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Ex Post Regret and the Decentralized Sharing of Information*

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Firms reveal private information about the value of investment through their investment decisions. As a consequence they may have ex post regret once they see other firms' investments. We define a notion of rational expectations equilibrium for games which imposes a "no-regret" property. In this equilibrium, all firms make the same investment decision, but despite the absence of ex post regret, the investment herd can be inefficient. In addition equilibrium might not exist. We introduce a notion of probabilistic existence, and identify conditions under which, if the number of firms is large, enough information comes out so that investment herds are first-best efficient with high probability, and equilibrium exists with high probability.
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1. INTRODUCTION

In competitive markets investment decisions are typically decentralized. While decentralization has well-known benefits, it also has costs; in par-

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ticular firms might have private information which, if pooled, would lead to more efficient investment. Despite the decentralization firms can learn something of each other's private information by observing each other's investment decisions. Since efficiency could generally be improved by sharing information, we would hope that decentralized choices reveal the aggregate information completely. In this paper we study an investment model where, on the contrary, information may be shared very incompletely.

The amount of information revealed through decentralized action depends in part on whether the firms can revise their decisions once the other firms have revealed their information. If not, then firms in equilibrium may have ex post regret; that is, their investment decisions might not be optimal in light of the information revealed.

An equilibrium notion that permits ex post regret is the sequential entry model, as introduced by Banerjee (1992) and Bikhchandani, Hirschleifer and Welch (1992). Firms make decisions in an order that is given exogenously. The first movers act on their private information, and reveal it. Early movers will regret their actions if it becomes clear that the preponderance of information is different than their own. An important conclusion of the literature on sequential entry is that firms eventually herd on a single action. The agents might herd on the "wrong" action, in the sense that they would prefer another action if the aggregate information were observable.

While one could imagine frictions that prevent firms from revising their investment decisions, these scenarios seem extreme. In this paper we ask whether the information is shared more completely if no such frictions are present. We study the opposite extreme in which actions can be revised frictionlessly so that in equilibrium no firm wishes to revise further. In equilibrium, firms do not have ex post regret. Since each firm's action must be optimal conditional on the information revealed by the other firms' actions, we call this a rational expectations equilibrium (REE).

We discover that eliminating frictions does not always improve the decentralized sharing of information. In REE the only possible equilibrium actions are herds and, again, the whole herd might take the "wrong" action. Although eliminating frictions increases the opportunities for firms to learn from each other's actions, in equilibrium there might be less to learn. The impediment to sharing information is more basic than suggested in the herd literature, namely: *By relying on the information of others, firms undermine how much of their own information can be revealed*. This is why the information is only partially revealed in equilibrium.

We compare welfare in REE with equilibrium in the sequential entry model, and find that, if the number of firms is "small," the welfare cannot be ranked. Both equilibria can be inefficient relative to the first best, since the information is not completely revealed. REE has the advantage that the revealed information is used efficiently (there is no ex post regret), but the sequential entry model has the advantage that early movers who set the herd in motion cannot be swayed by the strong reinforcement of seeing all the other firms taking a common action, which might be the inefficient one. They will therefore act on their own information, which is correlated with the true state.

Although herds can be inefficient for small numbers of firms, we give a condition under which, as the number of firms becomes large, the probability of inefficient herds goes to zero. The condition is that with positive probability there is a very strong signal of the true state. A similar result holds for the sequential entry model (Smith and Sorenson (1994)).

Our analysis exposes a similarity between two well-known equilibrium ideas: Rational expectations and equilibria of games, where actions are common knowledge. This relationship has recently been developed for a more general class of games by Minelli and Polemarchakis (1997), using a different definition of equilibrium than we use.[1] Following Aumann (1976) and Geanakoplos (1994), if agents' actions are common knowledge, then agents act in equilibrium as if they have the same information. We show that the same reasoning extends to games.

An underexplored problem is that REE in games might not exist.[2] In fact we show that REE typically does not exist in our herding model, and to deal with this, we introduce a notion of probabilistic existence.

In the next section we describe the model and define REE. In Sec. 3 we characterize REE. In Sec. 4 we give conditions under which it exists for large n. In Sec. 5 we compare REE with the sequential entry model.

2. THE MODEL

We consider a model of investment with n firms, indexed $i \in \{1, \cdots, n\}$, and two states of the world, $\omega = G, B$, each with probability $\frac{1}{2}$. Each firm has a signal σ^i of the state. These are distributed according to $F(\cdot|\omega)$, $\omega = B, G$, with support Σ. For simple arguments, we assume that $F(\cdot|B)$, $F(\cdot|G)$ are atomless with densities $f(\cdot|\omega)$, $\omega = B, G$, and $\Sigma = [0, 1]$. Our conclusions also hold for the discrete case, where $\Sigma = \{1, 2, \ldots, k\}$. We or-

[1] Minelli and Polemarchakis develop the connection between REE and common knowledge more generally than we do, but do not discuss existence or efficiency of equilibrium. See also Krishna (1986) and Diamantaras (1988). The "no-regret" feature has been imposed on mechanisms in the concept of "posterior implementable," defined by Green and Laffont (1987), and in self-fulfilling mechanisms, defined by Forges and Minelli (1998), who also discuss how such mechanisms apply to the game discussed here. However, the "REE" described there does not accord with the definition in this paper.

[2] Diamantaras discusses the existence problem, and finds a class of games for which he can exploit the techniques of REE in markets to show existence of equilibria that are fully revealing. That class of games does not extend to the herding example.

der and label the signals such that $f(\cdot|B)/f(\cdot|G)$ is decreasing (the Monotone Likelihood Ratio Property holds), and in addition we will sometimes require the following stronger condition:

DEFINITION 1. The distributions $F(\cdot|B), F(\cdot|G)$ satisfy the Strong Monotone Likelihood Ratio Property (SMLRP) if the likelihood ratio $f(\cdot|B)/f(\cdot|G)$ is decreasing, and in addition, $\lim_{\sigma\to 1} f(\sigma|B)/f(\sigma|G) = 0$ and $\lim_{\sigma\to 0} f(\sigma|B)/f(\sigma|G) = \infty$.

