

On the Evolution of Attitudes towards Risk in Winner-Take-All Games*

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A long-standing conjecture is that winner-take-all games such as patent races lead to the survival of risk-takers and the extinction of risk-averters. In many species a winner-take-all game determines the males' right to reproduce, and the same argument suggests that males will evolve to be risk-takers. Psychological and sociological evidence buttresses the argument that males are more risk-taking than females. Using an evolutionary model of preference-formation, we investigate to what extent evolution leads to risk-taking in winner-take-all environments. *Journal of Economic Literature* Classification Numbers: C7, D8. © 1999 Academic Press

1. INTRODUCTION

Economists typically take preferences as given. This sets them apart from other social scientists, such as psychologists, who often try to explain preferences. In this paper we explore an evolutionary model where preferences, in particular attitudes toward risk, are endogenously determined.

In economics, preferences are simply rules for choosing among feasible consumption bundles. If “successful” choice rules become more prevalent in

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society, then the distribution of choice rules, hence preferences, is an endogenous outcome. While there are many notions of “success” and various dynamic processes that can determine how successful rules proliferate, we consider an evolutionary process. That is, success means reproductive success, and choice rules (preferences) are inherited by offspring from their parents. Preferences can be described as inherited either if children emulate their parents’ choices or if preferences are genetically coded.

Of course, if preferences are endogenous, something else must be exogenous, namely the “game” in which reproductive success is determined. We investigate winner-take-all games, which are common in nature: the leading male(s) mate with (almost) all the females, and therefore only that male’s genes (hence preferences) are inherited by the subsequent generation. In a winner-take-all environment it pays to take risks, so one might expect that risk-taking preferences become dominant in the population. For example, Tirole [29, pp. 396–7] explains that R&D competitions favor riskier choices in the sense of mean-preserving spreads, since patent races resemble winner-take-all games. We explore the sense in which risk taking is selected in winner-take-all environments, and discover that the evolutionary outcome is subtler than suggested.

In particular, risk taking is not selected if the winner-take-all competition is in small groups, and in large groups the selected form of risk taking can depend on whether winner-take-all games are played simultaneously in many randomly matched groups from a large population, or whether there is only one match that comprises the entire population. In Sections 2 and 3 we present these two winner-take-all environments and show what form of risk taking evolves.

To apply our theory to humans, we need to argue that humans inherit risk-taking behavior from ancestors in whom risk-taking was evolutionarily selected via winner-take-all games. The conjecture seems more plausible for males than for females, both because there is considerable evidence from biologists and sociologists that males are more risk-taking, and because one can point to rituals in the animal world where males compete in tournaments for the right to mate. The evidence we have found, which is reviewed in Section 4, is at best indirect. To build a bridge between the observation that winner-take-all games lead to (a form of) risk taking and the conclusion that human males will therefore evolve to be risk takers, we argue that

- among species where winner-take-all games determine reproductive success, the winning behavior can be interpreted as analogous to risk taking in humans rather than something else;
- risk taking behavior in such species is genetically coded; and

- humans inherit such genes from evolutionary ancestors who played winner-take-all games.

Among economists, early proponents of the view that preferences are endogenous included Becker [4, 5], Hirschleifer [15, 16] and Rubin and Paul [26]. Their view has recently been revived. For example, Hansson and Stuart [14] and Rogers [24] give an evolutionary account of how discount rates are determined, and cite related work on the evolution of intertemporal preferences. Waldman [30] develops an explicitly sexual model, where evolution can lead to “second-best adaptation.” For example, he argues that such evolution may result in males overestimating their ability and suffering disutility from effort, instead of being “efficient” by correctly estimating ability and not suffering disutility of effort. Evolution of attitudes toward risk were explored by Karni and Schmeidler [18], Cooper [10] and Robson [22], who have used different evolutionary models to show that preferences with an expected-utility representation will be selected. Robson [22] extends his model to show how preferences conforming to non-expected utility can also evolve.

Papers more closely related to this one are Rubin and Paul [26] and recent (independent) papers of Robson [23] and Wärneryd [31]. Rubin and Paul argue in a model that is not explicitly dynamic that males may be risk taking if females only select males with income above a threshold. Robson [23] shows that if males choose lotteries over wealth with the objective to maximize offspring, and are selected by females according to their (relative) wealth, they will choose very risky lotteries if they choose any at all. Wärneryd [31] studies a rent-seeking contest which is similar to the model below in that the winner is rewarded, but differs in that many preferences can survive, since the rents are dissipated among the participants. In the (unusual) case where only one kind of preference survives, it is the one closest to risk neutrality. Skaperdas [27, 28] considers how different attitudes toward risk affect outcomes in rent-seeking games.

2. A RANDOM-MATCHING MODEL OF WINNER-TAKE-ALL GAMES

2.1. *The Model*

In both the random-matching model of this section, and the playing-the-field model of the next section, players are genetically coded to choose one of a finite number of lotteries, $\mathcal{F} \equiv \{F_1, \dots, F_L\}$. The set of lotteries is held fixed throughout the analysis. Each section examines a dynamic process on the proportions of the population choosing the different lotteries in \mathcal{F} . For each of the two dynamic processes discussed in this paper we say that a particular F_ℓ within \mathcal{F} is *selected* (or *evolves*) if the proportion of the

agents choosing it converges to 1. As is standard with revealed preference, we interpret a player's (genetically encoded) choice of a lottery in \mathcal{F} as that player's most preferred element in the set. We thus characterize the evolution of a choice rule, which specifies for a given finite \mathcal{F} the chosen, i.e., most preferred, lottery.

