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Economic Theory

Consumption externalities, rental markets and purchase clubs*

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Summary. A premise of general equilibrium theory is that private goods are rival. Nevertheless, many private goods are shared, e.g., through borrowing, through co-ownership, or simply because one person's consumption affects another person's wellbeing. I analyze consumption externalities from the perspective of club theory, and argue that, provided consumption externalities are limited in scope, they can be internalized through membership fees to groups. Two important applications are to rental markets and "purchase clubs," in which members share the goods that they have individually purchased.

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1 Introduction

Microeconomic theory generally treats private goods as rival in the sense that preferences depend only on an agent's own consumption. Nevertheless, consumption externalities are pervasive. If someone plants a tree to shade his garden, he may block his neighbor's view. If the neighbor buys a dog that barks, the whole neighborhood may suffer.

The term "externality" generally involves an assumption about market institutions. If the commodity space is defined so that the "externalities" are chosen

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voluntarily, they are no longer externalities. A better term might be "shared consumption." There are many formal and informal institutions that facilitate such sharing, or allow the coordination of consumption decisions. On the informal side, agents often benefit from their friends' purchases of books, CD's, or magazine subscriptions. An implicit condition of friendship may be reciprocity in such purchases. On the more formal side, shared living arrangements often impose restrictions that prohibit or require certain consumption or activities ("no smoking"), and those restrictions may be reflected in the price. Higher fees may entitle the member to more privileges or relieve him of obligations. Other formal institutions for sharing the consumption of private goods include rental markets and lending libraries. Rental prices may be higher at establishments where the rental good is less often in use, so that scheduling is easier. Rental fees may be seasonal.

Shared consumption can be handled in various ways in general equilibrium theory. Some sharing arrangements, such as rental markets, can be handled by re-interpreting traded goods in the theory of Arrow and Debreu. The treatment of Grodal and Vind (2001) and Vind and Grodal (2004) (see also Vind, 1983) extends that theory to allow each agent's preferences to depend on all agents' consumptions. The description of the economy includes a "coordination function" whereby agents are allowed to make joint decisions. Whether an equilibrium is efficient depends on the extent to which preferences are interdependent, and on the coordination function. Gersbach and Haller (2001) study economies where preferences are independent within households, and allow households to make joint consumption decisions using a combined budget.

This paper shows that shared private goods can also be handled through club theory. The essential idea in club theory is that, by forming a group called a "club," members share the services that the club provides, and also share the "externalities" conferred by the attributes or activities of the club's members. An important difference between the club approach and the approaches mentioned above is in the commodity space. In the club model, each agent purchases a vector of private goods, and also purchases memberships in clubs. When purchasing memberships in clubs, members anticipate the full suite of externalities, which are therefore internalized, since each member has the option not to join. The club model has wide-ranging applicability, comprising educational opportunities, firms, schools, social activities, academic departments, and many other human activities that take place in groups. See Ellickson, Grodal, Scotchmer and Zame (EGSZ 1999, 2001, 2003), Cole and Prescott (1999) and Prescott and Townsend (forthcoming). For how these ideas relate to local public goods, see Scotchmer (2002).

The question for club theory, just as for more traditional general equilibrium theory, is how to model the sharing of private goods, and how to provide for pricing. Different goods involve different protocols for sharing. Goods like power tools, ski equipment and sometimes cars are used only occasionally by each user. As long as the transactions costs are not exorbitant, it is more efficient to keep the good in use than to let it sit idle. Nevertheless, sharing may be inconvenient. If the good cannot be used simultaneously, then there must be a protocol for resolving conflicts or scheduling use. We would expect prices to reflect the priority that a member gets, or the overall inconvenience of the use, as measured, for example, by the ratio of total use to total goods.

For some goods there is rather little inconvenience due to sharing. Computer software and entertainment products (music, movies) can be used simultaneously when installed simultaneously on different users' computers. The only inconvenience is in keeping the sharing group small enough to avoid detection, since simultaneous use will typically violate the seller's intellectual property rights.

For shared consumption that generates pleasure for one person and discomfort for another, such as playing Beatles tunes at midnight or smoking cigarettes, the protocol of sharing might be to prohibit use at certain hours or in certain places.

In Section 2, I reprise the clubs model of EGSZ (1999, 2003), and show how it can accommodate consumption externalities. In Section 3, I extend that model to accommodate proprietary goods. This extension allows us to address policy concerns raised by the sharing of proprietary computer software and entertainment products. Section 4 shows that club theory leads to a useful model of rental markets, with peak and off-peak pricing, and prices that reflect the inconvenience of competing with other users.

2 Consumption externalities in groups

The most convenient clubs model for this purpose is that of EGSZ (1999, as extended in 2003). The commodity space includes private goods and memberships in clubs. In order to define the memberships, we must first define the types of groups that can form as clubs. A primitive of the economy is an exogenous set of *group types*. To define memberships in the group types, we need a set of *membership characteristics,* designated by elements of Ω, which is an abstract, finite set. In addition, the groups can have *activities,* designated by elements of Γ, which is also an abstract, finite set.

A *group type* is a triple (π, γ, y) consisting of a *profile* $\pi : \Omega \to \mathbf{Z}_+ = \{0, 1, \dots\}$, an activity $\gamma \in \Gamma$, and a vector of private goods $y \in \Re^N$. The negative elements of y represent net inputs, and the positive elements represent net outputs. For $\omega \in \Omega, \pi(\omega)$ is the number of members having the membership characteristic ω. A membership characteristic specifies the role in the group type that the membership entails (such as teacher or student), as well as the personal qualities required for the membership such as intelligence and personal habits, or (as discussed here) contributions of goods. In Sections 3 and 4, the membership characteristics are respectively contributions of a proprietary good that will be shared by members or the usage of a rental good. In those cases, the membership characteristics are described by real numbers, but in general no such structure is imposed on Ω.

We take as given a finite set of possible group types $\mathcal{G} = \{(\pi, \gamma, y)\}$.

A *membership* is an opening in a particular group type for an agent assuming a particular membership characteristic; i.e., $(\omega, (\pi, \gamma, y))$ such that $(\pi, \gamma, y) \in \mathcal{G}$ and $\pi(\omega) \geq 1$. We write $\mathcal{M}$ for the (finite) set of memberships. Each agent may choose many memberships in groups or none. A membership *list* is a function $\ell : \mathcal{M} \to \{0, 1, \dots\}$, where $\ell((\omega, (\pi, \gamma, y)))$ specifies the number of memberships of type $(\omega, (\pi, \gamma, y))$. The list may include memberships in firms ("jobs"), in schools (as "student" or "teacher"), in living groups, and (as below) in groups that share private goods. It is due to membership in groups that agents may be subject to externalities or confer externalities.