Each firm takes an action $a^i \in A^i = \{0, 1\}$, where 1 means that the firm invests in a project, and 0 means that it does not. We let $a \in A = \prod_{i=1}^n A^i$ refer to the vector of all firms' actions. Typically a firm will not know the aggregate information $\sigma = (\sigma_1, \ldots, \sigma_n)$, but will form a belief based on incomplete information.

When we say that an action a^i is "efficient," we mean that it is profit-maximizing conditional on the aggregate information, σ. We define a parameter $\rho > 0$ so that the action $a^i = 1$ is *efficient* if and only if $P(G|\sigma)/P(B|\sigma) \geq \rho$, where $P(G|\sigma)/P(B|\sigma) = (\prod_{i=1}^n f(\sigma^i|G))/(\prod_{i=1}^n f(\sigma^i|B))$ is the posterior likelihood ratio of $\omega = G$ to $\omega = B$.[3]

We define firm i's *strategy*, for $i = 1, \ldots, n$, as $\bar{\sigma}^i$: $A \to [0, 1]$ such that firm i takes action $a^i = 1$ if $\sigma^i \geq \bar{\sigma}^i(a)$ and takes action $a^i = 0$ if $\sigma^i < \bar{\sigma}^i(a)$.[4] The threshold strategy is justified because $f(\sigma^i|G)/f(\sigma^i|B)$ is increasing: Conditional on the information learned by observing other agents' actions, the firm's likelihood ratio of $\omega = G$ to $\omega = B$ increases with its own signal.

We refer to the vector of strategies, or the *strategy profile*, as $\bar{\sigma}$. We say that an action a is *consistent* with the strategy profile $\bar{\sigma}$ at a signal σ if $a^i = 1$ implies $\sigma^i \geq \bar{\sigma}^i(a)$ and $a^i = 0$ implies $\sigma^i < \bar{\sigma}^i(a)$.

In forming its posterior belief about ω, firm i uses its knowledge of other firms' strategies $(\bar{\sigma}^j(a))_{j\neq i}$ to draw inferences from their actions a. For each i, we will let $\ell^i(\sigma^i, a)$ represent firm i's posterior likelihood ratio of $\omega = G$ to $\omega = B$, conditional on (σ^i, a), $a \in A$. We say that firm i's belief ℓ^i is *consistent* with the strategy $\bar{\sigma}$ at (a, σ^i) if it is obtained from Bayesian

[3]Thus firms' profits are not affected by other firms' actions except insofar as they reveal information. For example, in the biotechnology industry, the main concern of investors may lie in the viability of the field as a whole rather than in the prospect of competition between as yet hypothetical end products. See for example Austin (1996).

[4]It might seem odd that the strategy of firm i should depend on the whole vector of actions $a \in A$, which includes the action of firm i. However this is demanded by the notion of REE. By including a^i we indicate that firm i knows what action the *other* firms see, hence what thresholds $(\bar{\sigma}^j(a))_{j\neq i}$ they use. Firm i must infer these thresholds in order to make an inference about the other firms' signals. The consistency condition on actions ensures there is no conflict between firm i's action as dictated by the strategy and firm i's action which is an argument to the strategy function.

updating:

$$\ell^i(\sigma^i, a) = \frac{f(\sigma^i|G)}{f(\sigma^i|B)} \frac{\prod_{k\neq i}\,[(1-F^k(\bar{\sigma}^k(a)|G))a^k + F^k(\bar{\sigma}^k(a)|G)(1-a^k)]}{\prod_{k\neq i}\,[(1-F^k(\bar{\sigma}^k(a)|B))a^k + F^k(\bar{\sigma}^k(a)|B)(1-a^k)]}. \tag{1}$$

A *rational expectations equilibrium (REE)* is a strategy profile $\bar{\sigma}$ such that for each signal σ there exists an action a and beliefs ℓ such that:

1. The beliefs ℓ are consistent with $\bar{\sigma}$ at (a, σ).
2. The action a is consistent with $\bar{\sigma}$ at σ.
3. $\ell^i(\bar{\sigma}^i(a), a) = \rho$ if $\bar{\sigma}^i(a) \in (0, 1)$
   $\ell^i(\bar{\sigma}^i(a), a) \geq \rho$ if $\bar{\sigma}^i(a) = 0$
   $\ell^i(\bar{\sigma}^i(a), a) \leq \rho$ if $\bar{\sigma}^i(a) = 1$.

Agents' strategies are, thus, defined as a function of other agents' actions and of the agent's private information. Each agent's beliefs about the state of the world must be consistent with Bayesian updating and the other agents' equilibrium strategies.

3. REE AND COMMON KNOWLEDGE OF ACTIONS

In a rational expectations equilibrium, all firms take the same action, either $a = \mathbf{0} = (0, \ldots, 0)$ or $a = \mathbf{1} = (1, \ldots, 1)$. As in the herding literature, we call such outcomes "herds." Herds arise in REE because in equilibrium all agents act as if they have the same information. This is because their actions and the dependence of actions on beliefs are both "common knowledge." The link between common knowledge of actions and common posterior beliefs was first suggested for bargaining and other contexts by Aumann (1976) and Geanakoplos (1994). In this section, we show how the same reasoning extends to rational expectations in the herding game.

A convenient consequence of how we define REE is that we can use Bayesian updating to prove the herd result, without discussing information partitions and their common refinements. Bayesian updating allows a particularly easy proof, which follows.