Although our model concerns selection within a given finite set of lotteries, we interpret it as a selection of preferences, i.e., a selection from all "relevant" sets \mathcal{F} , where a set \mathcal{F} is relevant if the agent is "likely" to face that set. The interpretation we have in mind is that Nature determines in each period the set of lotteries, \mathcal{F} , from which the agents choose. For any such set that appears "often enough," evolution will select the lottery we describe. This is not entirely satisfactory if there are uncountably many sets, in which case evolutionary forces will not have the opportunity to operate on every possible subset \mathcal{F} . On the other hand, one might argue that only finitely many lotteries are distinguishable by our coarse sensory capabilities, in which case there are only finitely many subsets \mathcal{F} among which agents choose.

We apply replicator dynamics to an infinite population. The precise dynamic process is specified below. Intuitively, in each period infinitely many groups of size m are matched from the population, and Nature independently determines an outcome for each player from his chosen lottery. In each group the player with the highest outcome "wins" and has m offspring, all of whom are coded to choose the same strategy as the winner. Losers have no offspring. We will describe the population in each time period by a probability vector $\sigma(t) = (\sigma^1(t), \dots, \sigma^L(t))$, where $\sigma^\ell(t)$ is the proportion of players in the population who choose F_ℓ when faced with \mathcal{F} . We call these the type- F_ℓ or type- ℓ players, or just F_ℓ players. Our objective is to characterize how the limit of $\sigma(t)$, $t = 1, 2, \dots$, depends on the set \mathcal{F} , starting from a population with full support, $\sigma^\ell(0) > 0$ for all ℓ .

Let $\mathcal{A}^m \equiv \{k: \sum_{\ell=1}^L k_\ell = m, k \geq 0\}$. For $k = (k_1, \dots, k_L) \in \mathcal{A}^m$, we will interpret k_ℓ as the number of type- F_ℓ players in a group of size m . If the group is drawn randomly from a population with relative frequencies $\sigma(t)$, then k is a random variable from a multinomial distribution with probabilities $\sigma(t) = (\sigma^1(t), \dots, \sigma^L(t))$. For each k in \mathcal{A}^m , let $f_\ell(k) \equiv k_\ell \times \int [F_\ell(x)]^{k_\ell-1} \prod_{i \neq \ell} [F_i(x)]^{k_i} dF_\ell(x)$. Thus $f_\ell(k)$ is the probability that any of the type- F_ℓ players wins in a match with $k = (k_1, \dots, k_L)$ players of the different types.

If m members of the population are selected randomly (with or without replacement), then the probability of k is $\binom{m}{k} \prod_i [\sigma_i(t)]^{k_i}$, where $\binom{m}{k} \equiv \binom{m}{k_1, \dots, k_L}$. We assume that the population proportions evolve according to the following dynamic process.

$$\sigma^\ell(t+1) = \sum_{k \in \mathcal{A}^m} \binom{m}{k} \prod_i [\sigma_i(t)]^{k_i} f_\ell(k)$$

This process describes replicator dynamics (e.g., Weibull [32, p. 72]), applied to groups of size m rather than groups of size 2. Following the literature, we justify this dynamic system by appeal to the law of large numbers. Thus, we assume that the proportion of matches with profile k equals the probability of drawing the profile k . We also appeal to the law of large numbers a second time, and assume that within the matches with profile k , the proportion of matches in which a type ℓ wins equals the probability of such a win, namely $f_\ell(k)$.

Justifying the dynamic process from a more fundamental story of random matching is regrettably not as natural or as straightforward an application of the law of large numbers as one would hope. If the population is a continuum, there are the well-known technical difficulties associated with selecting i.i.d. random variables. (See Feldman and Gilles [13] or McLennan and Sonnenschein [20, footnote 4], who discuss the implications in the context of random matching.) For countably large populations, there are other difficulties. (See Boylan [7, 8] for a discussion, and Propositions 2 and 5 in [7] for a matching process that embodies the strong-law-of-large-numbers intuition.)

2.2. Orderings of Lotteries

Our results characterize which lottery from \mathcal{F} is selected. We define a lottery F_ℓ as *avored within* \mathcal{F} if, for any $k \in \Delta^m$, the probability that any type F_ℓ player wins is more than proportional to their number, i.e., greater than k_ℓ/m . In the next subsection we show that when F_ℓ is favored within \mathcal{F} , then F_ℓ is selected.

DEFINITION 1. F_1 is *avored within* \mathcal{F} in a random-matching environment with matches of size m , if $f_1(k) > k_1/m$ for all $k \in \Delta^m$, $0 < k_1 < m$.

The condition of being favored is a partial ordering of lotteries. It is therefore of interest to investigate how it relates to standard orders, namely first- and second-order stochastic dominance. Recall that F_1 strictly FOSD F_ℓ if $F_1(x) \leq F_\ell(x)$ for all x , with strict inequality for some x ; and F_1 strictly SOSD F_ℓ if $\int_{-\infty}^t F_1(x) dx \leq \int_{-\infty}^t F_\ell(x) dx$ for all t , with strict inequality for some t .