The set of agents is a nonatomic measure space $(A, \mathcal{F}, \lambda)$. That is, A is a set, $\mathcal{F}$ is a σ-algebra of subsets of A and λ is a non-atomic measure on $\mathcal{F}$ with $\lambda(A) < \infty$. A complete description of an agent $a \in A$ consists of a consumption set, an endowment of private goods and a utility function.

The agents' *consumption sets* X_a, $a \in A$, take on a different role in this model than in, for example, the Arrow-Debreu model of general equilibrium. An agent's consumption set constrains the memberships available to him, *e.g.,* according to his innate abilities (not everyone can have a job as a professional basketball player), or by specifying collateral consumption of private goods or other memberships that must also be consumed in order to "learn" a membership characteristic. (A computer programmer must learn his skill either by owning a computer and programming books or by choosing a student membership in a programming school.) The consumption set would also prevent the agent from choosing memberships that are inconsistent, such as being simultaneously a sumo wrestler and a member of a ballet club. The consumption of private goods is always restricted to be nonnegative, but may additionally be restricted by the list of memberships. For technical convenience, there is a bound M on how many memberships an agent can consume.

Agent a's *endowment* is $(e_a, 0) \in X_a \subset \Re^N_+ \times \Re^{\mathcal{M}}$. Agents are endowed with private goods but not with group memberships. Agent a's *utility function* $u_a : X_a \rightarrow \Re$ is defined over private goods consumptions and lists of group memberships. The utility function is continuous and strictly increasing in the private goods. A *state* of an economy is a measurable mapping

$$(x, \mu) : A \rightarrow \Re^N \times \Re^{\mathcal{M}}$$

A state specifies choices of private goods and a list of group memberships for each agent.

Feasibility of a state of the economy entails consistent matching of agents, that every agent's consumption is in his consumption set, and that the aggregate resource constraint is satisfied. Consistent matching is a type of coordination of the agents' membership choices. It means that there are no partially filled groups. If an agent chooses a membership in a group of a certain type, then other agents must choose the complementary memberships to fill the group.

Consistent matching is expressed in terms of an *aggregate membership vector* $\bar{\mu} \in \Re^{\mathcal{M}}$, representing the total number of memberships of each type chosen by the agents collectively. We say that an aggregate membership vector $\bar{\mu} \in \Re^{\mathcal{M}}$ is *consistent* if for every qrouptype $(\pi, \gamma, y) \in \mathcal{G}$, there is a real number $\alpha(\pi, \gamma, y)$, representing the "number" (measure) of groups of type (π, γ, y), such that

$$\bar{\mu}(\omega, (\pi, \gamma, y)) = \alpha(\pi, \gamma, y)\pi(\omega)$$

for each $\omega \in \Omega$. Thus, associated with a feasible state of the economy is a collection $\{\alpha(\pi, \gamma, y) | (\pi, \gamma, y) \in \mathcal{G}\}$ which describes the measures of the groups of various types. For each $(\pi, \gamma, y) \in \mathcal{G}$, either $\alpha(\pi, \gamma, y) = 0$ (no such groups) or $\alpha(\pi, \gamma, y) > 0$ (a positive measure of such groups).

The state (x, μ) is *feasible* if it satisfies the following requirements:

(i) **Individual feasibility** $(x_a, \mu_a) \in X_a$ for each $a \in A$
(ii) **Material balance**

$$\int_A x_a \, d\lambda(a) \leq \int_A e_a \, d\lambda(a) + \int_A \sum_{(\omega,(\pi,\gamma,y))\in\mathcal{M}} \mu_a(\omega, (\pi, \gamma, y)) \frac{y}{|\pi|} \, d\lambda(a)$$

(iii) **Consistency** The aggregate vector of memberships $\int_A \mu_a \, d\lambda(a)$ is consistent.

Both private goods and group memberships are priced, so prices (p, q) lie in $\Re_+^N \times \Re^{\mathcal{M}}$. The vector of prices for private goods is p, and the vector of prices for group memberships is q. Prices of group memberships may be positive, negative or zero. Membership prices have different interpretations in different examples. They may be required to pay for the infrastructure of the group or its activities, to remunerate a member for his opportunity cost of membership, in particular, wages, or may, when negative, compensate him for a membership that other members value, but he himself dislikes. In Section 3 below, a negative price might mean that the member of a purchase club is partially reimbursed by other members for the purchases he contributes.

At prices (p, q), we say that a list $(x, \ell) \in X_a$ is *budget feasible* for $a \in A$ if

$$(p, q) \cdot (x, \ell) \leq p \cdot e_a$$

A *group equilibrium* consists of a feasible state (x, μ) and prices $(p, q) \in \Re_+^N \times \Re^{\mathcal{M}}, p \neq 0$ such that

(a) **Budget feasibility:** For almost all $a \in A$, (x_a, μ_a) is budget feasible.
(b) **Optimization:** For almost all $a \in A$, if $(x, \ell) \in X_a$ and $u_a(x, \ell) > u_a(x_a, \mu_a)$, then (x, ℓ) is not budget feasible.
(c) **Budget balance for group types:** For each $(\pi, \gamma, y) \in \mathcal{G}$:

$$\sum_{\omega \in \Omega} \pi(\omega) \, q(\omega, (\pi, \gamma, y)) + p \cdot y = 0$$

Thus, at an equilibrium individuals optimize subject to their budget constraints and the sum of membership prices in a given group type is exactly equal to the net cost or surplus generated by the use or production of private goods, $p \cdot y$.

By a simple extension of their (1999) arguments, EGSZ (2003) assert the first welfare theorem for this model as well as existence and core/competitive equivalence.[1] The (2003) paper differs from the (1999) paper in that private goods may be produced and membership characteristics can be acquired. There is a concept

[1] In club economies there are considerations beyond those known for exchange economies, in going from quasi-equilibrium to equilibrium; see example 3 of Gilles and Scotchmer (1997) and example 3.2 of EGSZ (1999). These difficulties are not emphasized here because this paper focuses on characterizations of equilibrium, and not existence.

of learning, which may be facilitated by memberships in schools or consumption of certain private goods.

The internalization of consumption externalities can be described by this model using the following adaptions, illustrated by the examples that follow.

- Using consumption sets, consumption of private goods can be restricted in a way that depends on group memberships.
- The activity vector γ can specify how private goods, modeled in y as inputs, are shared.
- The membership characteristic can obligate the member for certain purchases that must be shared with other members, with the terms of sharing specified by γ.
- The membership characteristic can entitle the member to certain specified usage of the shared good.

Example 1. Suppose that a group of friends share a house. The household is a type of club, (π, γ, y), and the house is an input in y. If the house seeks to limit the negative externalities that arise from late-night parties or playing Beatles tunes, the activity γ can consist of a commitment not to do those things. If it is a no-smoking house, the consumption set may prohibit agents from both belonging to such a house and consuming cigarettes.

