus we have assumed that the quality available initially to all consumers is the same. If $\Delta_0 = 0$, then the first innovation increases the quality available to the consumers with high willingness to pay, but does not increase the quality available to the consumers with low willingness to pay. If $\Delta_0 > 0$, then the value of the first innovation is larger than if $\Delta_0 = 0$ because the first innovation transfers the quality increment Δ_0 to the consumers with low willingness to pay. Thus the socially optimal cutoff is decreasing in Δ_0. By choosing $\Delta_0 = 0$ to define Δ^{opt}, we are biasing the socially optimal cutoff upwards. If $\Delta_0 > 0$, there would be an even greater discrepancy between market outcome $\tilde{\Delta}^*$ and the optimal cutoff. The spirit of Proposition 5 remains intact: If $\tilde{\Delta}^*$ represents underinvestment relative to an optimum with $\Delta_0 = 0$, then it also represents underinvestment relative to an optimum with $\Delta_0 > 0$.

To allow for $\Delta_0 > 0$ in comparing W^K with W^T in Proposition 7, we must add the terms

$$\frac{\Gamma(K)}{r+\Gamma(K)}\frac{\ell\,\Delta_0}{r} \quad \text{and} \quad e^{-rT}\frac{\ell\,\Delta_0}{r}$$

to W^K and W^T respectively. Using equation (12), the addition to W^K is smaller than the addition to W^T, and thus Proposition 7 also remains intact with $\Delta_0 > 0$.

Thus the welfare comparisons in the text are unaffected by changing our assumption $\Delta_0 = 0$ to any $\Delta_0 > 0$, including its expected value.

Appendix C: Details of the Natural-Oligopoly Model

We now show that the axioms on the output market in Section 4 are justified (with some caveats) by the natural-oligopoly model in which each consumer consumes one unit of the quality-differentiated product. What follows is the version of that model developed by Anderson et al. (1988, Ch. 8) based on the model of vertical differentiation presented by Mussa and Rosen (1978). As stressed above, the inefficiency that arises in this oligopoly is not that too few units of the good are consumed (each person always consumes one unit), but that some consumers consume an inferior product.

Letting q represent the quality of the good consumed, p represent the price, and θ represent the consumer's marginal willingness to pay for quality, a consumer's utility is $q\theta - p$. The support of consumers' tastes is $[\theta_0, \theta^0]$, where the measure of consumers with tastes on each unit interval is one. Firms are risk-neutral and maximize discounted expected profit. The marginal cost of producing each product is zero.

We first consider the outcome of price competition between two firms, assuming that their qualities q_H and q_L are fixed. We then consider how those two competitors are chosen.

The main result of the natural-oligopoly model is that the number of firms that can capture positive market share is finite and depends only on the domain of preferences. If $\theta^0/\theta_0 > 2$, then a second firm will always capture positive market share, and as long as $\theta^0/\theta_0 \leq 4$, a third firm will never capture positive market share. To justify that the market can accommodate exactly two firms, we assume $4 \geq \theta^0/\theta_0 > 2$, and $\infty > \theta^0 > \theta_0 > 0$. If a third firm were to enter the market, then the lowest-quality firm would receive zero market share. We assume there is an $\varepsilon > 0$ cost per unit time of being in the market (e.g., there is a small fixed cost), so no third firm will enter.[11] Given the quality gap $\Delta = q_H - q_L$, we find that the prices p_L, p_H, the profits $\pi_L\Delta$, $\pi_H\Delta$, and the numbers of buyers of the low- and high-quality products, n_L, n_H, are

$$p_L = \frac{\Delta}{3}(\theta^0 - 2\theta_0),$$

$$p_H = \frac{\Delta}{3}(2\theta^0 - \theta_0),$$

$$n_L = \frac{1}{3}(\theta^0 - 2\theta_0),$$

$$n_H = \frac{1}{3}(2\theta^0 - \theta_0),$$

$$\pi_L\Delta = \frac{(\theta^0 - 2\theta_0)^2}{9}\Delta,$$

$$\pi_H\Delta = \frac{(2\theta^0 - \theta_0)^2}{9}\Delta.$$

11. Without this assumption a third firm might enter the market and receive zero market share. Its equilibrium price would be zero, but the prices of the other two firms would be lower than if the third firm were absent. With the assumption of an ε flow cost of market participation, the incumbents' prices are not constrained by potential competition.