PROPOSITION 1. *The only actions consistent with REE are herds*, *namely*, $a \in \{\mathbf{0}, \mathbf{1}\}$.

Proof. We will show this for atomless distributions. Suppose that $a \neq \mathbf{0}, \mathbf{1}$, and suppose in particular that $a_1 = 1$, $a_2 = 0$. Then equilibrium requires that:

$$\ell^1(\bar{\sigma}^1(a), a) \geq \ell^2(\bar{\sigma}^2(a), a).$$

Substituting the expression in Eq. (1) for these posteriors when $a_1 = 1$ and $a_2 = 0$ and simplifying gives:

$$\frac{f(\bar\sigma^2(a)|B)}{f(\bar\sigma^2(a)|G)}\left(\frac{1-F(\bar\sigma^1(a)|B)}{1-F(\bar\sigma^1(a)|G)}\right) \geq \frac{f(\bar\sigma^1(a)|B)}{f(\bar\sigma^1(a)|G)}\,\frac{F(\bar\sigma^2(a)|B)}{F(\bar\sigma^2(a)|G)}. \qquad (2)$$

But the MLRP implies that for all σ

$$F(\sigma|B) = \int_0^\sigma \frac{f(\tilde\sigma|B)}{f(\tilde\sigma|G)}\, f(\tilde\sigma|G)\, d\tilde\sigma > \frac{f(\sigma|B)}{f(\sigma|G)}\, F(\sigma|G).$$

This implies the second inequality of (3), and the first also follows from the MLRP:

$$\frac{1-F(\sigma|B)}{1-F(\sigma|G)} < \frac{f(\sigma|B)}{f(\sigma|G)} < \frac{F(\sigma|B)}{F(\sigma|G)}. \qquad (3)$$

But (3) contradicts (2). ■

Intuition for this result can be seen in the following example.

EXAMPLE 1. [REE actions are herds.] Suppose $n = 2$ and $a = (1, 0)$. Firm 1 invests and firm 2 does not invest. We will show that there are no $\bar\sigma^1(1, 0)$, $\bar\sigma^2(1, 0)$ such that $(1, 0)$ is an equilibrium action. Suppose there were such an equilibrium, and consider the signal $\sigma = (\bar\sigma^1(1, 0),\ \bar\sigma^2(1, 0))$. When firm 1 observes $a = (1, 0)$, what it knows about the signals are that $\sigma^1 = \bar\sigma^1(1, 0)$ and the expected value of σ^2 is $\int_0^{\bar\sigma^2(1,0)} (\sigma^2 f(\sigma^2))/(F(\bar\sigma^2(1, 0)))d\sigma^2$, where $f(\cdot)$ is the unconditional distribution of σ. What firm 2 knows is that $\sigma^2 = \bar\sigma^2(1, 0)$ and the expected value of σ^1 is $\int_{\bar\sigma^1(1,0)}^1 (\sigma^1 f(\sigma^1))/(1 - F(\bar\sigma^1(1, 0)))d\sigma^1$. Hence firm 1 believes that both signals are lower in expectation than firm 2 believes. This is inconsistent with the fact that firm 1 is investing and firm 2 is not. ■

The herding result is also implied by common knowledge arguments of Aumann (1976), Geanakoplos (1994), Minelli (1995), and Minelli and Polemarchakis (1997). These arguments tell us that if firms know each other's strategies and actions, they must act in equilibrium as if they have the same information. Below we indicate how to prove the herding result using a Proposition of Geanakoplos. A complete development of the argument requires a discussion of information partitions, and how they are refined in equilibrium. We avoid this for simplicity. In general, we prefer to base our arguments on Bayesian updating because this exposes the role played by monotonicity in firms' strategies.

Proof of Proposition 1, *using a theorem of Geanakoplos* (*1994*, *Sec.* 6). The theorem assumes that a firm's action (invest or not invest) is described by an "external action rule" that maps the set of all possible events into actions. In our model an event E is a set of signals, $E \subset \Sigma^n$. Consider the following rule: *For each $E \subset \Sigma^n$, let $a^i = 1$ if and only if the posterior probability of $\omega = G$, conditional on E, is at least ρ.* This rule satisfies the sure thing principle,[5] as required by the theorem, and generates the strategies described for REE. The theorem says that if firms' actions a are common knowledge, then there is some single event E such that each firm's action is given by the external action rule evaluated at E. This theorem implies the herding result since the external action rule is the same for all firms. ■

A consequence of Proposition 1 is that since all firms have the same profit function, they all obey the same threshold in equilibrium, either $\bar{\sigma}(\mathbf{1})$ or $\bar{\sigma}(\mathbf{0})$, depending on whether the herding is on $a = \mathbf{1}$ or $a = \mathbf{0}$.

The following example shows that REE can be inefficient; that is, the "wrong" herd can arise in equilibrium.

EXAMPLE 2. Consider a discrete distribution on the support $\Sigma = \{1, 2, 3\}$, defined by

$$f(1|G) = 1/8 \qquad f(1|B) = 3/8$$

$$f(2|G) = 1/2 \qquad f(2|B) = 1/2$$

$$f(3|G) = 3/8 \qquad f(3|B) = 1/8$$

and suppose $\rho = 9/8$. Equilibrium strategies are $\bar{\sigma}(\mathbf{0}) = \bar{\sigma}(\mathbf{1}) = 2$. That is, a herd on $\mathbf{1}$ can arise if all firms have signals 2, 3, and a herd on $\mathbf{0}$ can arise if all firms have signals 1, 2.