Example 2. Suppose that members of the household share tasks. Someone must be the cook, someone must bring sports equipment, and someone must do the members' collective homework. These commitments could be built into the membership characteristics ω. Some characteristics, like being the cook, could be acquired skills. The cookbooks can be an input in y, but alternatively, the cook can be required to bring them. The prices will be different in these two arrangements. The homework membership may require innate abilities and also learned skills. Whether the homework membership is feasible for a certain student is reflected in his consumption set. Similarly, the person who contributes the sports equipment must presumably invest in it, and his membership price should reflect this investment. If different members bring different sports equipment, their personal characteristics will reflect their contributions. The activity γ must specify the organizational arrangements under which they decide how to ration the sports equipment.

Example 3. The friends may band together for the dedicated purpose of sharing music or software CD's, in order to avoid purchasing duplicate copies. This is a purchase club, described in Section 3.

Example 4. Suppose that the shared good is partially rival, in the sense that intense use creates inconvenience. This is described in Section 4. A membership price will then reflect the overall usage, modeled in γ, and will also reflect the member's usage, modeled in ω. The membership price may also reflect time. Other things equal, the rental price of a sailboat on a balmy Saturday in August might be higher than on a random Tuesday in January, when the price must be low to keep the sailboat in use. The vector y represents the input vector of shared goods themselves.

3 Proprietary pricing and purchase clubs

Copyright owners have argued for many years that their profits are undermined when users share. Their calculation of the loss usually involves the assumption that every unauthorized user would otherwise purchase a legitimate copy at the prevailing price. Both common sense and the economics literature challenge this view. What is argued in the literature (Besen and Kirby, 1989; Varian, 2000; Bakos et al., 1999) is that proprietors will anticipate the sharing behavior, and set different prices if the good is sold to individual users than if sold to users who are expected to share it. These papers argue, somewhat provocatively, that sharing may actually *increase* the proprietor's profit. We revisit this question, using a variant of the club model that allows for proprietary pricing.

We begin with an example to show what the club model adds to previous discussions of purchase clubs. Whether sharing enhances profit depends on the groups that form. However the whole point of club theory is that group formation is an equilibrium phenomenon. Instead of taking group formation as exogenous, the club perspective recognizes that groups will form in a way that is collectively efficient – efficient for the buyers, that is. Group formation that is efficient for the buyers is probably not efficient for the sellers. Indeed, this is more or less what the theorem in Section 3.2 shows. In the example of Section 3.1, the sellers' profits may be enhanced if group formation is, for example, random, but profit will not be enhanced if group formation is systematic in some way that serves the interests of the buyers. The theorem shows that the profit available to the sellers is exactly the same with sharing of purchases as without, provided the purchase groups form efficiently in equilibrium, and the selling price can depend on the size of the group.

This result would not survive in the form given if the shared goods involved marginal costs of supply, as sharing would then reduce industry costs, and the proprietor would presumably share in the benefits. This is the focus of the related work by Besen and Kirby (1989) and Varian (2000).

3.1 Purchase clubs: an example

We use this example to show two things: that if the groups form exogenously prior to the setting of prices, then group formation may indeed increase the proprietors' profit, but if the groups can re-form conditional on the prices, that result is nullified. The example suggests the theorem of the next section, which is that the maximum profit, anticipating group formation, is the same as if consumers do not share the proprietary goods.

We will consider purchase clubs that share CD's of two kinds, classical and jazz. Assume that for each CD, half the population has willingness to pay (WTP) equal to a and the other half has WTP x, $x < a/2$. Our benchmark will be the profitability of selling to single buyers. The most profitable price is $p = a$, so half the agents buy. We compare this benchmark with a situation where groups of size 2 can share CD's.

Suppose that there are four types of consumers, with different willingnesses to pay for the two CDs: $\{v_1, v_2, v_3, v_4\} = \{(a,a),(a,x),(x,a),(x,x)\}$. (No such structure is used in Section 3.2.)

Suppose first that each taste vector occurs in 1/4 of the population, and that the agents are randomly and exogenously matched into groups. The groups occur with the following frequencies:

Group	Frequency	WTP classical WTP jazz	Group	Frequency	WTP classical WTP jazz
(v_1, v_1)	$1/16$	$a+a$ $a+a$	(v_1, v_3)	1/8	$a+x$ $a+a$
(v_2, v_2)	$1/16$	$a+a$ $x+x$	(v_1, v_4)	1/8	$a+x$ $a+x$
(v_3, v_3)	1/16	$x+x$ $a+a$	(v_2, v_3)	1/8	$a+x$ $a+x$
(v_4, v_4)	1/16	$x+x$ $x+x$	(v_2, v_4)	1/8	$a+x$ $x+x$
(v_1, v_2)	1/8	$a+a$ $a+x$	(v_3, v_4)	1/8	$x+x$ $a+x$

The most profitable price is $p_c = p_j = (a+x)$, which entails selling to 3/4 of the groups. It is not optimal to charge price $2a$ for either CD, because only 1/4 of the groups would purchase it.

But at prices $p_c = p_j = (a+x)$, groups will want to re-form. To see that the randomly matched groups cannot be optimal for all agents, notice that, if they re-form into homogeneous groups ((v_1, v_1), (v_2, v_2), etc.) there is more total consumers' surplus to divide. With random matching, only 1/4 of the groups receive positive consumers' surplus for a given CD, those with 2 agents who have WTP equal to a. If agents reorganize into homogeneous groups, then for each CD, half the groups have two agents with WTP equal to a for a given CD. But while total consumers' surplus goes up, profit goes down, since only half the groups buy each CD rather than 3/4 of them.

But once the groups are reorganized into homogeneous groups, the proprietors can do better by charging $p_c = p_j = 2a$. With these prices, proprietors earn the same profit as selling to individual agents. Although this is not obvious, they cannot do better. There are no prices they could choose which would generate more profit once the consumers reorganize into the best groups conditional on those prices. That is the theorem in the next section.

The optimum in this example entails groups with the same tastes. But in the model below there is no reason that tastes should be duplicated in the population. It is not the homogeneity of tastes that erases any profit advantage to selling to groups, but rather the fact that groups form endogenously in a way that is collectively efficient for the members, conditional on the proprietary prices.

3.2 Purchase clubs: a Theorem

In developing the example, it was convenient to describe the members of groups by their tastes. However membership prices cannot depend on tastes, as tastes are unobservable. The membership characteristics will be the contributions of proprietary goods.

We will make the notation easier by supposing that the only group types are purchase clubs. (Otherwise we must distinguish memberships in purchase clubs from other types of memberships.) Suppose there are C goods that can be purchased and shared. Proprietors market these goods at prices $r = (r_1, ..., r_C) > 0$, anticipating the groups that will form. In the example, $C = 2$, jazz and classical.

The set Ω will serve various purposes in the model that follows. Most importantly, the elements $\omega \in \Omega$ will represent the contributions that a member might make to a group. For convenience, let

$$\Omega = \left\{ z \in Z_+^C \mid z \leq (Mk, Mk....Mk) \right\}$$

for a given $k > 1$ where M is the maximum number of memberships in an agent's consumption set.