Actually these are the prices that would prevail if $p_L\Delta < \theta_0 q_L$, which means that the consumer with lowest willingness to pay would purchase the lower-quality product rather than stay out of the market, and justifies the assumption that the whole market is served. If Δ is very large or q_L is very small, this assumption is not satisfied. Even though q_L will eventually be large, it is still possible in each of the above patent regimes that there is a long string of infringing innovations before a noninfringing innovation, such that the market gap Δ becomes too large to justify the assumption that in equilibrium the whole market is served. The probability that this happens is small, and our axioms ignore it for simplicity.[12]

Assuming that the whole market $[\theta_0, \theta^0]$ is served, it is partitioned between the low-quality firm and high-quality firm such that the consumers with willingness to pay $\theta \in [\theta_0, \frac{1}{3}(\theta_0 + \theta^0)]$ purchase from the low-quality firm, and consumers with willingness to pay $\theta \in [\frac{1}{3}(\theta_0 + \theta^0), \theta^0]$ purchase from the high-quality firm. The division of consumers between the two firms, given by the willingness to pay $\theta = \frac{1}{3}(\theta_0 + \theta^0)$, does not depend on the quality gap Δ. In our welfare calculations above,

$$B = \int_{\theta_0}^{\theta^0} \theta \, d\theta,$$

$$\ell = \int_{\theta_0}^{\frac{1}{3}(\theta^0+\theta_0)} \theta \, d\theta.$$

The natural-oligopoly model establishes that two firms will be in the market at any time, but does not indicate which firms or what qualities. In every patent regime the quality of the market leader was assumed to be the quality of the most recent innovation.

In the policy regimes of no leading breadth and finite leading breadth, (K, ∞), we assumed that the market follower is the firm that was market leader prior to the most recent noninfringing innovation, and he sells the product he invented, say q_i. Since he has patent rights

12. Once that point is passed, the marginal infringing innovation becomes less profitable, and incentives to invest in it are somewhat reduced. Allowing for this effect would dampen the incentive to invest in R&D overall, and reduce the social surplus available from each innovation, since there is some possibility that not all the market would be served. We cannot see how these complications would change our comparison of the two patent regimes.

on all the qualities between the previous innovation and his own, an alternative assumption would be that he sells a product of quality $q_{i-1} + \delta$, where δ is small. With the market leader's product fixed at q_{i+1}, this would increase the market gap and increase the profit of both sellers. If we modified the profit functions to reflect this alternative assumption, the cutoff ideas in both regimes would fall, and the rate of innovation would be larger. However, the qualitative conclusions remain the same: if λ is low, then firms may overinvest due to the foot-in-the-door effect; if λ is high, firms may underinvest and leading breadth may be a useful supplement to lagging breadth; and small leading breadth can be counterproductive.

Since the market follower is a random firm in the regime (∞, T), the analogous issue does not arise.

References

Anderson, S., A. de Palma, and J.-F. Thisse, 1992, *Discrete Choice Theory of Product Differentiation,* Cambridge, MA: The MIT Press.

Aghion, P. and P. Howitt, 1992, "A Model of Growth through Creative Destruction," *Econometrica,* 60, 323–352.

Chang, H.F., 1995, "Patent Scope, Antitrust Policy and Cumulative Innovation," *The Rand Journal of Economics,* 26 (Spring), 34–57.

Cheong, I., 1994, "Invent or Develop? On Sequential Innovation and Patent Policy," Mimeograph, Department of Economics, Princeton University.

Chou, T. and H. Haller, 1995, "The Division of Profit in Sequential Innovation Reconsidered," Working Paper E95-02, Department of Economics, VPI & State University.

Denicolo, V., 1996, "Patent Races and Optimal Patent Breadth and Length," *The Journal of Industrial Economics,* 44, 249–265.

Gabszewicz, J.J. and J.-F. Thisse, 1980, "Entry (and Exit) in a Differentiated Industry," *Journal of Economic Theory,* 22, 327–338.

Gallini, N.T., 1992, "Patent Policy and Costly Imitation," *The Rand Journal of Economics,* 23 (Spring), 52–63.

Gilbert, R. and C. Shapiro, 1990, "Optimal Patent Length and Breadth," *The Rand Journal of Economics,* 21 (Spring), 106–112.