Suppose that $\sigma^i = 2 = \sigma^j$. Then conditions 1., 2., and 3. hold at both $a = \mathbf{0}$ and $a = \mathbf{1}$. The signal 2 is uninformative, so the likelihood ratio of $\omega = G$ to $\omega = B$, conditional on the aggregate signal $\sigma = (2, 2)$, is the same as the prior, namely 1. Since $\rho > 1$ and $P(G|\sigma^1, \sigma^2)/P(B|\sigma^1, \sigma^2) = 1$, only the action $a = \mathbf{0}$ is efficient.

For intuition about what is going on in an inefficient herd, consider the herd $a = \mathbf{1}$. Each firm can infer that no other firm has the low signal, 1. Such a firm would not invest because its posterior likelihood ratio would be too low. The absence of a low signal suggests that the state is more likely G rather than B, and that is why the inefficient action $\mathbf{1}$ is self-confirming. ■

4. EXISTENCE OF REE

In this section we discuss what is required for equilibrium to exist. We first point out that our definition of equilibrium is so demanding that equi-

[5]An external action rule satisfies the sure thing principle if and only if for any two disjoint events E, $E' \subset \sum^n$, if $E \cap E' = \varnothing$ and if the actions taken at E and E' are the same action a, then the action taken at $E \cup E'$ is also a. See Geanakoplos (1994), p. 1453.

librium in an exact sense typically does not exist. We then give a condition under which a less exact notion of existence is assured. We show that the analogue of this condition for discrete distributions guarantees existence in the exact sense.

Equilibrium is a demanding concept because conditions 1., 2., and 3. in the definition must hold for *every* signal $\sigma \in \Sigma^n$ at either $a = \mathbf{0}$ or $a = \mathbf{1}$. Equilibrium fails to exist if two firms might have strong signals that conflict. In Example 2 above, if $\sigma^i = 1$ (a strong signal of $\omega = B$) and $\sigma^j = 3$ (a strong signal of $\omega = G$), then nothing can persuade firm i to invest, and nothing can persuade firm j to not invest. But in equilibrium the firms cannot take different actions, so equilibrium does not exist.

In fact, Example 2 illustrates a general problem of existence, stated in the following proposition. (A proof is available from the authors.) An informal intuition for Proposition 2 is this: When two firms have extreme signals that conflict, there must be some herd consistent with both signals. The herd must then be consistent with all the intermediate signals: That is, firms choose the common action no matter what signals they receive. Hence, firm i learns nothing from the actions of the other firms. That is, firm i bases its action purely on its own signal and is willing to follow the herd no matter what this signal is. But this is a stringent requirement that might conflict with optimizing behavior. If condition (5) holds, firm i will find it optimal to invest if its signal is sufficiently high, but not if its signal is very low, even if all the other firms are taking the same action. Thus there exist signals for which firm i will contradict the herd. Since such signals exist, equilibrium does not exist in the very demanding sense that conditions 1., 2., and 3. are satisfied for *every* aggregate signal σ.

Proposition 2. *Suppose that $F(\cdot|G)$ and $F(\cdot|B)$ have common support Σ, that the densities $f(\cdot|G)$ and $f(\cdot|B)$ are continuous, and that there exist $\sigma_L, \sigma_H \in \Sigma$ such that for all $\sigma^i \in \Sigma$*

$$\frac{f(\sigma_L|G)}{f(\sigma_L|B)} \leq \frac{f(\sigma^i|G)}{f(\sigma^i|B)} \leq \frac{f(\sigma_H|G)}{f(\sigma_H|B)} \tag{4}$$

$$\frac{f(\sigma_L|G)}{f(\sigma_L|B)} < \rho < \frac{f(\sigma_H|G)}{f(\sigma_H|B)} \tag{5}$$

Then rational expectations equilibrium does not exist: *For some aggregate signals σ, conditions* 1., 2., *and* 3. *are not satisfied*.

However nonexistence should be less troubling if for *most* signals $\sigma \in \Sigma^n$ there is an action $a = \mathbf{0}$ or $a = \mathbf{1}$ that is consistent with equilibrium strategies. We therefore weaken the notion of equilibrium.

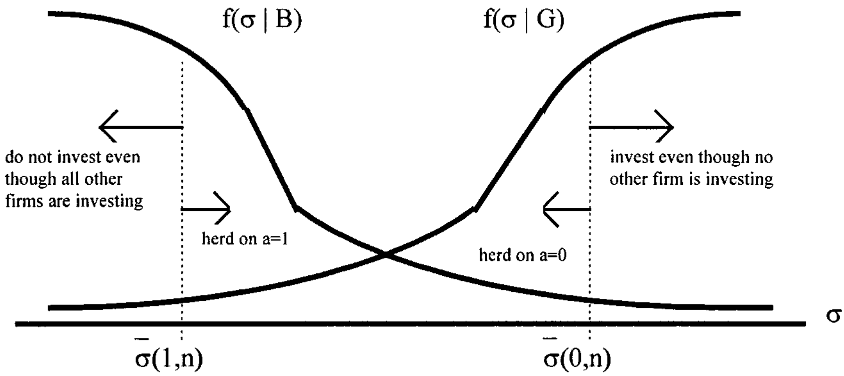

FIGURE 1

We say that *REE exists with probability* p if there exist strategies $\bar{\sigma}$ and a set of signals $M \subset \Sigma^n$ such that for each $\sigma \in M$ there exists an action a such that REE conditions 1., 2., and 3. hold, and in addition

4. $p \leq \int_{\sigma \in M} \frac{1}{2}(\prod_{i=1}^{n} f(\sigma^i|G) + \prod_{i=1}^{n} f(\sigma^i|B))\, d\sigma$. (The probability that $\sigma \in M$ is at least p.)

In this section we will write strategies $\bar{\sigma}$ so that they depend on n, the number of firms, since we will consider the case that n becomes large.