A *purchase club type* (π, γ, y) is a club type such that the membership characteristics $\omega \in \Omega$ are interpreted as contributions, and $\sum_{\omega \in \Omega} \pi(\omega)\omega$ is the vector of proprietary goods shared by members of the group. (Recall that, in general, the expression $\sum_{\omega \in \Omega} \pi(\omega)\omega$ has no meaning, as the characteristic ω need not be a number.) If a member chooses a membership for which $\omega > 0$, then he contributes at least one shared good, and may be paid in equilibrium by members who choose $\omega = 0$.

To isolate the points of interest, we make some special assumptions. We shall assume there is a single private good, the numeraire, and shall refer to equilibrium as $(x, \mu), q$. We assume that the set Γ is a singleton which specifies that members will share their purchases. We thus suppress the activity $\gamma \in \Gamma$ in the description of the group type. We also suppress y in the description of a group type, since the shared goods are described in the membership characteristics. Thus, a group type is only described by the profile π indicating how many members contribute each vector ω of shared goods. We will also assume that there is an exogenous bound k on the size of sharing groups. This is the k in the definition of Ω. (In the example, $k = 2$). Since γ is a singleton and $y = 0$, the sets of possible group types and memberships are

$$\mathcal{G} = \{\pi : \Omega \to Z_+ | \, |\pi| \leq k\}$$
$$\mathcal{M} = \{(\omega, \pi) | \omega \in \Omega, \pi \in \mathcal{G}\}$$

The *contributions* of an agent who consumes a list ℓ of memberships are

$$\sum_{(\omega,\pi) \in \mathcal{M}} \ell(\omega, \pi)\omega.$$

We will use the notation ω^ℓ to refer to the *consumption* of an agent (distinct from the *contributions* of the agent) if he consumes a list ℓ:

$$\omega^\ell = \sum_{(\omega,\pi) \in \mathcal{M}} \ell(\omega, \pi) \left(\sum_{\omega \in \Omega} \pi(\omega)\omega \right) \tag{1}$$

The group equilibrium defined in Section 2 must be altered to account for the cost of contributing proprietary goods, which is the second term in (2). At prices (p, q) and r, we say that a list $(x, \ell) \in X_a$ is *budget feasible* for $a \in A$ if

$$(p, q) \cdot (x, \ell) + \sum_{(\omega,\pi)\in\mathcal{M}} \ell(\omega, \pi)\, r \cdot \omega \quad \leq p \cdot e_a \tag{2}$$

A *purchase-club equilibrium* at prices r consists of a feasible state (x, μ) and prices $(p, q) \in \Re_+^N \times \Re^{\mathcal{M}}, p \neq 0$, such that (a), (b) and (c) of group equilibrium hold, with budget feasibility defined as (2). This equilibrium can be understood as involving absentee proprietors who collect the profit on the proprietary goods.

Utility functions $u_a : X_a \to \Re$, are defined by

$$u_a(x, \ell) = U_a(x, \omega^\ell) \tag{3}$$

where $U_a : \tilde{X}_a \to \Re$ represents utility as a function of the goods themselves, and

$$X_a = \Re_+ \times \left\{\ell \in Z_+^{\mathcal{M}} : |\ell| \leq M\right\}$$
$$\tilde{X}_a = \left\{(x, \omega^\ell) | (x, \ell) \in X_a\right\}$$

A1: Preferences can be defined as in (3), where for all $a \in A$, (i) U_a is strictly increasing in its first argument, and (ii) if $z \notin \Omega$, then there exists $z' \in \Omega$, $z' \leq z$, such that $U_a\,(x,\ z') \geq U_a\,(x,\ z)$ for all $x \geq 0$.

Part (ii) of this assumption is satisfied if consumers can reduce their consumption of a shared good from a number larger than Mk to a smaller number, without reducing utility. If members of each group can use the shared good simultaneously, as in the example where they were assumed to install digital music separately on all their computers, one unit of each shared good is sufficient.

The next three claims characterize a purchase-club equilibrium. Claim 1 describes prices such that, in equilibrium, agents are indifferent as to which membership they have in a group type that is used in equilibrium. Members who contribute shared goods pay low prices (perhaps negative prices), and members who contribute no shared goods pay high prices, to just an extent that they are indifferent.

Claim 1 *Suppose that* A1 *holds. Let* $(x, \mu), q$ *be a purchase-club equilibrium at prices* $r > 0$. *Then*

(i) *For each group type* π, q *satisfies* (4) *for at least one membership* (ω, π).

$$q(\omega, \pi) \leq \frac{r}{|\pi|} \cdot \sum_{\omega\in\Omega} \pi(\omega)\omega - r \cdot \omega \tag{4}$$

(ii) *If the group type* π *is used in equilibrium* $(\alpha(\pi) > 0)$ *and* $(\pi(\omega) > 0)$, *then* q *satisfies (4) with equality.*

(iii) If the group type π *is used in equilibrium and* $\pi(\omega_1), \pi(\omega_2) > 0$, *then for all* $a \in A$

$$q(\omega_1, \pi) + r \cdot \omega_1 = q(\omega_2, \pi) + r \cdot \omega_2 = \frac{r}{|\pi|} \cdot \sum_{\omega \in \Omega} \pi(\omega)\omega$$

Proof.

(i) If (4) holds with equality, budget balance is satisfied. (Multiply both sides of (4) by $\pi(\omega)$ and sum on ω.) If (4) does not hold with equality, then by budget balance, (4) holds as an inequality for at least one membership in a given group type π.

(ii) Suppose to the contrary that for a given π such that $\alpha(\pi) > 0$,

$$q(\omega_1, \pi) > \frac{r}{|\pi|} \cdot \sum_{\omega \in \Omega} \pi(\omega)\omega - r \cdot \omega_1$$
$$q(\omega_2, \pi) < \frac{r}{|\pi|} \cdot \sum_{\omega \in \Omega} \pi(\omega)\omega - r \cdot \omega_2$$

where $\pi(\omega_1), \pi(\omega_2) > 0$. Using (2), an agent's total payments when he chooses a membership are the cost of the contributions plus the membership fee, $q(\omega, \pi) + r \cdot \omega$. Since all memberships in a given group type π give access to the same shared goods $\sum_{\omega \in \Omega} \pi(\omega)\omega$, and, by A1, agents care only about their consumption (x, ω^ℓ), every agent is better off with the cheaper membership (ω_2, π) than with the more expensive (ω_1, π). This is a contradiction, since $\pi(\omega_1) > 0$.

(iii) follows from (ii). □

Claim 2 *Suppose that* A1 *holds. Let* $(x, \mu), q$ *be a purchase-club equilibrium at prices* $r > 0$. *Let* $\{\omega^{\mu_a} | a \in A\}$ *be the consumptions of shared goods defined by* (1). *Then*

(i) $\omega^{\mu_a} \in \Omega$ *for almost every* $a \in A$

(ii) If the group type π *is used in equilibrium* ($\alpha(\pi) > 0$), *then* $|\pi| = k$.