Green, J. and S. Scotchmer, 1995, "On the Division of Profit in Sequential Innovation," *The Rand Journal of Economics,* 26 (Spring), 20–33.

Grossman, G.M. and E. Helpman, 1991, "Quality Ladders in the Theory of Growth," *Review of Economic Studies,* 58, 43–61.

Klemperer, P., 1990, "How Broad should the Scope of Patent Protection Be?" *The Rand Journal of Economics,* 21 (Spring), 113–130.

Lanjouw, J., 1993, "Patent Protection in the Shadow of Infringement: Simulation Estimations of Patent Value," Mimeograph, Department of Economics, Yale University.

Levin, R.C., A.K. Klevorick, R.R. Nelson, and S.G. Winter, 1987, "Appropriating the Returns from Industrial Research and Development," *Brookings Papers on Economic Activity,* 3 (Special Issue), 783–831.

Luski, I. and D. Wettstein, 1995, "An Optimal Patent Policy in a Dynamic Model of

Innovation," Discussion Paper 95-3, Monaster Center for Economic Research, Ben Gurion University of the Negev, Beer Sheva.
Mansfield, E., 1984, "R&D and Innovation: Some Empirical Findings," in Zvi Griliches, ed., *R&D, Patents and Productivity,* Chicago: University of Chicago Press for the National Bureau of Economic Research.
Matutes, C., K.E. Rockett, and P. Regibeau, 1996, "Optimal Patent Protection and the Diffusion of Innovation," *The Rand Journal of Economics,* 27 (Spring), 60–83.
Merges, R.P. and R.R. Nelson, 1990, "On the Complex Economics of Patent Scope," *Columbia Law Review,* 90(4), 839–916.
Mussa, M. and S. Rosen, 1978, "Monopoly and Product Quality," *Journal of Economic Theory,* 18, 301–317.
Nordhaus, W., 1969, *Invention, Growth and Welfare,* Cambridge, MA: MIT Press.
O'Donoghue, T., 1996, "Patent Protection When Innovation Is Cumulative," Ph.D. Thesis, Department of Economics, University of California.
O'Donoghue, T., 1997, "A Patentability Requirement for Sequential Innovation," Northwestern University, Math Center Discussion Paper No. 1185.
Pakes, A., 1986, "Patents as Options: Some Estimates of the Value of Holding European Patent Stocks," *Econometrica,* 54, 755–785.
Pakes, A. and M. Schankerman, 1984, "The Rate of Obsolescence of Patents, Research Gestation Lags, and the Private Rate of Return to Research Resources," in Zvi Griliches, ed., *R&D, Patents, and Productivity,* Chicago: University of Chicago Press, 73–88.
Reinganum, J., 1989, "The Timing of Innovation: Research, Development and Diffusion," in R. Schmalensee and R.D. Willig, eds., *Handbook of Industrial Organization,* Amsterdam: Elsevier, 849–908.
Romer, P., 1994, "Increasing Returns and Long-Run Growth," *Journal of Political Economy,* 94, 1002–1037.
Schankerman, M. and A. Pakes, 1986, "Estimates of the Value of Patent Rights in European Countries during the Post–1950 Period," *Economic Journal,* 96, 1052–1076.
Schmitz, J.A., Jr., 1989, "On the Breadth of Patent Protection," Mimeograph, State University of New York at Stony Brook.
Scotchmer, S., 1991, "Standing on the Shoulders of Giants: Cumulative Research and the Patent Law," *Journal of Economic Perspectives,* Symposium on Intellectual Property Law, Winter, 29–41.
Scotchmer, S., 1996, "Protecting Early Innovators: Should Second Generation Products be Patentable?" *The Rand Journal of Economics,* 27, 117–126.
Scotchmer, S. and J. Green, 1990, "Novelty and Disclosure in Patent Law," *The Rand Journal of Economics,* 21 (Spring), 131–146.
Shaked, A. and J. Sutton, 1983, "Natural Oligopolies," *Econometrica,* 51, 1469–1483.
Tandon, P., 1982, "Optimal Patents with Compulsory Licensing," *Journal of Political Economy,* 90, 470–486.
Van Dijk, T., 1996, "Patent Height and Competition in Product Improvements," *Journal of Industrial Economics,* 44, 151–167.