PROPOSITION 1. *If F_1 strictly FOSD F_ℓ for $\ell = 2, \dots, L$ then F_1 is favored within \mathcal{F} .*

Proof. $f_1(k) = k_1 \cdot \int [F_1]^{k_1-1} \prod_{\ell=1} [F_\ell(x)]^{k_\ell} dF_1(x) > k_1 \int F_1(x)^{m-1} dF_1(x) = k_1/m$, where the last integral is $1/m$ because it equals the probability that one of m players, all of whom are type F_1 , wins. ■

EXAMPLE 1 (Being favored does not imply FOSD). Suppose that $m = 2$, that G places probability 1 on the outcome 1 and F places probabilities $\frac{1}{3}$ and $\frac{2}{3}$ respectively on 0 and $\frac{3}{2}$. We assume that if two players have the same outcome, each wins with probability $\frac{1}{2}$. Clearly F is favored over G , even though it does not strictly FOSD G .

The previous example might suggest that “riskier” lotteries (those that are second-order-stochastically dominated) are always favored. However, the following example shows otherwise. Thus the result in the next section that evolution selects for favored lotteries does not confirm the intuition that it should select riskier lotteries.

EXAMPLE 2 (A lottery may be favored even if it SOSD (is less risky than) another). Suppose that $m = 2$, that G places probability 1 on the outcome 1 and \hat{F} places probability $\frac{2}{3}$ on $\frac{1}{4}$ and $\frac{1}{3}$ on 2.5. While G strictly SOSD \hat{F} , G is favored over \hat{F} .

Remark 1(Intransitivity). Examples 1 and 2 can also be used to show that a pairwise definition of favored does not give a transitive order: F is favored over G which is favored over \hat{F} , and it can be checked that \hat{F} is favored over F . Both examples can be generalized to any m by changing the probability from $\frac{1}{3}$ to $1/n$ for n sufficiently large.¹

The result that evolution selects for favored lotteries can be shown to imply that for large m evolution selects for a condition we call *tail dominance*.

DEFINITION 2. F_1 tail dominates F_ℓ if there exists \bar{x}_ℓ such that $F_1(x) \leq F_\ell(x)$ for all $x \geq \bar{x}_\ell$ with strict inequality for $x = \bar{x}_\ell$.

A lottery F_1 tail dominates F_ℓ if it first-order dominates F_ℓ in an interval at the top of the supports; see Fig. 1. If F_1 tail dominates F_ℓ for each F_ℓ in \mathcal{F} , then we say that F_1 tail dominates \mathcal{F} . For a finite set of lotteries the tail-dominance order corresponds to an expected-utility ranking where the utility of the highest outcome in the supports of the lotteries is “much” larger than the utility of the second highest outcome, which is “much” larger than the next highest outcome, and so on.² Proposition 2 links tail

¹ This intransitivity is similar to Gale’s roulette wheel example, see, e.g., Binmore (1992, p. 90). If $\mathcal{F} = \{F, G, \hat{F}\}$ then there is a “mixed-strategy equilibrium,” i.e., a polymorphic steady state of the dynamic process, in which $\frac{2}{3}$ of the population is type F , $\frac{1}{3}$ is type \hat{F} , and $\frac{1}{3}$ is type G . We have not verified whether or not this steady state is locally stable.

² How much larger depends on the probabilities that the various lotteries assign to the different outcomes. Thus, for an infinite set of lotteries on a finite set, the same can be done if the utility function is non-standard and the utility of the highest outcome is infinitely larger than that of the second highest, and so on. Otherwise the tail-dominance order need not correspond to any expected (or non-expected) utility ranking.

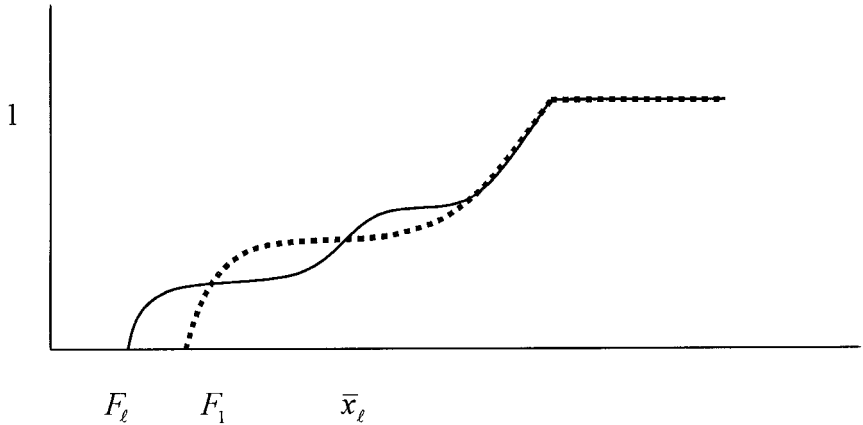


FIGURE 1

dominance to FOSD and SOSD, while Proposition 3 shows that for large m tail-dominating lotteries are favored.

Remark 2 (Completeness of the tail-dominance order). If we restrict attention to lotteries with finite support, tail dominance is a complete, transitive order. However, if we compare two lotteries that do not have finite support then it can happen that neither is tail dominant. Thus the order is not complete without the restriction to finite support.

It remains an open question whether some other ordering would determine the selection within a set \mathcal{F} for which no $F \in \mathcal{F}$ tail dominates \mathcal{F} .

EXAMPLE 3. A sequence $\{F^n\}$ of lotteries that tail dominate a lottery G can converge (in the weak convergence topology on lotteries) to a lottery F that is tail dominated by G . For example suppose F^n gives probabilities $\frac{1}{2}, (\frac{1}{2} - 1/n), 1/n$ to the outcomes 0, 1, $1/n$ respectively. Then $F^n \rightarrow F$ where F gives probabilities $\frac{1}{2}, \frac{1}{2}$ to the outcomes 0, 1 respectively. Suppose G gives probability 1 to the outcome 1. Then each F^n tail dominates G and G tail dominates F . Note however, that there is no finite group size m^* such that Proposition 3 below applies for all n in the sequence (i.e., such that F^n will be favored over G for all n). The minimum required group size grows with n .