The existence problem can be seen in Fig. 1, which shows $F(\cdot|B)$ and $F(\cdot|G)$ with two equilibrium threshold values. If all firms' signals are higher than $\bar{\sigma}(\mathbf{1}, n)$, then the action $a = \mathbf{1}$ is consistent with the strategy $\bar{\sigma}$. If all firms' signals are lower than $\bar{\sigma}(\mathbf{0}, n)$, then the action $a = \mathbf{0}$ is consistent with the strategy $\bar{\sigma}$. Any firm with a signal above $\bar{\sigma}(\mathbf{1}, n)$ will invest even though its signal is relatively low, provided all the other firms are investing. The investment of other firms makes the firm optimistic that $\omega = G$, but a signal below $\bar{\sigma}(\mathbf{1}, n)$ is so low that it overcomes such optimism. Symmetric reasoning applies to $\bar{\sigma}(\mathbf{0}, n)$.

Let M be the union of the signals for which equilibrium exists; that is, all vectors such that every firm's signal is at least as high as $\bar{\sigma}(\mathbf{1}, n)$ or no higher than $\bar{\sigma}(\mathbf{0}, n)$. This means that if $\sigma \in M$, no two firms have signals that are respectively lower and higher than $\bar{\sigma}(\mathbf{1}, n)$ and $\bar{\sigma}(\mathbf{0}, n)$. If p is the probability of M, then REE exists with probability p, which is typically less than 1.

LEMMA 1. *As* $n \to \infty$, $\bar{\sigma}(\mathbf{1}, n)$ *converges to* 0 *and* $\bar{\sigma}(\mathbf{0}, n)$ *converges to* 1.

Proof. The equilibrium satisfies:

$$\frac{f(\bar{\sigma}(\mathbf{1}, n)|B)}{f(\bar{\sigma}(\mathbf{1}, n)|G)} \left[\frac{1 - F(\bar{\sigma}(\mathbf{1}, n)|B)}{1 - F(\bar{\sigma}(\mathbf{1}, n)|G)}\right]^{n-1} = \frac{1}{\rho} \tag{6}$$

If $\bar{\sigma}(\mathbf{1}, n) \not\to 0$, then the lefthand side would converge to 0 as $n \to \infty$, a contradiction. A similar argument applies for $\bar{\sigma}(\mathbf{0}, n)$. ■

One might hope that convergence of $\bar{\sigma}(\mathbf{1}, n)$ to 0 and $\bar{\sigma}(\mathbf{0}, n)$ to 1 would guarantee existence of an equilibrium with high probability. This is not the case. For every finite n there is a strictly positive probability that some signal is below $\bar{\sigma}(\mathbf{1}, n)$ and that another is above $\bar{\sigma}(\mathbf{0}, n)$. To guarantee that this happens with smaller and smaller probability as $n \to \infty$, we must guarantee that $\bar{\sigma}(\mathbf{1}, n) \to 0$ and $\bar{\sigma}(\mathbf{0}, n) \to 1$ at a fast rate.

Proposition 3 shows that if the SMLRP holds, an REE exists with probability approaching 1 as $n \to \infty$. An intuitive interpretation of the condition is that signals close to 0 or 1 are an "almost sure" indicator that $\omega = B$ or $\omega = G$, respectively. For every n, the minimum signal that elicits investment, $\bar{\sigma}(\mathbf{1}, n)$, is close to zero. This might seem wrong, since investment does not reveal that a firm has a high signal. However the threshold is large enough so that if $\omega = B$ there is a high probability of yet a lower signal among the n firms. The absence of a lower signal in equilibrium (revealed by the fact that all firms are playing $a^i = 1$) makes them confident that $\omega = G$.

To put it more succinctly, the absence of a strong signal contrary to the herd permits the herd to exist. Equilibrium exists if strong signals contrary to the herd are unlikely to arise by accident, and for this it is required that $\bar{\sigma}(\mathbf{0}, n)$ and $\bar{\sigma}(\mathbf{1}, n)$ converge to 1 and 0 rapidly.

PROPOSITION 3 (existence of REE for large n). *Suppose that $F(\cdot|B)$, $F(\cdot|G)$ satisfy the SMLRP. If $\omega = G$ (respectively, $\omega = B$), the probability that there is a herd on $a = \mathbf{1}$ (respectively, $a = \mathbf{0}$) approaches 1 as $n \to \infty$.*

Proof. We argue for the atomless case, and for $\omega = G$. Using (3) and (6), write

$$\frac{1}{\rho} = \frac{f(\bar{\sigma}(\mathbf{1}, n)|B)}{f(\bar{\sigma}(\mathbf{1}, n)|G)} \left[1 - \frac{F(\bar{\sigma}(\mathbf{1}, n)|B) - F(\bar{\sigma}(\mathbf{1}, n)|G)}{1 - F(\bar{\sigma}(\mathbf{1}, n)|G)}\right]^{n-1}$$
$$\leq \frac{F(\bar{\sigma}(\mathbf{1}, n)|B)}{F(\bar{\sigma}(\mathbf{1}, n)|G)} \left[1 - \frac{F(\bar{\sigma}(\mathbf{1}, n)|B) - F(\bar{\sigma}(\mathbf{1}, n)|G)}{1 - F(\bar{\sigma}(\mathbf{1}, n)|G)}\right]^{n-1} \tag{7}$$

Suppose, in contradiction to the proposition, that $[1 - F(\bar{\sigma}(\mathbf{1}, n)|G)]^n \not\to 1$. We will consider a subsequence along which $nF(\bar{\sigma}(\mathbf{1}, n)|G) \to c$ for some $c > 0$, where it could hold that $c = \infty$. Such a subsequence exists because otherwise $nF(\bar{\sigma}(\mathbf{1}, n)|G) \to 0$ for all subsequences, hence

$$\left[1 - F(\bar{\sigma}(\mathbf{1}, n)|G)\right]^n = \left(\left[1 - F(\bar{\sigma}(\mathbf{1}, n)|G)\right]^{1/F(\bar{\sigma}(\mathbf{1},n)|G)}\right)^{nF(\bar{\sigma}(\mathbf{1},n)|G)}$$
$$\to \left(\frac{1}{e}\right)^0 = 1$$