(iii) For almost every $a \in A$, *the consumption of private goods satisfies*

$$x_a = e_a - \frac{r}{k} \cdot \omega^{\mu_a} \tag{5}$$

Proof. Using Claim 1(ii) and budget feasibility, equilibrium consumption (x_a, μ_a) satisfies the following for almost every $a \in A$:

$$\begin{aligned} x_a &= e_a - \sum_{(\omega,\pi)\in\mathcal{M}} \mu_a(\omega, \pi) \left[r \cdot \omega + q(\omega, \pi) \right] \\ &= e_a - \sum_{(\omega,\pi)\in\mathcal{M}} \mu_a(\omega, \pi) \frac{r}{|\pi|} \cdot \sum_{\omega\in\Omega} \pi(\omega)\omega \\ &\leq e_a - \frac{r}{k} \cdot \sum_{(\omega,\pi)\in\mathcal{M}} \mu_a(\omega, \pi) \sum_{\omega\in\Omega} \pi(\omega)\omega = e_a - \frac{r}{k} \cdot \omega^{\mu_a}. \end{aligned} \tag{6}$$

Before proving the claim, we observe that, given $\omega \in \Omega$, and provided x satisfies $e_a - (r/k) \cdot \omega \geq x$, we can find a list ℓ such that $(x, \ell) \in X_a$ is budget-feasible and $\omega^\ell = \omega$. Choose a group type with $|\pi| = k$ and $\omega = \sum_{\hat{\omega} \in \Omega} \pi(\hat{\omega})\hat{\omega}$, and choose the list ℓ with a single membership in that group type such that (4) holds. Then

$$\begin{aligned}
0 \leq x \leq e_a - \frac{r}{k} \cdot \omega^\ell &= e_a - \sum_{(\omega,\pi) \in \mathcal{M}} \ell(\omega, \pi) \frac{r}{k} \cdot \sum_{\omega \in \Omega} \pi(\omega)\omega \\
&= e_a - \sum_{(\omega,\pi) \in \mathcal{M}} \ell(\omega, \pi) \frac{r}{|\pi|} \cdot \sum_{\omega \in \Omega} \pi(\omega)\omega \\
&\leq e_a - \sum_{(\omega,\pi) \in \mathcal{M}} \ell(\omega, \pi) \left[\, r \cdot \omega + \ q(\omega, \pi)\right]
\end{aligned}$$

Thus, (x, ℓ) is budget feasible.

(i) Suppose $\omega^{\mu_a} \notin \Omega$ for a set of agents of positive measure, say $\bar{A} \subseteq A$. For each $a \in \bar{A}$, by A1 there exists $\omega' \in \Omega$, $\omega' \leq \omega^{\mu_a}$ such that $U_a(x_a, \omega^{\mu_a}) \leq U_a(x_a, \omega')$. Using (6), and since $\omega'_i < \omega_i^{\mu_a}$ for at least one proprietary good i, $0 \leq x_a \leq e_a - (r/k) \cdot \omega^{\mu_a} < e_a - (r/k) \cdot \omega'$. Then, as observed above, there is a list ℓ such that $\omega^\ell = \omega'$ and $(x_a, \ell) \in X_a$ is budget feasible. Further, there is a budget-feasible $(x, \ell) \in X_a$ such that $e_a - \frac{r}{k} \cdot \omega^\ell \geq x > x_a$. But then $U_a(x_a, \omega^{\mu_a}) \leq U_a(x_a, \omega^\ell) < U_a(x, \omega^\ell)$, which contradicts equilibrium.

(ii) Suppose that there is a group type π such that $|\pi| < k$ and $\alpha(\pi) > 0$. Let $\bar{A} \subseteq A$ be the set of agents with memberships in this group type π. Let $a \in \bar{A}$. Using (i), we can assume without loss of generality that $\omega^{\mu_a} \in \Omega$. Since (6) holds as a strict inequality, $e_a - \frac{r}{k} \cdot \omega^{\mu_a} > x_a \geq 0$. As observed above, there is a list ℓ with a single membership in a group type π, $|\pi| = k$, such that $\omega^\ell = \omega^{\mu_a}$ and (x_a, ℓ) is budget feasible for a. For this ℓ there is a budget-feasible (x, ℓ), $e_a - (r/k) \cdot \omega^\ell \geq x > x_a$, such that $U_a(x, \omega^\ell) = U_a(x, \omega^{\mu_a}) > U_a(x_a, \omega^{\mu_a})$, which contradicts equilibrium.

(iii) But if $|\pi| = k$, then, using Claim 1(ii), (5) holds because

$$x_a = e_a - \sum_{(\omega,\pi) \in \mathcal{M}} \mu_a(\omega, \pi) \left[\, r \cdot \omega + \ q(\omega, \pi)\right] = e_a - (r/k) \cdot \omega^{\mu_a}. \qquad \square$$

Our objective is to compare the group equilibrium at prices r to a market in which proprietors sell to individual agents at prices r/k. To study the market with individual buyers, we define the agents' demand sets. For each $a \in A$ and $r > 0$

$$D^a(r) = \{f \in \Omega \mid e_a - r \cdot f \geq 0 \text{ and for all } \omega \in \Omega, \text{ either} \tag{7}$$

$$U_a\left(e_a - r \cdot f,\ f\right) \geq U_a\left(e_a - r \cdot \omega,\ \omega\right) \text{ or } e_a - r \cdot \omega < 0\}$$

By A1, there is no loss of generality in restricting to demand vectors in Ω.

Aggregate demand is the integral of a selection from individual demand sets. A *demand selection at prices r* is an integrable function $f : A \to \Omega$ such that $f(a) \in D^a(r)$ for each $a \in A$. The *aggregate demand correspondence* is

$$D(r) = \left\{ \int_A f(a) d\lambda(a) | f \text{ is a demand selection at prices } r \right\}$$

Claim 3 *Suppose that* A1 *holds. Let* $(x, \mu), q$ *be a purchase-club equilibrium at prices* $r > 0$, *and let* $\{\omega^{\mu_a}\}_{a \in A}$ *be the associated consumptions of shared goods. Then (8) holds for almost every agent* $a \in A$:

$$U_a\left(x_a, \omega^{\mu_a}\right) \geq U_a\left(e_a - \frac{r}{k} \cdot \omega,\ \omega\right) \tag{8}$$
$$\textit{for all } \omega \in \Omega \ \textit{ such that } e_a - \frac{r}{k} \cdot \omega \geq 0$$

Proof. Suppose (8) does not hold for a set of agents of positive measure, $\bar{A} \subseteq A$. We will find budget-feasible consumptions $\{(\tilde{x}_a, \tilde{\mu}_a)\}_{a \in A}$ for which

$$\begin{aligned} U_a\left(\tilde{x}_a, \omega^{\tilde{\mu}_a}\right) &\geq U_a\left(x_a, \omega^{\mu_a}\right) \text{ for all } a \in A \\ U_a\left(\tilde{x}_a, \omega^{\tilde{\mu}_a}\right) &> U_a\left(x_a, \omega^{\mu_a}\right) \text{ for all } a \in \bar{A}. \end{aligned} \tag{9}$$

This contradicts equilibrium.

