Asymmetric Stable Equilibria of Non-Neutral Plasmas

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A pure electron plasma, confined within an azimuthally symmetric boundary by a coaxial magnetic field, has an equilibrium shape which is cylindrical. We apply perturbations which break the azimuthal symmetry and deform the plasma into a noncircular shape that is stationary in the laboratory frame. These asymmetric equilibria form a broad new class of stable equilibria. A theoretical model correctly predicts the plasma shapes and explains their stability.

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Pure electron plasmas exhibit many unusual confinement properties, and have been extensively studied [1]. The typical plasma confinement geometry is azimuthally symmetric, and, consequently, the plasmas assume an azimuthally symmetric equilibrium shape. We find that when the wall boundary conditions are asymmetric, that is, when the boundary is not an equipotential surface, the plasma equilibrium shapes are themselves asymmetric. These new equilibria exhibit several remarkable features, and require a new confinement paradigm to explain their long confinement times.

Our plasma trap (Fig. 1) consists of a set of three conducting cylinders (radii of 1.9 cm). The plasma is confined within the center cylinder. The electrons are radially confined by a strong (1 kG) coaxial magnetic field, and axially confined in the potential well formed by negatively biasing the two end cylinders. The grounded center cylinder is equipped with three electrically isolated angular patches. Biasing these patches produces the asymmetric wall boundary conditions which result in the asymmetric equilibria [2]. The trap described above is similar to other pure electron plasma traps [3].

The plasma electron dynamics is characterized by several time scales. The cyclotron orbit time scale \( (r \sim 10^{-10} \text{ s}) \) is so fast that we can use the guiding center approximation. The axial bounce motion along the magnetic field is also fast \( (r \sim 10^{-7} \text{ s}) \); consequently, the electric field experienced by the particle can be approximated by the bounce-averaged electric field. Thus, the dynamics is primarily the two-dimensional, bounce-averaged \( \mathbf{E} \times \mathbf{B} \) drifts. For example, when the angular patches are grounded, the plasma's self-electric-field is entirely radial, so that the plasma rotates due to the azimuthal \( \mathbf{E} \times \mathbf{B} \) drifts.

The experimental procedure for obtaining noncircular equilibria is as follows. First, one of the negatively biased end cylinders is momentarily grounded, allowing a cylindrical plasma to flow from a thermionically emitting tungsten filament into the central confining cylinder. Next, the plasma is trapped by restoring the grounded end cylinder to its usual negative bias. The boundary is then perturbed by slowly biasing the desired patches. (Here "slow" means that the biases change by a small fraction during the plasma \( \mathbf{E} \times \mathbf{B} \) drift period.) The plasma responds to these biases by assuming an asymmetric shape. Finally, this shape is imaged by briefly grounding one of the negatively biased end cylinders. The unconfined electrons then rapidly spill out along the magnetic field lines, and are accelerated onto a phosphor screen. The resulting image is recorded with a charge-coupled-device camera. Some typical images are shown in Fig. 2. In these experiments the electron density is \( n \approx 3 \times 10^{17} \text{ cm}^{-3} \), the temperature is \( T \approx 3 \text{ eV} \), the plasma length is \( l \approx 3 \text{ cm} \), the initial plasma radius is \( r_0 = 1.2 \text{ cm} \), and a typical patch bias is 20 V.

The asymmetric plasma equilibria have many interesting properties. For example, although the plasma itself is stationary in the laboratory frame, individual electrons follow long and convoluted paths. Since these paths are determined by the \( \mathbf{E} \times \mathbf{B} \) drifts, the electrons move perpendicular to the electric field, i.e., along the equipotential contours. These contours result from the sum of the self-consistent plasma potentials and the externally applied boundary wall potentials, and may be very irregu-
tours become unclosed. As shown in Fig. 4, a separatrix divides the closed and unclosed equipotentials. Electrons which cross this separatrix are immediately swept to the wall and lost. This effect is seen in long time confinement studies of these asymmetric equilibria. For several seconds the plasma expands slowly (perhaps by scattering off of background gas molecules) without loss of total charge. After the plasma has expanded to the separatrix, further expansion leads to immediate charge loss. Alternatively, if a sufficiently large perturbing bias is slowly applied to an angular patch, the separatrix constrains to enclose the same area as the initial plasma. Any larger bias causes the separatrix to cross into the plasma, resulting in the rapid loss of those electrons outside of the separatrix.

When a bias is applied to the angular patch, sufficient charge density is necessary for plasma confinement. In the limit of infinitesimal plasma density, a small patch potential makes all equipotential contours open. To obtain closed equipotentials, the plasma must have sufficient charge density to produce a local minimum in the combined plasma and boundary potential. Note that the minimum charge density is a complicated function of both the perturbing voltage and the initial plasma radius. Asymmetric plasma equilibria can be used to fix the initial conditions for observing the subsequent evolution of an initially noncircular plasma. When the perturbing voltages on the angular patches are instantly reduced to zero, the plasma shape is no longer stationary; it proceeds to evolve within the symmetric confining voltages. Since the noncircular shapes can be decomposed into $I \geq 1$ diocotron [4,5] mode amplitudes, this technique can be used to observe the evolution of a diocotron mode of prescribed initial amplitude and phase.

The long confinement times of these asymmetric plasmas (typically 10 s, compared with 200 s for the unperturbed case) are not predicted by the usual non-neutral plasma confinement paradigm. Traditionally, the confinement of non-neutral plasmas is attributed to the azimuthal symmetry of the confinement geometry [6]. Such symmetry assures that the canonical angular momentum $P_\theta$ is conserved. In the limit of a strong magnetic field, $P_\theta = -eB/2\pi \sum_i \rho_i^2$, where the sum is over all plasma electrons. Thus, the constancy of angular momentum limits the radial expansion of the plasma. In the experiments described here, there is no such symmetry, and consequently angular momentum conservation cannot be used to explain the long confinement times. Future experiments will investigate the effect of asymmetries on the plasma confinement time.