Write

$$v(n) \equiv \frac{F(\bar{\sigma}(\mathbf{1}, n)|B) - F(\bar{\sigma}(\mathbf{1}, n)|G)}{1 - F(\bar{\sigma}(\mathbf{1}, n)|G)}$$
$$= \left(\frac{F(\bar{\sigma}(\mathbf{1}, n)|G)}{1 - F(\bar{\sigma}(\mathbf{1}, n)|G)}\right)\left(\frac{F(\bar{\sigma}(\mathbf{1}, n)|B)}{F(\bar{\sigma}(\mathbf{1}, n)|G)} - 1\right)$$

The inequality (7) can be written

$$\frac{1}{\rho} \leq \frac{F(\bar{\sigma}(\mathbf{1}, n)|B)}{F(\bar{\sigma}(\mathbf{1}, n)|G)}\left([1 - v(n)]^{1/v(n)}\right)^{(n-1)v(n)} \tag{8}$$

It is not the case that $v(n) \to \infty$, since $v(n) \leq 1$.
If $v(n) \to 0$, then $[1 - v(n)]^{1/v(n)} \to \frac{1}{e}$.
If $v(n) \to$ constant, then $[1 - v(n)]^{1/v(n)} \to d$, $0 < d < 1$.
It follows from (6) that $F(\bar{\sigma}(\mathbf{1}, n)|G) \to 0$, hence, using the SMLRP and (3), that $F(\bar{\sigma}(\mathbf{1}, n)|B)/F(\bar{\sigma}(\mathbf{1}, n)|G) \to \infty$ and

$$nv(n) = \left(\frac{n\, F(\bar{\sigma}(\mathbf{1}, n)|G)}{1 - F(\bar{\sigma}(\mathbf{1}, n)|G)}\right)\left(\frac{F(\bar{\sigma}(\mathbf{1}, n)|B)}{F(\bar{\sigma}(\mathbf{1}, n)|G)} - 1\right)$$
$$\to c\left(\frac{F(\bar{\sigma}(\mathbf{1}, n)|B)}{F(\bar{\sigma}(\mathbf{1}, n)|G)} - 1\right)$$

Thus $nv(n) \to \infty$ and $(n-1)v(n) \to \infty$, so the righthand side of (8) goes to 0 whether c is finite or infinite. ■

The examples in Sec. 5 use discrete distributions of signals rather than atomless distributions. In those examples, equilibrium exists. Existence follows from the discrete analog to Proposition 3, which is presented in our (1995) working paper. The existence theorem in that paper supposes that with some probability there is a "sure" signal of the state of nature; that is, a signal that reveals ω with probability one. Sure signals are the discrete analog to the SMLRP. The sure signal might have low probability, but if there are many firms, then such signals will assure existence, as follows.

If an agent receives a sure signal, then he knows ω for sure, and will act on the sure signal without taking account of other players' actions. Suppose that $\omega = G$. Then no firm can have the sure signal that $\omega = B$. With enough firms there will be a herd equilibrium with action $a = \mathbf{1}$. Each firm reveals by its willingness to play $a^i = 1$ that it does not have a sure signal of $\omega = B$. If no firm in a large number of firms has a sure signal of $\omega = B$, then the state is probably $\omega = G$. On that basis, all firms are willing to follow the herd and play $a^i = 1$. A symmetric story applies if the true state is $\omega = B$, and with high probability, the herd will play $a = \mathbf{0}$. Thus, whatever the true state, equilibrium exists, and the equilibrium action is efficient.

In our (1995) working paper, we also discussed whether mixed strategies guarantee existence. Mixed strategies allow that for some (σ^i, a), firm i

can randomize on whether to invest. By the monotonicity of strategies, such randomization is only optimal at a single σ^i (for a fixed a), and such a point arises with probability 0 if F is atomless. Hence, mixed strategies cannot help very much.

5. EFFICIENCY OF REE: A COMPARISON WITH TWO OTHER EQUILIBRIUM CONCEPTS

Example 2 shows that herds in REE can be inefficient. Of course the "right" comparison for evaluating the social losses in REE is not necessarily to compare with the first best, since the first best cannot always be achieved with asymmetric information. Two natural comparisons are with the single-firm decision problem, where a firm acts only on its own signal, and with the sequential entry model, in which firms use each other's information, but only in a fixed order of play.

5.1. *Sequential Entry Model and the Single-Firm Decision Problem*

The single-firm decision problem is particularly simple. There is a common investment threshold σ^S such that $\rho = f(\sigma^S|G)/f(\sigma^S|B)$. Firm i will invest if and only if $\sigma^i \geq \sigma^S$. Of course the resulting action vector a is typically not efficient, since some firms invest and others do not. The efficient action vector is either $a = \mathbf{0}$ or $a = \mathbf{1}$.

To describe the sequential entry model, we will redefine our notation, hopefully without confusion. The literature on sequential play assumes that the firms are ordered exogenously so that a firm's index, say i, gives its place in the order of play. We redefine $a^{-i} = (a^1, a^2, \ldots, a^{i-1})$ and $A^{-i} = \prod_{j<i} A^j$. We let h^i: $\Sigma \times A^{-i} \to \{0, 1\}$ represent firm i's strategy, i.e., whether or not to invest, conditional on its own signal and on the actions of previous firms in the order of play.