PROPOSITION 2. *If F strictly FOSD G or if G strictly SOSD F , then F tail dominates G .*

Proof. Obvious.

PROPOSITION 3. *Suppose that F_1 tail dominates all other lotteries in \mathcal{F} , and that the support of F_1 is an interval.³ There exists m^* such that for $m > m^*$, $f_1(k) > k_1/m$.*

Proof. Let $\{\bar{x}_\ell\}$ be as in the definition of tail dominance. The rough idea of the proof is that, by taking m large, with high probability the winner will have value above \bar{x}_ℓ for each ℓ , and in the region above \bar{x}_ℓ , F_1 strictly FOSD F_ℓ so that the intuition underlying Proposition 1 can be applied.

The probability that one of the F_1 players wins is at least the probability that the winner is an F_1 player and has outcome greater than some $y(k)$, which is the probability of the winner having value at least $y(k)$ times the probability of an F_1 winning conditional on the winner being above $y(k)$.

Let

$$K \equiv \left\{ k \in \mathbf{Z}_+^L \mid k_1 > 0, \sum_{\ell=1}^L k_\ell = m \right\}$$

$$y(k) \equiv \max \{ \bar{x}_\ell \mid F_\ell \in \mathcal{F}, k_\ell \neq 0 \} \quad \text{for each } k \in K$$

$$K_+ \equiv \{ k \in K \mid F_1(y(k)) > 0 \}$$

If $F_1(y(k)) < 1$,

$$\begin{aligned} f_1(k) &\geq \left(1 - \prod_{\ell=1}^L F_\ell(y(k))^{k_\ell} \right) \times \frac{k_1 \int_{y(k)}^\infty F_1(x)^{k_1-1} \prod_{\ell=2}^L F_\ell(x)^{k_\ell} dF_1(x)}{1 - \prod_{\ell=1}^L F_\ell(y(k))^{k_\ell}} \\ &= \left(1 - \prod_{\ell=1}^L F_\ell(y(k))^{k_\ell} \right) k_1 \\ &\quad \times \frac{\int_{y(k)}^\infty F_1(x)^{k_1-1} [\prod_{\ell=2}^L F_\ell(x)^{k_\ell} + F_1(x)^{m-k_1} - F_1(x)^{m-k_1}] dF_1(x)}{1 - \prod_{\ell=1}^L F_\ell(y(k))^{k_\ell}} \\ &= \left(1 - \prod_{\ell=1}^L F_\ell(y(k))^{k_\ell} \right) k_1 \times \frac{\int_{y(k)}^\infty F_1(x)^{m-1} dF_1(x)}{1 - \prod_{\ell=1}^L F_\ell(y(k))^{k_\ell}} \\ &\quad + k_1 \int_{y(k)}^\infty F_1(x)^{k_1-1} \left[\prod_{\ell=2}^L F_\ell(x)^{k_\ell} - F_1(x)^{m-k_1} \right] dF_1(x) \\ &\geq \frac{k_1}{m} + k_1 \times \left\{ \int_{y(k)}^\infty F_1(x)^{k_1-1} \left[\prod_{\ell=2}^L F_\ell(x)^{k_\ell} - F_1(x)^{m-k_1} \right] dF_1(x) \right. \\ &\quad \left. - \frac{\prod_{\ell=1}^L F_\ell(y(k))^{k_\ell}}{m} \right\}. \end{aligned}$$

³ It is straightforward to adapt this proof to the case of finite support; we present the more difficult case.

So $f_1(k) > k_1/m$ if the term in large curly brackets is positive.

Choose $m^*, \delta > 0$ such that (2.2) below holds for each ℓ and each $k \in K_+$ (a finite set).

$$F_1(y(k))^{m^* - k_\ell} [F_1(y(k) + \delta) - F_1(y(k))] \left[1 - \left(\frac{F_1(y(k) + \delta)}{F_\ell(y(k))} \right)^{k_\ell} \right] - \frac{1}{m^*} > 0 \quad \text{if } k_\ell \geq 1$$

We now demonstrate that the term in large curly brackets is positive for $m > m^*$. This is immediate for $k \in K \setminus K_+$ (i.e., k such that $F_1(y(k)) = 0$), since the integral term is positive by tail dominance, and the last term vanishes. For $k \in K_+$ the term in curly brackets above is positive if

$$F_1(y(k))^{m - k_\ell} \int_{y(k)}^\infty [F_\ell(x)^{k_\ell} - F_1(x)^{k_\ell}] dF_1(x) - \frac{F_\ell(y(k))^{k_\ell}}{m} > 0 \quad \text{for some } \ell.$$