We construct the state $(\tilde{x}, \tilde{\mu})$ from a demand selection f at prices r/k. Using the definition of f and Claim 1(i)(iii), the following holds for all $a \in A$ and holds strictly for almost all $a \in \bar{A}$.

$$U_a\left(e_a - \frac{r}{k} \cdot f(a),\ f(a)\right) \geq U_a\left(e_a - \frac{r}{k} \cdot \omega^{\mu_a},\ \omega^{\mu_a}\right) = U_a\left(x_a, \omega^{\mu_a}\right) \tag{10}$$

Therefore we can complete the proof by constructing $(\tilde{x}, \tilde{\mu})$ such that for each $a \in A$, $\omega^{\tilde{\mu}_a} = f(a)$, $\tilde{x}_a = e_a - (r/k) \cdot f(a)$. Since there are a finite number of possible demands (those in Ω), we can match consumers with the same demands into size-k groups. For each $\bar{\omega} \in \Omega$, let $A^{\bar{\omega}} \equiv \{a \in A \mid f(a) = \bar{\omega}\}$. If $A^{\bar{\omega}}$ has positive measure, assign each $a \in A^{\bar{\omega}}$ to a single membership in a group type $\pi^{\bar{\omega}} \in \mathcal{G}$ defined such that $\bar{\omega} = \sum_{\omega \in \Omega} \pi^{\bar{\omega}}(\omega)\omega = f(a)$ and $|\pi^{\bar{\omega}}| = k$. Since (9) holds for $(\tilde{x}, \tilde{\mu})$, and the consumptions are budget-feasible, the state (x, μ) cannot be an equilibrium. □

The inequality (8) characterizes agents' consumption of shared goods in a group equilibrium. Combined with (5), it becomes

$$U_a\left(e_a - \frac{r}{k} \cdot \omega^{\mu_a},\ \omega^{\mu_a}\right) \geq U_a\left(e_a - \frac{r}{k} \cdot \omega,\ \omega\right) \tag{11}$$
$$\text{for all } \omega \in \Omega \ \text{ such that } e_a - \frac{r}{k} \cdot \omega \geq 0$$

which looks very much like the definition of the demand correspondence (7) for individual purchases at prices r/k. This is the basis of the argument that follows, which says that proprietors have the same profit opportunities in both market circumstances.

A complication, however, is that neither the individual demand correspondence nor group equilibrium is necessarily unique. Consumers may be indifferent between these equilibria, but the proprietors will not be. Assuming that the proprietors price above marginal cost, they prefer more sales to fewer. Similarly, the several group equilibria at prices r will generate the same total utility for agents, but will generate different total profit for the proprietors. We must define a notion of equivalence that accounts for the problem of multiple equilibria.

We show that, despite the multiple equilibria, the profit possibilities are the same whether the proprietors sell to individuals or to groups. Aggregate sales in the group equilibrium can be defined as

$$\omega(x,\mu) = \int_A \sum_{(\omega,\pi)\in\mathcal{M}} \mu_a(\omega,\pi)\,\omega \;\; d\lambda(a) \in Z_+^C$$

If z represents an aggregate demand vector at prices r/k and $\omega(x,\mu)$ represents aggregate sales to members of groups in a group equilibrium $(x,\mu), q$ at prices r, then the profits in the two situations are the same if (12) holds. The following proposition says that there is always an equivalence of that type.

$$z = k\omega\,(x,\mu) \tag{12}$$

Proposition 1 [Profit Equivalence] *Suppose that* A1 *holds.*

(i) *Let* $(x,\mu), q$ *be a purchase-club equilibrium at prices* $r > 0$. *Then* $k\omega(x,\mu) \in D(r/k)$.

(ii) *Let* $z \in D(r/k)$ *be an aggregate demand vector at prices* $r/k > 0$. *Then there exists a purchase-club equilibrium* $(x,\mu), q$ *at prices* r*, with aggregate purchases* $\omega(x,\mu) = z/k$.

Proof.
(i) If $\{\omega^{\mu_a}\}_{a\in A}$ are the consumptions associated with the group equilibrium, they satisfy

$$\frac{1}{k}\int_A \omega^{\mu_a}\; d\lambda(a) = \int_A \sum_{(\omega,\pi)\in\mathcal{M}} \mu_a(\omega,\pi)\,\omega \;\; d\lambda(a) = \omega(x,\mu) \tag{13}$$

This is because there are k agents consuming every purchased good. Since $\{\omega^{\mu_a}\}_{a\in A}$ satisfy (11), they are also a demand selection at prices r/k. Hence $k\omega(x,\mu) \in D(r/k)$.

(ii) The aggregate demand can be written $z = \int_A f(a)\, d\lambda(a)$ for a demand selection f at prices r/k. Construct an equilibrium $(x,\mu), q$ from the demand selection f as in the proof of Claim 1, using prices q described by (4) with equality. Budget feasibility for agents and budget balance for group types are satisfied by construction of q. We also need to show that for almost every $a \in A$,

$$U_a(x_a, \omega^{\mu_a}) \geq U_a(x, \omega^\ell) \text{ for every budget feasible } (x, \ell) \in X_a. \tag{14}$$

By construction of $(x, \mu), q$ and the definition of f, for each $a \in A$,

$$U_a(x_a, \omega^{\mu_a}) = U_a(e_a - \frac{r}{k} \cdot f(a), f(a)) \geq U_a(e_a - \frac{r}{k} \cdot \omega, \omega) \tag{15}$$
$$\text{for all } \omega \in \Omega \text{ such that } e_a - \frac{r}{k} \cdot \omega \geq 0.$$

But if $(x, \ell) \in X_a$ satisfies $\omega^\ell \in \Omega$, (15) implies (14). If (x, ℓ) is budget feasible, it satisfies $e_a - \frac{r}{k} \cdot \omega^\ell \geq x \geq 0$, so $U_a(x_a, \omega^{\mu_a}) \geq U_a(e_a - \frac{r}{k} \cdot \omega^\ell, \omega^\ell) \geq U_a(x, \omega^\ell)$. Suppose that $\omega^\ell \notin \Omega$, and $U_a(x_a, \omega^{\mu_a}) < U_a(x, \omega^\ell)$ for some budget feasible (x, ℓ), so (14) does not hold. Using A1, choose a list ℓ' such that $\omega^{\ell'} \in \Omega$, $\omega^{\ell'} \leq \omega^\ell$ and $U_a(z, \omega^\ell) \leq U_a(z, \omega^{\ell'})$ for all $z \geq 0$. Since $\omega_i^{\ell'} < \omega_i^\ell$ for at least one proprietary good i, $e_a - (r/k) \cdot \omega^\ell < e_a - (r/k) \cdot \omega^{\ell'}$. Thus, if there is a budget feasible (x, ℓ), there is a budget-feasible (x', ℓ'), $x < x'$. Hence, $U_a(x_a, \omega^{\mu_a}) < U_a(x, \omega^\ell) < U_a(x', \omega^{\ell'})$. But this is a contradiction, since we have already shown that, if $\omega^{\ell'} \in \Omega$ and (x', ℓ') is budget feasible, then $U_a(x_a, \omega^{\mu_a}) \geq U_a(x', \omega^{\ell'})$. Thus (14) holds, and $(x, \mu), q$ is a purchase-club equilibrium at prices r. □