The collection of strategies $(h^j)_{j=1}^n$ induces a partition $\mathcal{P}^i$ on the space of signals Σ^n for each i. This means that since firm i knows all the firms' strategies, it can deduce from the actions a^{-i} played by the first $i-1$ firms what their signals could possibly have been. Each element of the partition, say $q(\sigma^i, a^{-i}) \in \mathcal{P}^i$, contains all the signals $\sigma \in \Sigma^n$ that would have permitted the actions a^{-i} as a consequence of the equilibrium strategies $(h^j)_{j=1}^n$.

A *sequential entry equilibrium* (SEE) is a collection of strategies $(h^j)_{j=1}^n$ and a collection of partitions $(\mathcal{P}^j)_{j=1}^n$, each a partition of Σ^n, such that each firm's strategy maximizes profit conditional on the information that has been revealed: For each $i = 1, \ldots, n$, $h^i(\sigma^i, a^{-i}) = 1$ if and only if the posterior likelihood ratio, conditional on $q(\sigma^i, a^{-i}) \in \mathcal{P}^i$ is at least ρ.

5.2. *Welfare Comparisons*

We first compare profits in REE and SEE to the single person decision problem. By Blackwell's Theorem we have the following observation.

Observation. The ex ante expected profit of each firm, conditional on knowing only its own signal, is greater (no smaller) in both REE and SEE than if each firm uses only its own private signal.

We do not provide a formal proof, but the intuition is as follows. For SEE, the information possessed by the first mover is its own signal and nothing else, hence there is no advantage. But every subsequent firm knows more than its own signal. It makes inferences about the signals of previous movers based on their equilibrium strategies and the actions they took. Since the information partition used by firms in SEE is a refinement of their initial partition, they make better decisions.

The argument for REE (when it exists) is more subtle. For some σ, the equilibrium action is not unique (see Fig. 1 and Example 2). To calculate the expected profit of a firm, we must have a selection rule, either the "optimistic" rule ($a = \mathbf{1}$ when both are possible) or the "pessimistic" rule ($a = \mathbf{0}$ when both are possible). In expectation, firms make better decisions in REE conditional on each rule.

To evaluate the efficiency of REE when it exists with probability $p < 1$, we will assume that if there is no consistent herd, the actions are the same as the single-firm decisions; hence the comparison turns on those σ for which REE exists, and the reasoning still applies. Of course for a particular vector σ, the outcome of the single-firm decision problem might dominate either REE or SEE. We claim only that, conditional on σ^i, firm i's *expected* profit is greater than when it acts only on its own private information.

We now compare efficiency in REE and SEE. A priori, their ranking is unclear. REE and SEE lead to different actions. Neither equilibrium reveals the information σ completely, and each is therefore inefficient relative to the first best. We will consider the case as n gets large separately from the case of small n.

For the case of large n, we already know something about efficiency of the sequential entry model. Banerjee (1992) and Bikhchandani *et al.* (1992) showed that inefficient herds persist with positive probability as $n \to \infty$. However Smith and Sorensen (1994) showed that if agents' private information contains signals that are correct with probability arbitrarily close to 1, then (almost surely) the herd eventually takes the efficient action. The following Corollary to Proposition 3 states a similar result for REE.

COROLLARY 1 (Herds in REE are efficient for large n.) *Suppose that* $F(\cdot|B)$, $F(\cdot|G)$ *satisfy the SMLRP. Then if* $\omega = G$ (*respectively* $\omega = B$), *the probability of a herd on* $a = \mathbf{0}$ (*respectively* $a = \mathbf{1}$) *goes to 0 as* $n \to \infty$.

For finite n, the comparison between SEE and REE is not definitive. The following example shows that REE might or might not dominate SEE. This is true whether the selection among REE actions is optimistic or pessimistic.

EXAMPLE 3 (Comparison of SEE and REE). Instead of writing $\Sigma = \{1, 2, 3, 4\}$ we shall write $\Sigma = \{L_o, L, H, H^o\}$. L_o is a perfect signal that the state is B and H^o is a perfect signal that the state is G. Thus it is impossible that $\sigma^i = H^o$ and $\sigma^j = L_o$ for firms i and j. The probability that the state is revealed to a firm with certainty is ε.

$$f(\sigma^i|G) = \begin{cases} \varepsilon & \text{if } \sigma^i = H^o \\ (1-\varepsilon)2/3 & \text{if } \sigma^i = H \\ (1-\varepsilon)1/3 & \text{if } \sigma^i = L \\ 0 & \text{if } \sigma^i = L_o. \end{cases}$$

$$f(\sigma^i|B) = \begin{cases} \varepsilon & \text{if } \sigma^i = L_o \\ (1-\varepsilon)2/3 & \text{if } \sigma^i = L \\ (1-\varepsilon)1/3 & \text{if } \sigma^i = H \\ 0 & \text{if } \sigma^i = H^o. \end{cases}$$

In this example, an REE exists with probability 1 provided[6]

$$1 - \frac{1}{1+2(1-\varepsilon)^{n-1}} < \frac{\rho}{1+\rho} < \frac{1}{1+2(1-\varepsilon)^{n-1}}.$$

In the equilibrium, $\bar{\sigma}(\mathbf{1}) = L$ and $\bar{\sigma}(\mathbf{0}) = H^o$. That is, when all other firms are investing, firm i invests if and only if $\sigma^i = L, H, H^o$. And when no other firms are investing, firm i invests if and only if $\sigma^i = H^o$.

Let $\Pi(\omega)$ be the profit of each firm if the state is ω. Then $(\rho/1+\rho)\Pi(G) + (1/1+\rho)\Pi(B) = 0$, with $\Pi(B) < 0$. In this example, the difference between firms' first best (full information) expected profits and total REE expected profits is $n\frac{1}{2}(1-\varepsilon)^n|\Pi(B)|$ with optimistic selection and $n\frac{1}{2}(1-\varepsilon)^n\Pi(G)$ with pessimistic selection.