To verify this, first notice that $F_\ell(x)^{k_\ell} \geq F_1(x)^{k_\ell}$ for all $x \geq y(k)$ and all ℓ , using the fact that if $y(k) \leq x < \bar{x}_\ell$, then $k_\ell = 0$. Then

$$\begin{aligned} & \left\{ \int_{y(k)}^\infty F_1(x)^{k_1 - 1} \left[\prod_{\ell=2}^L F_\ell(x)^{k_\ell} - F_1(x)^{m - k_1} \right] dF_1(x) - \frac{\prod_{\ell=1}^L F_\ell(y(k))^{k_\ell}}{m} \right\} \\ &= \left\{ \int_{y(k)}^\infty F_1(x)^{k_1 - 1} \left[\prod_{\ell=2}^L F_\ell(x)^{k_\ell} - \prod_{\ell=2}^L F_1(x)^{k_\ell} \right] dF_1(x) - \frac{\prod_{\ell=1}^L F_\ell(y(k))^{k_\ell}}{m} \right\} \\ &= \left\{ \int_{y(k)}^\infty F_1(x)^{-1} \left[\prod_{\ell=1}^L F_\ell(x)^{k_\ell} - \prod_{\ell=1}^L F_1(x)^{k_\ell} \right] dF_1(x) - \frac{\prod_{\ell=1}^L F_\ell(y(k))^{k_\ell}}{m} \right\} \\ &\geq \left\{ \int_{y(k)}^\infty \left[\prod_{\ell=1}^L F_\ell(x)^{k_\ell} - \prod_{\ell=1}^L F_1(x)^{k_\ell} \right] dF_1(x) - \frac{\prod_{\ell=1}^L F_\ell(y(k))^{k_\ell}}{m} \right\} \\ &\geq \left\{ \int_{y(k)}^\infty F_1(x)^{m - k_{\ell'}} [F_{\ell'}(x)^{k_{\ell'}} - F_1(x)^{k_{\ell'}}] dF_1(x) - \frac{\prod_{\ell=1}^L F_\ell(y(k))^{k_\ell}}{m} \right\} \\ &\geq F_1(y(k))^{m - k_{\ell'}} \int_{y(k)}^\infty [F_{\ell'}(x)^{k_{\ell'}} - F_1(x)^{k_{\ell'}}] dF_1(x) - \frac{F_{\ell'}(y(k))^{k_{\ell'}}}{m}. \end{aligned}$$

Since ℓ' was chosen arbitrarily, the term in curly brackets is therefore positive if (2.2) holds for some ℓ .

By considering the integral from $y(k)$ to $y(k) + \delta$, we can conclude that

$$\begin{aligned} & F_1(y(k))^{m-k_\ell} \int_{y(k)}^{\infty} [F_\ell(x)^{k_\ell} - F_1(x)^{k_\ell}] dF_1(x) \\ & - \frac{F_\ell(y(k))^{k_\ell}}{m} \geq F_1(y(k))^{m-k_\ell} [F_1(y(k) + \delta) - F_1(y(k))] \\ & \times [F_\ell(y(k))^{k_\ell} - F_1(y(k) + \delta)^{k_\ell}] - \frac{F_\ell(y(k))^{k_\ell}}{m} \end{aligned}$$

But the latter term is positive for $m > m^*$ by choice of δ, m^* . ■

2.3. Selection

We now use the orders defined above to illuminate the outcome of evolution. Our first result is that evolution selects favored lotteries.

PROPOSITION 4. *Consider a random-matching winner-take-all environment with matches of size $m \geq 2$, and suppose that F_1 is favored within \mathcal{F} . Given ψ s.t. $0 < \psi < 1$, if $\sigma^1(0) > 0$, then there exists t^* such that $\sigma^1(t) > 1 - \psi$ for $t > t^*$.*

Proof. For k such that $m > k_1 > 0$, we have $\sigma^1(t+1) = \sum_{\{k \in \mathcal{A}^m\}} \Pr(k) f_1(k) > \sum_{\{k \in \mathcal{A}^m\}} \Pr(k) k_1/m = k_1/m = \sigma^1(t)$. So $\sigma^1(t)$ increases over time and, by continuity of 2.1, it converges to a steady state, which requires $k_1 = 0$ or $k_1 = m$. The former is ruled out by assumption. ■

Propositions 1 and 4 imply that evolution selects for preferences that are monotonic in the sense that they respect FOSD. Proposition 4 and Example 1 imply that evolution leads to a more refined order than FOSD, but Example 2 shows that this order does not respect SOSD. Thus our intuition that evolution selects for risk-taking in winner-take-all environments is not true in general. However Proposition 5 below shows that for large m the intuition is correct, and that the appropriate notion of risk taking is tail dominance. Combined with Proposition 2, Remark 2 and Example 3, we see that for large m evolution selects for preferences that are complete and transitive, and that respect FOSD and SOSD, provided the set of feasible lotteries is finite and each has finite support. However, if the set is infinite, the order need not be continuous, and if the set includes lotteries that do not have finite support, the order may not be complete.

PROPOSITION 5. *Consider a random-matching winner-take-all environment, where $F_1 \in \mathcal{F}$ tail dominates all other lotteries in \mathcal{F} , and the support*

of F_1 is an interval.⁴ Given ψ s.t. $0 < \psi < 1$, if $\sigma^1(0) > 0$, then there exists m^* and t^* such that for $m > m^*$ and $t > t^*$, $\sigma^1(t) > 1 - \psi$ for all $t > t^*$.

Proof. This follows from Propositions 4 and 3.

3. A PLAYING-THE-FIELD MODEL OF A WINNER-TAKE-ALL GAME

In this section we show that the preferences selected in winner-take-all games with random matching are different than those selected in winner-take-all games where each player is pitted against the entire population, which we call playing the field. Even though playing the field might correspond more immediately to the intuitive notion of a winner-take-all game, the results are (surprisingly) weaker than in random matching, and less supportive of the intuition that winner-take-all games should lead to risk taking.

In particular, we show that F can tail dominate G (as defined in the previous section) and still not be selected. However, there is a strengthening of the notion of tail dominance such that F will be selected, and it is satisfied if F is riskier than G in a particularly strong sense.