The stability of these energy-conserving non-neutral plasmas has been investigated by O’Neil and Smith [7] using variational techniques. The plasma is locally stable to $E \times B$ dynamics if its energy is extremal when compared to the energy of all the nearby accessible states. Below, we derive an expression for the distortion of the unperturbed equilibrium as a function of the boundary.

FIG. 2. Three experimentally observed noncircular plasma equilibrium shapes.
perturbation [Eq. (4)]. We have also shown [8] that, in
the limit of small perturbations, the distortions predicted
by Eq. (4) yield energy maxima. That is, the plasma de-
forms to the shape that maximizes the energy consistent
with keeping its area invariant. Consequently, E×B
mechanisms are not responsible for the observed slow loss
of plasma. This maximal energy principle also explains
why the negatively charged plasma is attracted to nega-
tive imposed potentials and repelled from positive im-
posed potentials; such movements clearly maximize its
energy.

The model which predicts the shapes of these plasmas
is based on the fact that the guiding centers drift along
the equipotential contours. The stationary shape condi-
tion is satisfied if the plasma surface lies on an equipoten-
tial contour:

$$\phi(r_p(\theta), \theta) = \phi_p(r_p(\theta), \theta) + \phi_a(r_p(\theta), \theta) = \text{const},$$  \hspace{1cm} (1)

where \( r_p(\theta) \) specifies the shape of the uniform density
plasma, \( \phi_p \) is the potential resulting from the charge of
the plasma itself, and \( \phi_a \) is the potential due to the ap-
pplied patch perturbations. Although \( \phi_a \) can be exactly
expressed in integral form as a convolution of the density
and a Green function, it is, for our parameters, well ap-
proximated by the potential of a circular plasma with the
appropriate surface charge density, \( \sigma(\theta) = n \delta r(\theta) \).
Here \( \delta r(\theta) \) is the deviation of the asymmetric plasma from a
 Circular plasma of radius \( r_0 \), and may be Fourier decomposed

$$r_p(\theta) = r_0 + \delta r(\theta) = r_0 + \text{Re} \left[ \sum_{l=1}^{\infty} \delta r_l e^{i l \theta} \right].$$  \hspace{1cm} (2)

This approximation is valid when \( \delta r \ll r_0 \). The potential
field can then be easily found both inside and outside of
the cylindrical column by employing Gauss's law to
match the discontinuous electric field across the surface
charge.

The applied electrostatic potential \( \phi_a \) can also be ex-
pressed as a Fourier series,

$$\phi_a(r, \theta) = A_0 + \text{Re} \left[ \sum_{l=1}^{\infty} A_l \left( \frac{r}{R} \right)^l e^{il \theta} \right],$$  \hspace{1cm} (3)

where the complex coefficients \( A_l \) are determined from
the known patch biases. Thus the total field, \( \phi = \phi_p + \phi_a \),
can be expressed in terms of the applied perturbation,
\( \phi_a(R, \theta) \), and the shape of the plasma column, \( \delta r_i \). By
employing the equilibrium condition [Eq. (1)], and keep-
ing only the lowest-order terms in \( \delta r \), we find that the
shape of the asymmetric plasma column in terms of the
boundary perturbation is

$$\delta r_l = -\frac{(r_0/R)^l}{2mn r_0 [1 - (1 - (r_0/R)^2)^{2l}]} A_l.$$  \hspace{1cm} (4)

Figure 3 shows the plasma shape predicted by this ap-
proximate model. The model's success is illustrated by
the close agreement of this shape with the corresponding
shape of Fig. 2. The length of the angular patches is
slightly less than the diameter of the confining cylinder;
to compensate for this three-dimensional effect within the
two-dimensional theory, the values of the patch potential
used in the analytic calculations are reduced by a factor
of 0.6 from the experimental values. This numerical fac-
tor was obtained by comparing the effect of an infinitely
long sector to the bounce-averaged effect of a finite sec-
tor.

In addition to the simple analytic theory presented
here, a two-dimensional vortex-in-cell electrostatic code
has been used to simulate the experiments. An assembly
of 4096 line charges is used to model the plasma. The
simulation is time advanced by first weighting the plasma
to a uniform polar grid, then solving the Poisson equa-
tion, and finally, moving the line charges according to the
E×B dynamics. The noncircular shape is obtained by
slowly applying the external fields to the initially circular
plasma. A comparison of Figs. 2 and 4 shows the good
agreement between the experimental and simulated plas-
ma shapes.

In conclusion, we find that stable asymmetric plasma
equilibria are easily observed experimentally. The shapes
agree with both theoretical models and numerical simula-
tions. The long lifetimes of these asymmetric equilibria

\[\text{FIG. 3. Plasma edge as determined by the theoretical model.}\]

\[\text{FIG. 4. Equipotential lines and particle positions obtained from the simulation. The separatrix lies between the closed and unclosed equipotential lines.}\]
challenge the basic confinement paradigm of non-neutral plasmas, but may partially be explained by their stability within the $E \times B$ model. Since two-dimensional $E \times B$ plasma dynamics are identical to two-dimensional, inviscid, incompressible fluid dynamics, many of our results can be generalized to vortex dynamics. Our results may also explain the long confinement times observed in the square traps used in Fourier-transform mass spectrometers [9,10].

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[2] Note that we define equilibrium to imply local thermal equilibrium, i.e., long-lived equilibrium along field lines involving no cross field transport.