4 Rental markets

Example 4 in Section 3 suggests that the club model can be interpreted as a rental market. Our objective here is to elaborate that example, and to show circumstances in which sharing groups are equivalent to how we would conceive of a rental market in ordinary general equilibrium theory.

The easiest way to think of rental markets is that there is an amortized cost of keeping the rental good continuously in use. The competitive price of using it will reflect this amortized cost. If this is all there is to it, then general equilibrium theory as conceived by Arrow and Debreu can account for rental markets, even if demand depends on time. If, for example, there are peak and off-peak demand periods (in the case of sailboats, balmy summer days and dark winter days), then we might think of rentals in the two periods as jointly produced, but different, goods. Price cannot equal "marginal cost" in both periods, since the price in the two periods will be different.

We now show how the club model accommodates rental markets, allowing the quality of the rentals (in the sense of inconvenience due to congestion) to be endogenous, and differentiating prices according to peak and off-peak periods.

Pricing in the club model is more flexible than in a rental market. Prices in an ordinary rental market are linear on units of usage, although possibly different in peak and off-peak periods. We show conditions under which rental prices in a group equilibrium can also be interpreted as linear prices on usage.

Let elements of Ω represent *usage*. For fixed k, represent usage by

$$\Omega = \{(\omega_p, \omega_o) | \omega_p \in \{0, 1, 2, ...k\}, \omega_o \in \{0, 1, 2, ...k\}\},$$

where the membership characteristic $(\omega_p, \omega_o) \in \Omega$ represents the number of units of rental of each type, peak and off-peak. As in the model of the previous section, this model specializes Ω to be a space of numbers rather than an abstract space.

A *rental group type* is (π, γ, y), where y represents the rental goods bought in a competitive market, and $\gamma = (\gamma_p, \gamma_o) \in \Gamma$ specifies the total usage offered by the rental group at both peak and offpeak times. In particular, let $\Gamma = \{\{1, 2, ...\bar{\gamma}_p\} \times \{1, 2, ...\bar{\gamma}_o\}\}$. A *feasible* rental group type (π, γ, y) satisfies $\pi \in \Pi(\gamma_p, \gamma_o)$, where

$$\Pi(\gamma_p, \gamma_o) = \left\{ \pi : \Omega \to Z_+ \;\mid \sum_{(\omega_p, \omega_o) \in \Omega} \pi(\omega_p, \omega_o)\omega_p = \gamma_p, \right.$$
$$\left. \sum_{(\omega_p, \omega_o) \in \Omega} \pi(\omega_p, \omega_o)\omega_o = \gamma_o \right\}$$

In order to interpret total usage $(\gamma_p, \gamma_o) \in \Gamma$ as congestion in the two periods, assume that all group types have the same input vector y, say, one sailboat. We shall thus leave y out of the description of a group type, although it remains in the budget balance condition for each rental group type. The feasible set of group types and memberships are

$$\mathcal{G} = \{(\pi, \gamma) \mid \gamma = (\gamma_p, \gamma_o) \in \Gamma, \pi \in \Pi(\gamma_p, \gamma_o)\}$$
$$\mathcal{M} = \{((\omega_p, \omega_o), (\pi, \gamma)) \mid (\omega_p, \omega_o) \in \Omega, \; (\pi, \gamma) \in \mathcal{G}\}$$

Let $\omega^\ell = (\omega_p^\ell, \omega_o^\ell) : \Gamma \to Z_+ \times Z_+$ represent *usage* associated with ℓ. For each $(\gamma_p, \gamma_o) \in \Gamma$,

$$\omega_p^\ell(\gamma_p, \gamma_o) = \sum_{\pi \in \Pi(\gamma_p, \gamma_o)} \sum_{(\omega_p, \omega_o) \in \Omega} \ell((\omega_p, \omega_o), (\pi, \gamma)) \;\omega_p \tag{16}$$
$$\omega_o^\ell(\gamma_p, \gamma_o) = \sum_{\pi \in \Pi(\gamma_p, \gamma_o)} \sum_{(\omega_p, \omega_o) \in \Omega} \ell((\omega_p, \omega_o), (\pi, \gamma)) \;\omega_o$$

Consumption sets are

$$X_a = \left\{ x \in \Re_+^N, \; \ell \in Z_+^{\mathcal{M}} | \, |\ell| \leq M \right\}$$
$$\tilde{X}_a = \left\{ \left(x, \omega^\ell\right) \mid (x, \ell) \in X_a \right\}.$$

The following assumption says that utility depends only on usage.

A2. For each $a \in A$, the utility function $u_a : X_a \to \Re$ is defined by $u_a(x, \ell) = U_a(x, \omega^\ell)$ for a utility function $U_a : \tilde{X}_a \to \Re$. U_a is increasing in its first argument.

We say that a group equilibrium $(x, \mu), (p, q)$ is *equivalent to equilibrium in a rental market* if there exist rental prices $(\theta_p, \theta_o) : \Gamma \to \Re_+ \times \Re_+$ such that the price of each list ℓ satisfies

$$(\theta_p, \theta_o) \cdot \left(\omega_p^\ell, \omega_o^\ell\right) = \ell \cdot q \tag{17}$$

The important feature of rental markets is that they impose a restriction on prices. The membership price $q((\omega_p, \omega_o\ ), (\pi, \gamma))$ will reflect the member's peak usage and offpeak usage, as well as the congestion, but there is no prior restriction that the price $q((\omega_p, \omega_o\ ), (\pi, \gamma))$ can be conceived as a linear price on usage, and that the linear price is the same as that of other users, scaled by usage. However:

Proposition 2 *Suppose that* A2 *holds. Let* $(x, \mu), (p, q')$ *be a group equilibrium. Then there is another group equilibrium* $(x, \mu), (p, q)$ *that is equivalent to an equilibrium in a rental market.*