We now assume there are n players in the population who are encoded to choose between the lotteries F and G . The extension to more than two lotteries is straightforward. In each time period, each player has a random draw from his or her chosen distribution, F or G , and the player with the highest realization reproduces the next generation of n players. If there were no mutations, the dynamic process would end in the second period, since all players—and hence all future generations—would be of one type. We therefore assume that each offspring has a probability μ of mutating. (In the random matching game we did not have mutations. We argue below that the differences between playing-the-field and random-matching environments are due to whether competition is in small independent groups or in the whole group, and not due to mutations.) We will define a Markov process with three states: An “ F state” means that all n players are coded to play F , a “ G state” means that all players are coded to play G , and an “ M state” means that the population is mixed because there was at least one mutation.

There is a unique ergodic distribution of the Markov process denoted by $(\tilde{x}^F, \tilde{x}^G, \tilde{x}^M)$. (This distribution depends on n and on μ , but we suppress the notation for simplicity.) We present below an example which explains why

⁴ It is straightforward to adapt this proof to the case of finite support; we present the more difficult case.

tail-dominating lotteries, which were selected in the random-matching environment, need not be selected in the playing-the-field model. Thus, tail dominance is not a sufficiently strong notion of risk taking for selection in playing-the-field environments. However when the population n is large, and the mutation rate, μ , is small, if F “strongly” tail dominates G as defined below, then F will be selected (\tilde{x}^F is close to 1).

The following example shows that two of the main results from the random-matching model do not extend to the playing-the-field environment.

- In the playing-the-field environment, first-order stochastically dominating distributions need not be selected.
- In the playing-the-field environment, tail dominating distributions need not be selected.

Thus in the playing-the-field environment, preferences will not necessarily evolve to be monotonic or risk taking.

EXAMPLE 4. Suppose that F places probabilities $\frac{1}{2}, \frac{1}{2}$ on the outcomes 1, 2 respectively, while G places probabilities $\frac{1}{2}, \frac{1}{2}$ on 0, 2. We assume that if n players have the same outcome, each wins with probability $1/n$. Clearly F tail dominates G , so F is selected in the random-matching environment of the previous section. However, F will not be selected in the playing-the-field environment. This is because for large n the probability that an F population turns into a G population is almost the same as the probability that a G population turns into an F population, hence the two populations will alternate at approximately equal intervals. If n is large, then since each player has probability $\frac{1}{2}$ of obtaining the outcome 2, it follows that with high probability the winner in any generation (whether F or G) wins with outcome 2, and will win according to the tie-breaking rule. In order that an F population turns into a G population, there must be at least one mutation of an F player to G , and the G player must win, and similarly in reverse. Conditional on the winner having outcome 2, these are equally likely events. The probability that a G population turns into an F population is actually slightly higher than the reverse because the F 's have an advantage in those outcomes where no player has outcome 2.

In the notation of the following proof, “ c/d ” does not converge to zero in the example as n becomes large, and therefore the proportion of the time that the population is F does not converge to 1.

Example 4 shows the contrast between playing the field and random matching. If we pit F against G in random matches, the G players will eventually disappear. In every period there are many matches and in most matches the winner will win with outcome 2 using the tie-breaking rule.

The winners of tied matches will split evenly between F and G , with no aggregate effect on the population proportions. However in the other matches F will win more often than G . Therefore, in every period the proportion of F grows, albeit slowly if m is large. To highlight this, imagine that F and G both gave the outcome 2 with probability 1. Then in the random-matching environment, the population distribution between F and G would be constant: $\sigma(t) = \sigma(0)$ for all t . On the other hand, in the playing-the-field model, in every period the population would either be (almost) all F s or (almost) all G s.

The playing-the-field game differs from the random-matching game in three ways: It has a finite population, it has mutations, and it has competition within the whole population. We now argue that the difference in results is due to the latter. First, Example 4 can be modified to show that the difference in results is not due to population size. If there were a continuum of players, in every period after an F wins, the proportion of F types would be $1 - \mu$, and half of them would obtain the outcome 2; similarly a proportion μ would be G types, half of whom would obtain the outcome 2. The probability of shifting from an F state to a G state or conversely would then be $\mu/(1 - \mu)$ so that half the time would be spent in each state. In every period the winner would have outcome 2, so the difference between F and G would never be relevant.

Second, the difference in outcomes is apparently not due to the fact that mutations are permitted in playing-the-field, but not in random-matching. The intuition is that if independent mutations occur in random matches with equal probabilities for both F and G players, then if there are continuously many random matches, the mutations can have no aggregate effect.

It seems then that the difference between the outcomes of random matching and playing the field derives from the different forms of interaction, that is, competition within small random groups or within the whole group. Reasoning from the random-matching model, one might have guessed that in Example 4 above, the population should be mostly type F most of the time. This is wrong for the following reason. In almost all periods when the population switches from almost all F s to almost all G s, the switch has nothing to do with the outcomes other than 2, which are equally probable under the two distributions, whereas in the random-matching model there is movement in the population proportions precisely because of the outcomes other than 2.

The following definition strengthens tail dominance so as to exclude Example 3. The reason that *strong* tail dominance suffices is that if there are "many" players with the strongly tail-dominant lottery, some such player will have an outcome higher than the maximum in the support of dominated distributions, and will win.

DEFINITION 3. F strongly tail dominates G if there is an \bar{x} such that $F(\bar{x}) < 1$ and $G(\bar{x}) = 1$.