To calculate expected profits in SEE, we must characterize the sequences of play that can arise in equilibrium, as well as their probabilities. See our (1995) working paper for details. For $n = 6$, three possible sequences are $(0, 0, 0, 1, 1, 1)$, $(0, 1, 0, 1, 0, 1)$ and $(1, 1, 1, 0, 0, 0)$. (This type of characterization is derived in the early sequential entry papers. See also Simons and Bhattacharya (1996).) It turns out that when $2+\varepsilon/4-\varepsilon > \rho/1+\rho > \frac{1}{2}$

[6]This inequality is easier to satisfy both for larger ε and larger n, because both of these help to insure that at least one person in the herd knows the true state of the world.

the difference between full information profits and profits in an SEE is given by:

$$\frac{1}{6}(\Pi(G) - \Pi(B))(1-\varepsilon)\left[1 + \varepsilon \sum_{k=0}^{n-2} k(1-\varepsilon)^k + (n-1)(1-\varepsilon)^{(n-1)}\right].$$

This analysis leads to:

- SEE is welfare superior to REE under optimistic selection if $n = 6$, $\rho = 1.052545$ and $\varepsilon = 0.1386$, and also when optimistic selection and pessimistic selection are equally likely.
- REE is welfare superior to SEE under pessimistic selection when $n = 6$, $\rho = 1.052545$ and $\varepsilon = 0.1386$.
- SEE is welfare superior to REE under pessimistic selection when $n = 6$, $\rho = 1.0000002$ and $\varepsilon = 0.1386$.
- REE is welfare superior to SEE when $n = 6$, $\rho = 1.052545$ and $\varepsilon = 0.14$.

In all these examples, $\rho > 1$, which means that the loss from investing when $\omega = B$ exceeds the loss from not investing when $\omega = G$. As a consequence, REE achieves higher welfare when the equilibrium selection is pessimistic than when it is optimistic. The first two examples make this point. However the third example shows that even when the equilibrium selection is pessimistic the SEE may do better than REE. The last example illustrates that increasing ε tends to improve REE relative to SEE because it increases the chance of a sure signal.

A question suggested here is whether SEE is a tatonnement process by which REE can be reached.[7] For each REE outcome, one could ask whether the signal $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ could be ordered such that the firms in SEE eventually take the action consistent with REE. An extension of Example 3 shows this conjecture to be false:

Example 3 (continued). Let $\rho = 7/3$, $\varepsilon = .85$, $n = 2$. In REE a firm with signal H^o always invests, a firm with signal H invests if and only if it sees the other firm invest, and a firm with signal L or L_o does not invest whether or not the other firm invests. Both actions $a = \mathbf{0}, \mathbf{1}$ are consistent with REE at the signal $\sigma = (H, H)$. However the action $a = (1, 1)$ is not an SEE. The firm that moves first calculates a likelihood ratio $P(G|H)/P(B|H) = 2$, which is less than ρ.

Each firm is willing to invest in the REE outcome $a = (1, 1)$ only because it sees the other firm investing, but would not invest otherwise.

[7]We thank Lones Smith for pointing us to this question and for noticing the analogy with market tatonnement processes and market equilibrium.

This cannot happen in the SEE because the first firm cannot see the other firm's investment decision. ■

6. CONCLUSION

This paper shows that welfare is not necessarily improved by eliminating ex post regret. Instead, frictionless learning makes herding behavior inevitable, and herds may be on the wrong outcome. The logic behind this herding result is essentially the same as the logic behind Aumann's and Geanakoplos' observation that common knowledge of actions leads agents to behave as if they have the same information.

Our result that information is not fully revealed contrasts with the "strong efficient markets" hypothesis of the finance literature. According to this hypothesis, strong efficient markets (i.e., those in which prices convey a sufficient statistic for the privately held information in the system) exist almost surely if (and only if) the dimension of the space of prices exceeds the dimension of the space of signals. (See Radner (1979), Allen (1982), Jordan and Radner (1982), and Jordan (1983). Bray (1985) has a nice survey.)[8] There are also some results for sequential entry models that relate the efficiency of equilibrium herding to the structure of the spaces of actions and signals (Lee (1993) and Zhang and Zhang (1995).) Our results for rational expectations equilibrium emphasize instead the role of strong signals and the number of firms, as do Smith and Sorensen (1994). The equilibrium herd becomes increasingly likely to be efficient as the number of firms gets large provided the signals are sufficiently informative.

REE and SEE do not exhaust the possible assumptions about timing and flexibility of investment decisions. For example, Chamley and Gale (1994) investigate a model in which investment decisions are irrevocable as in SEE, but the timing of investment is endogenous. It is costly to delay investment, but firms may nevertheless delay investment in order to avoid irrevocable mistakes. This generates a bias towards underinvestment. When information is not fully revealed in their model, it is usually a situation in which no firms are investing.

Our main point is that information is shared incompletely because a firm cannot both use others' information and simultaneously reveal its own information, at least completely. At one extreme is the individual decision problem, where firms reveal much of their own information, but cannot use the information revealed in the actions of others. In SEE and REE, firms have an opportunity to use the information revealed in the actions

[8]For other foundational work on REE in markets, see also Green (1973) and Anderson and Sonnenschein (1982).

of others, but by using it, they undermine the amount of information that they themselves reveal.

In an early paper on rational expectations in markets, Green (1973) interpreted the possible inefficiency of rational expectations equilibrium as arising from externalities. In a rational expectations equilibrium, each firm confers an externality on other firms by revealing information. Since the firm does not account for the value of this externality in taking its action, there is no reason to think that the amount of information revealed will be optimal.

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