Proof. We first define a distinguished set of group types that offer single-usage memberships, rather than selling usage in bulk. For each $\gamma \in \Gamma$, let (π^γ, γ) be a group type such that $\pi^\gamma(0,1) = \gamma_o$, $\pi^\gamma(1,0) = \gamma_p$, and $\pi^\gamma(\omega_p, \omega_o) = 0$ for $(\omega_p, \omega_o) \notin \{(0,1), (1,0)\}$. For example, a membership $((0,1), (\pi^\gamma, \gamma))$ is a single off-peak use. For each $(\omega, (\pi, \gamma)) \in \mathcal{M}$, let

$$q(\omega, (\pi, \gamma)) = \quad \omega_p\ \ q'((1,0), (\pi^\gamma, \gamma)) + \omega_o\ \ q'((0,1), (\pi^\gamma, \gamma))$$

To define the prices in the rental market, for each $\gamma \in \Gamma$ let

$$\begin{aligned} \theta_p(\gamma) &= q'((1,0), (\pi^\gamma, \gamma)) = q((1,0), (\pi^\gamma, \gamma)) \\ \theta_o(\gamma) &= q'((0,1), (\pi^\gamma, \gamma)) = q((0,1), (\pi^\gamma, \gamma)) \end{aligned} \tag{18}$$

To prove the proposition, we only need to show that (x, μ),(p, q) is an equilibrium. We first show that almost all the agents are in their budget sets, and then show that they are optimizing.

We show that, for almost all $a \in A$, $p \cdot e_a = x_a + q \cdot \mu_a = x_a + q' \cdot \mu_a$. Construct a consistent list assignment $\tilde{\mu}$ with the same individual usage as in the equilibrium lists μ, but with single-usage memberships. That is, for each $a \in A$, $\gamma \in \Gamma$, let

$$\begin{aligned} \tilde{\mu}_a((1,0), (\pi^\gamma, \gamma)) &= \omega_p^{\mu_a}(\gamma) \\ \tilde{\mu}_a((0,1), (\pi^\gamma, \gamma)) &= \omega_o^{\mu_a}(\gamma) \\ \tilde{\mu}_a((\omega_p, \omega_o), (\pi, \gamma)) &= 0 \qquad \text{for all other memberships} \end{aligned}$$

Then the following holds by construction for $a \in A$.

$$q \cdot \tilde{\mu}_a = q' \cdot \tilde{\mu}_a = (\theta_p, \theta_o) \cdot \left(\omega_p^{\mu_a}, \omega_o^{\mu_a}\right) \tag{19}$$

Since $u_a(x_a, \mu_a) = u_a(x_a, \tilde{\mu}_a)$ and $(x, \mu), (p, q')$ is an equilibrium, it holds that $p \cdot e_a = x_a + q' \cdot \mu_a \leq x_a + q' \cdot \tilde{\mu}_a$ for almost all $a \in A$, so that

$$q' \cdot \mu_a \leq q' \cdot \tilde{\mu}_a = q \cdot \tilde{\mu}_a \tag{20}$$

The assignment $\tilde{\mu}$ is consistent because μ is consistent. Since $\omega^{\mu_a}(\gamma) = \omega^{\tilde{\mu}_a}(\gamma)$ for each $\gamma \in \Gamma$ and every $a \in A$, the number of groups associated with $\tilde{\mu}$ is the same as the number associated μ, and each has the same cost $p \cdot y$. Since (p, q') balances the budget for each group type, μ and $\tilde{\mu}$ must generate the same revenue in aggregate.

$$\int_A q' \cdot \mu_a \, d\lambda(a) = \int_A q' \cdot \tilde{\mu}_a \, d\lambda(a) \tag{21}$$

But this proves that the inequality in (20) cannot be strict for a set of agents with positive measure. Hence we can conclude that for almost all $a \in A$,

$$p \cdot e_a = x_a + q' \cdot \mu_a = x_a + q' \cdot \tilde{\mu}_a = x_a + q \cdot \tilde{\mu}_a = x_a + q \cdot \mu_a.$$

We now show that agents are optimizing at the prices (p, q). Suppose that $(x, \ell) \in X_a$ and $u_a(x, \ell) > u_a(x_a, \mu_a)$. We show that (x, ℓ) is not budget feasible at prices (p, q). Construct a new list ℓ' from ℓ by substituting single-usage memberships in the same amount. For each membership $(\omega, (\pi, \gamma)) \in \mathcal{M}$, let $\ell'(\omega, (\pi, \gamma)) = 0$ except let $\ell'((0,1), (\pi^\gamma, \gamma)) = \omega_o^\ell(\gamma)$, $\ell'((1,0), (\pi^\gamma, \gamma)) = \omega_p^\ell(\gamma)$. Then the lists ℓ and ℓ' provide the same usage $\omega^\ell = \omega^{\ell'}$, and $u_a(x, \ell') = u_a(x, \ell) > u_a(x_a, \mu_a)$. Because $(x, \mu), (p, q')$ is an equilibrium, $p \cdot x + q' \cdot \ell > p \cdot e_a$ and $p \cdot x + q' \cdot \ell' > p \cdot e_a$. But since $q \cdot \ell' = q' \cdot \ell' = q \cdot \ell$, this implies that $p \cdot x + q \cdot \ell' > p \cdot e_a$ and $p \cdot x + q \cdot \ell > p \cdot e_a$. □

5 Conclusion

The club model can account for consumption externalities in various ways. By consumption externalities, we mean that each member of a club cares about the private-goods consumption of other members. The models above elaborate that idea by introducing different technologies of sharing, and showing how the technologies of sharing can be reflected in group types and membership characteristics.

The term "consumption externality" suggests that each agent makes a consumption decision without considering its impact on others. The club model forces him to consider the impact. Groups that want to avoid negative externalities that arise from private consumption decisions will have memberships that involve a commitment to avoid consumption of certain private goods. Groups that want to generate positive externalities due to private consumption decisions will have membership characteristics that require certain kinds of consumption. These commitments can be built into feasible consumption sets, which can constrain the consumption of private goods in a way that is linked to memberships in groups.

The technology of sharing private goods was more precise in what we called purchase clubs and rental clubs. In the case of automobiles, sailboats and other durable goods which cannot be used simultaneously by all members of a group, the terms of sharing must be specified in the group type and the membership characteristics. Nothing requires that congestible durable goods be shared in rental groups rather than purchase groups; in fact, there is no clear distinction between those two concepts. We only chose those terms to suggest familiar market institutions, and to give different names to models based on different sharing technologies. The key point is that, if users care about total congestion as well as their own usage, then membership prices must reflect both. And membership prices may also reflect the externality-producing private goods that a member brings as part of his membership.

In the purchase-club and rental-club models, we respectively treated proprietary pricing and congestion costs. Of course proprietary pricing and congestion can be combined in the same model: Goods that are purchased at proprietary prices can nevertheless be subject to congestion. A group equilibrium will be efficient for the users conditional on the proprietary prices, but this is a conditional notion of efficiency. Each copy of a proprietary good that is subject to congestion may be used "too much" in equilibrium, to conserve on paying the proprietary price.

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