PROPOSITION 6. Suppose that F strongly tail dominates G . Let $\tilde{x}^F(n, \mu_n)$ be the stationary proportion of type- F players in a playing-the-field game of size n when the probability that each offspring mutates is μ_n . Then given $\varepsilon > 0$ there exist n^* and a sequence $\mu_n \rightarrow 0$ such that for $n > n^*$ we have $\tilde{x}^F(n, \mu_n) > 1 - \varepsilon$.

Proof. We will drop the arguments (n, μ_n) to $(\tilde{x}^F, \tilde{x}^M, \tilde{x}^G)$ for simplicity. We first show that we can choose (n, μ) such that $\tilde{x}^M < \varepsilon/2$. In the transition matrix below, let N (for “no mutation”) be either an F state or G state, and let M represent mixed generations in which at least one mutation has occurred.

$$\begin{array}{cc} & \begin{array}{c} N \\ M \end{array} \\ \begin{array}{c} N \\ M \end{array} & \begin{pmatrix} (1 - \mu_n)^n & 1 - (1 - \mu_n)^n \\ (1 - \mu_n)^n & 1 - (1 - \mu_n)^n \end{pmatrix} \end{array}$$

The probability of changing from N to M is the same whether the N generation was comprised of F players or G players. Similarly, the probability of changing from M to N does not depend on the mixture of G and F in the M generation, since the “winner” will produce n offspring, and each has the same probability of mutation irrespective of the parent’s type. The probability of no mutation is $(1 - \mu_n)^n$. Recall that $\tilde{x}^N = \tilde{x}^F + \tilde{x}^G$, and the stationary probabilities are $(\tilde{x}^N, \tilde{x}^M) = ((1 - \mu_n)^n, 1 - (1 - \mu_n)^n)$. Consider a sequence $\mu_n \rightarrow 0$ such that $n\mu_n \rightarrow \varepsilon/2$. Expanding $(1 - \mu_n)^n$ as a Taylor series around $\mu_n = 0$, $(1 - \mu_n)^n$ is equal to $1 - \mu_n n$ plus a positive series. Thus, since $(1 - \mu_n)^n = \tilde{x}^N \geq 1 - \mu_n n$ and $\mu_n n \rightarrow \varepsilon/2$, \tilde{x}^N is bounded below for large n by, say, $1 - \varepsilon$. That is, the population is in a mutant state at most a fraction ε of the time.

It is straightforward to adapt this proof to the case of finite support; we present the more difficult case.

We now restrict attention to the N states. Let \hat{x}^F and \hat{x}^G represent the stationary probabilities of being in the F state and G state respectively, conditional on being in one or the other: $\hat{x}^F \tilde{x}^N = \tilde{x}^F$ and $\hat{x}^G \tilde{x}^N = \tilde{x}^G$. The steady state probabilities for the Markov process described in the table below satisfy $\hat{x}^F / \hat{x}^G = d/c$. Since $\hat{x}^F + \hat{x}^G = 1$, this implies that $\hat{x}^F = d/(c + d)$, and $\tilde{x}^F = \tilde{x}^N d/(c + d)$. Thus to prove the theorem it is enough to show that as n grows large, and μ_n becomes small, d is bounded above zero and c converges to zero. We complete the proof by showing these two claims.

	F	G
F	$1 - c$	c
G	d	$1 - d$

We first show that c converges to zero as n becomes large and μ_n becomes small. To get from an F state to a G state or vice versa there must be at least one mutation, and a mutant must win. We will describe a mutation state with mixed F and G players by an integer k which represents the number of type- G players it contains. The number of type- F players is therefore $n - k$. A player wins if he has the highest order statistic among the random draws of all n players. The probability that a G player wins depends on the number of G players, k . Denoting this probability by $g(k)$, the probability c of changing from the F state to the G state is $\sum_{k \geq 1} \binom{n}{k} \mu_n^k (1 - \mu_n)^{n-k} g(k)$.

We can calculate an upper bound on $g(k)$ as follows. The event that a G player wins is the same event as that every F player loses. Thus we want an upper bound on the event that every F player loses. Letting $q \equiv 1 - F(\bar{x})$, such a bound is $(1 - q)^{n-k}$, which is the probability that no F player has a random draw in the upper tail of F where G puts no probability. The event that no F player has a random draw in the upper tail is necessary for all the F 's to lose (if any F had a draw in the upper tail it would win), but is not sufficient. Thus the probability that all the F players lose is smaller than $(1 - q)^{n-k}$. Thus, $g(k) < (1 - q)^{n-k}$ and $c < \sum_{k \geq 1} \binom{n}{k} \mu_n^k (1 - \mu_n)^{n-k} (1 - q)^{n-k} = [\mu_n + (1 - \mu_n)(1 - q)]^n - (1 - \mu_n)^n (1 - q)^n$. The equality follows because the sum would be a polynomial expansion if it included the $k = 0$ term. Thus c goes to zero as n becomes large.

We turn now to the parameter d , the probability that a G state becomes an F state. We will let $f(k)$ denote the probability that one of the F players wins when there are k G 's. Thus $d = \sum_{k \geq 1} \binom{n}{k} \mu_n^{n-k} (1 - \mu_n)^k f(k)$. We want a lower bound for d , the probability that one of the F players wins. F will surely win if one of them has a random draw in the upper tail where G has no weight. The probability this does not happen is $(1 - q)^{n-k}$, and therefore $1 - (1 - q)^{n-k}$ is a lower bound for the probability that one of the F players wins.

That is, $f(k) \geq 1 - (1 - q)^{n-k}$. Thus

$$\begin{aligned}
 d &\geq \sum_{k < n} \binom{n}{k} \mu_n^{n-k} (1 - \mu_n)^k [1 - (1 - q)^{n-k}] \\
 &= \sum_{k < n} \binom{n}{k} \mu_n^{n-k} (1 - \mu_n)^k - \sum_{k < n} \binom{n}{k} \mu_n^{n-k} (1 - \mu_n)^k (1 - q)^{n-k} \\
 &= 1 - (1 - \mu_n)^n - [(1 - \mu_n) + \mu_n(1 - q)]^n + [1 - \mu_n]^n \\
 &= 1 - [1 - \mu_n q]^n
 \end{aligned}$$

Since $\mu_n q = n\mu_n q/n$ and $n\mu_n \rightarrow \varepsilon/2$, the second term converges to $e^{-\varepsilon q/2} < 1$. Thus, d is bounded above zero. ■

If F is riskier than G in the sense that F is created by adding strictly positive noise at every point to G , then F strongly tail dominates G . In this weak sense, risk-taking preferences are selected in the playing-the-field environment. But it is a weaker sense than the selection in the random-matching environment.

4. BIOLOGICAL AND BEHAVIORAL EVIDENCE ABOUT RISK-TAKING

We now ask whether the ideas discussed above are relevant to the evolution of preferences in humans, and in particular, in males. Before addressing the three bullet items in the introduction, we note some of the evidence that male humans are more risk taking than females. Much of the evidence is experimental. Arch [1] summarizes it, and reports

Research clearly supports the existence of differences between females and males in the extent of their ... risk taking. (Ellis [11]). It appears that the differences in behavior do not occur simply because males perceive the physical world as less threatening ... but because they are actually more likely to seek out and enjoy risk-laden situations (Zuckerman [34]).

The same conclusion is corroborated by many other authors, e.g., Zinkhan and Karande [33] report a study in which “Men ... showed more risk-taking behavior than women;” Levin, Snyder and Capman [19] quote Hudgens and Fatkin [17], who concluded that “men are more inclined than women to take risks,” and also describe a study in which “males responded more favorably to the hypothetical gambling options than did females.” Avnery [2] comes to the same conclusion based on computer experiments with young children. Even if the evidence is confounded by environmental influences, it is nevertheless provocative.

Since our hypothesis is that risk taking among males is inherited rather than learned, we turn to the three questions identified in the introduction.

Can a meaningful concept of risk taking be distinguished from other behaviors in non-humans, so that risk taking in humans has a logical evolutionary antecedent?

Naturally it is hard to measure risk taking in nonhumans, but researchers have identified behaviors that seem observationally equivalent both to risk

taking and to sensation seeking. For example, Fairbanks [12] reports variation in the willingness of vervet monkeys to enter new environments, approach new food sources and approach strangers. Possingham, Houston and McNamara [21] report that “patterns of risk-sensitive foraging in bumble bees are now well documented,” where they define an animal to be risk sensitive if both the mean and the variance of the energy contributed by a reward influence their revealed preference. Battalio, Kagel and MacDonald [3] examine rats’ preferences over lotteries that differ by mean preserving spreads and find evidence of risk aversion.

Do non-human males play winner-take-all games?

The reproductive games played by males are complex. An exhaustive description can be found in [9], of which one section is devoted to mammals, and in particular to deer, elephant seals, lions, vervet monkeys and baboons. While none of the examples exhibits the extreme feature of Section 3, where a single male fathers the entire next generation, they all have dominance hierarchies, which might be interpreted as winner-take-all games in small groups, as in Section 2. The behavior of elephant seals is most clear-cut: Of the sample followed by the researchers, only 8.8% of the males bred during their lifetimes.

Is risk taking genetically coded? If so, there should be biological correlates with this behavior.

Roy, De Jong and Linniola [25] report in their abstract that “Pathological gamblers may have a disturbance of their central nervous system noradrenergic functioning,” and that their “results suggest that the disturbance ... may be partly reflected in their personality.” The survey in “Biology of Brain May Hold Key for Gamblers,” the *New York Times* C1, Oct. 3, 1989, summarizes this study as finding that “The psychological forces that propel so many chronic gamblers ... may spring from a biological need for risk and excitement The study showed that gamblers had lower levels than usual of the brain chemicals that regulate arousal, thrill and excitement.”

Even if risk taking is heritable, is there any connection between different species, in particular a connection between current species and their ancestors?

The evidence here is obviously circumstantial, but nevertheless plausible. Ellis [11] reports

Research over the past 25 years has established that androgens—and, to a lesser degree, other sex hormones—profoundly affect brain functioning, and thereby, behavior... . When one finds males on average behaving differently from females, therefore, one should suspect neuroandrogenic etiology, especially if the sex differences are evident in several species.

This report has demonstrated that average sex differences in several human behavior patterns have close parallels in other mammals. In addition, experiments with nonhuman mammals for many of these behavior patterns have shown that androgenic effects on brain functioning highly influence these behavior patterns. Together, these observations strongly imply that average sex differences in such behavior even among humans are at least partially the result of neuroandrogenic factors.

... at least three additional behavior patterns could be neuroandrogen influenced in light of persistent tendencies for males to display the behavior more than females, both in humans and in several other primate species. These were ... sensation seeking

This literature supports a biological basis for behaviors such as risk taking, and if such a basis is shared by mammals with common ancestors, then reproduction according to winner-take-all games among our ancestors, retained in other contemporary species, might explain a preference for risk taking among human males.

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